LRFD Design Example #1:

Prestressed Precast Concrete Beam Bridge Design

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About this Design Example

Description

This document provides guidance for the design of a precast, prestressed beam bridge utilizing the AASHTO LRFD Bridge Design Specifications.

The example includes the following component designs:

- Traditional deck design
- Prestressed beam design
- Composite Neoprene bearing pad design
- Multi-column pier design
- End bent design

The following assumptions have been incorporated in the example:

- Two simple spans @ 90'-0" each, 20 degree skew.
- Minor horizontal curvature
- Multi-column pier on prestressed concrete piles.
- No phased construction.
- Two traffic railing barriers and one median barrier.
- No sidewalks.
- Permit vehicles are not considered.
- Design for jacking is not considered.
- Load rating is not addressed.
- No utilities on the bridge.
- For purposes of wind load calculation, the bridge is located in an area with a basic wind speed of 150 mph.

Since this example is presented in a **Mathcad** document, a user can alter assumptions, constants, or equations to create a customized application.

Standards

The example utilizes the following design standards:

- Florida Department of Transportation Standard Specifications for Road and Bridge Construction (2010 edition) and applicable modifications.
- AASHTO LRFD Bridge Design Specifications, 5th Edition, 2010.
- Florida Department of Transportation Structures Design Guidelines, January 2011 Edition.
- Florida Department of Transportation Structures Detailing Manual, January 2011 Edition.
- Florida Department of Transportation Design Standards, 2010 Interim Edition.

Acknowlegements

The Tampa office of HDR Engineering, Inc. prepared this document for the Florida Department of Transportation. The Structures Design Office of the Florida Department of Transportation updated the example in 2011.

Notice

The materials in this document are only for general information purposes. This document is not a substitute for competent professional assistance. Anyone using this material does so at his or her own risk and assumes any resulting liability.

General Notes

Design Method................. Load and Resistance Factor Design (LRFD) except that the Prestressed Beams and Prestressed Piles have been designed for Service Load.

Earthquake........................ No seismic analysis required (SDG 2.3.1). Must meet minimum support length requirement (LRFD 4.7.4.4).

Concrete............................

Class	Minimum 28-day Compressive Strength (psi)	Location
II	f`c = 3400	Traffic Barriers
II (Bridge Deck)	f`c = 4500	CIP Bridge Deck
IV	f`c = 5500	CIP Substructure
V (Special)	f`c = 6000	Concrete Piling
VI	f`c = 8500	Prestressed Beams

Environment...................... The superstructure is classified as slightly aggressive. The substructure is classfied as moderately aggressive.

Reinforcing Steel............. ASTM A615, Grade 60

Concrete Cover.................

Superstructure	
Top deck surfaces	2.5" (Long bridge)
All other surfaces	2"
Substructure	
External surfaces not in contact with water	3"
External surfaces cast against earth	4"
Prestressed Piling	3"
Top of Girder Pedestals	2"

Concrete cover does not include reinforcement placement or fabrication tolerances, unless shown as "minimum cover". See FDOT Standard Specifications for allowable reinforcement placement tolerances.

Assumed Loads................

Item	Load
Live Load	HL-93
Traffic Railing (plf)	420
Wearing Surface (psf)	0
Utilities (plf)	0
Stay-In-Place Metal Forms (psf)	20
Median Traffic Railing (plf)	485
Bridge Deck Sacrificial Thickness (in)	0.5

Dimensions........................ All dimensions are in feet or inches, except as noted.

PROJECT INFORMATION

Design Parameters

Description

This section provides the design input parameters necessary for the superstructure and substructure design.

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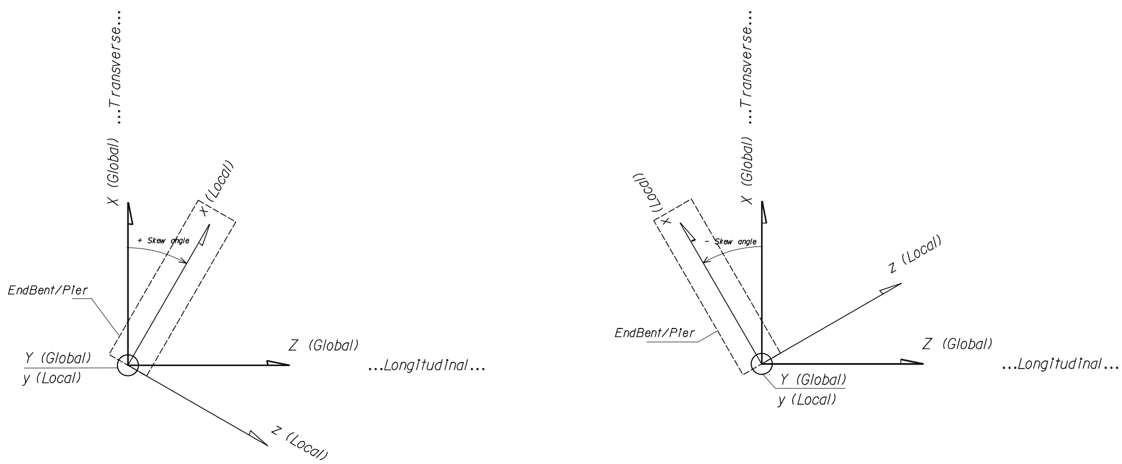
A. General Criteria

This section provides the general layout and input parameters for the bridge example.

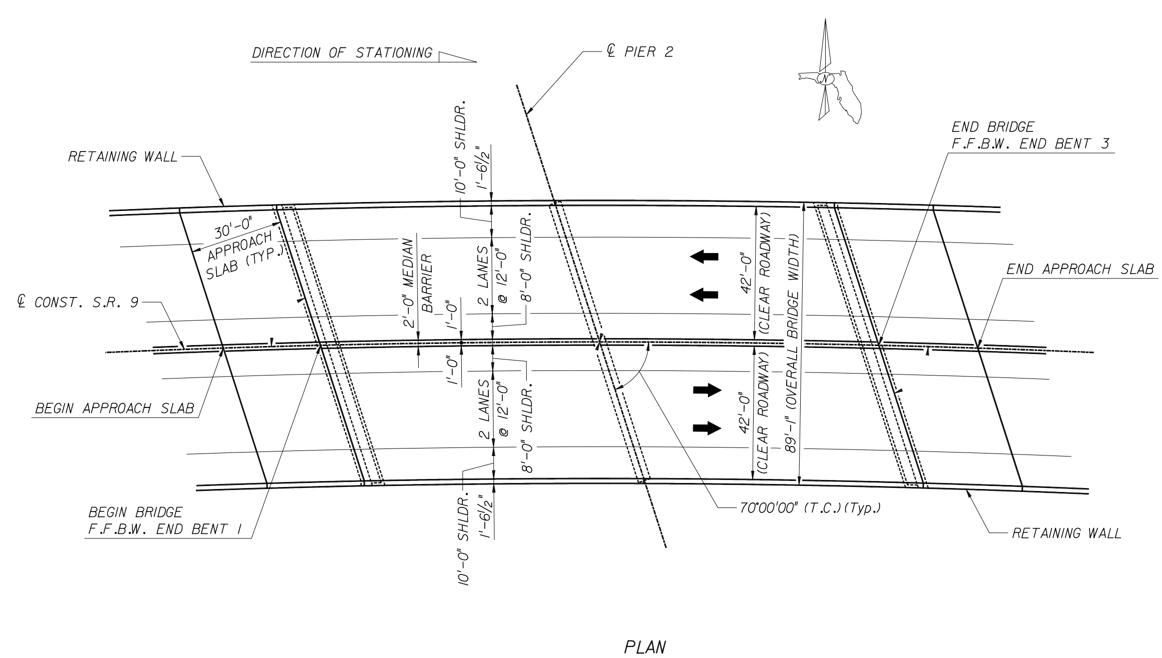


Coordinate System (except footings)

A1. Bridge Geometry

Horizontal Profile

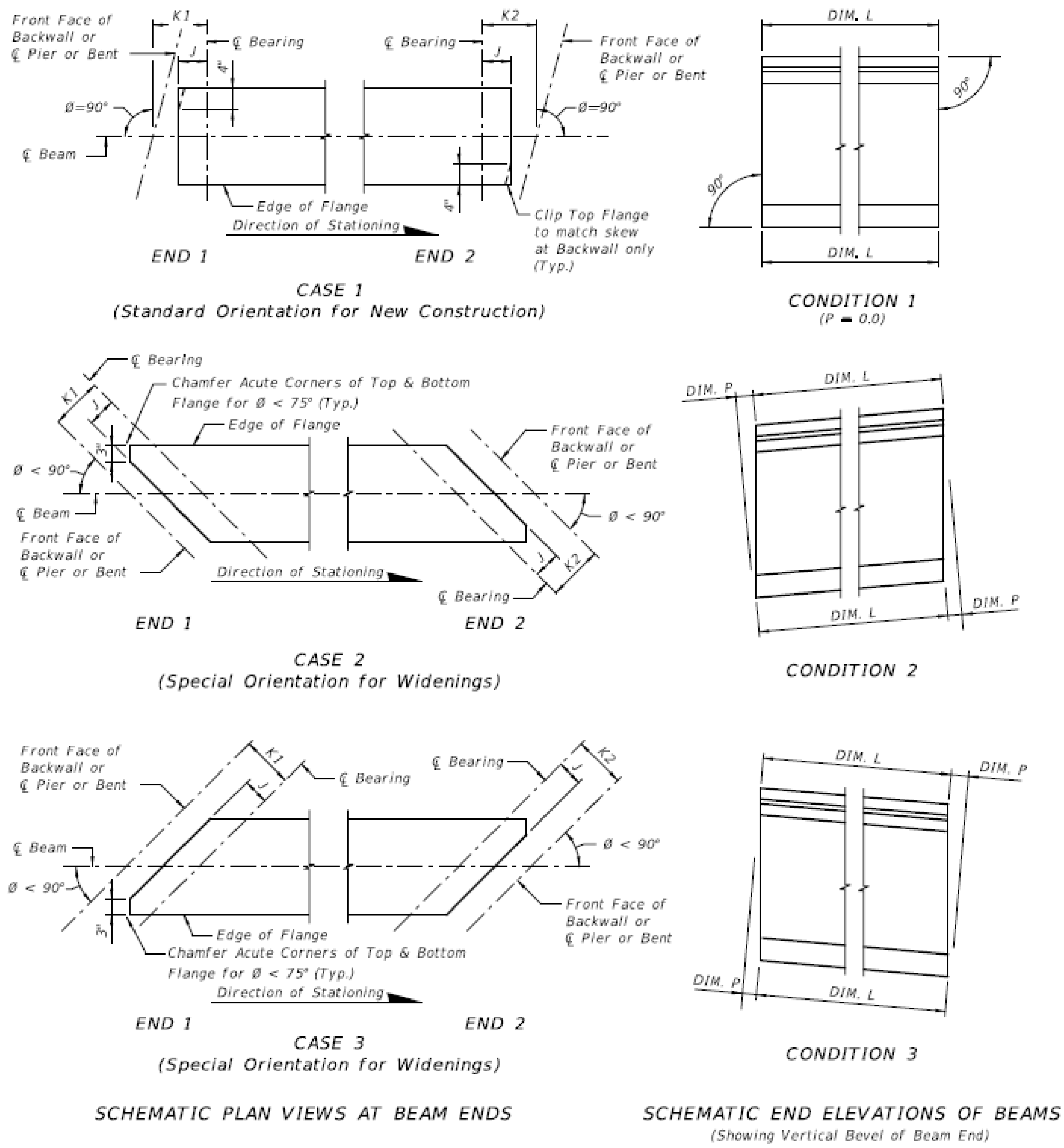
A slight horizontal curvature is shown in the plan view. This curvature is used to illustrate centrifugal forces in the substructure design. For all other component designs, the horizontal curvature will be taken as zero.



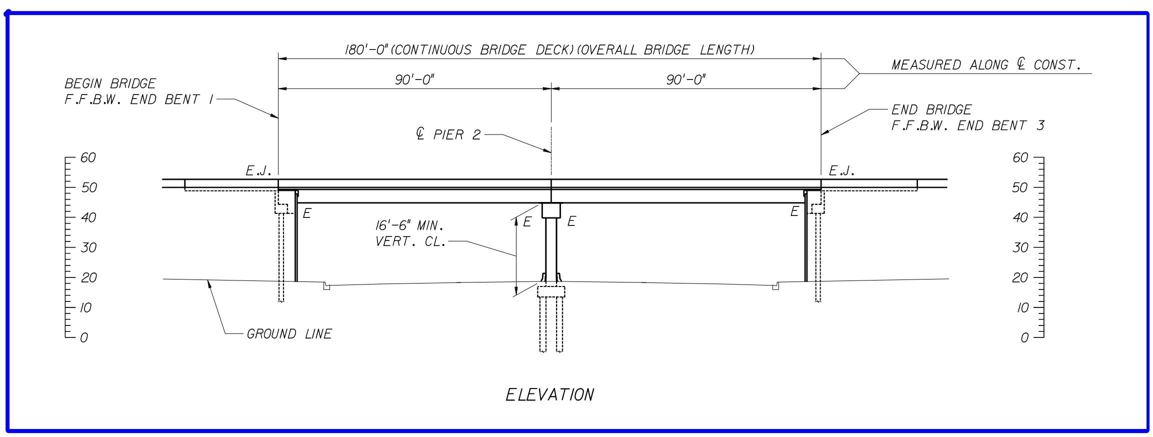
HORIZONTAL CURVE DATA

R = 3,800'

In addition, the bridge is also on a skew which is defined as $Skew := -20deg$



Vertical Profile



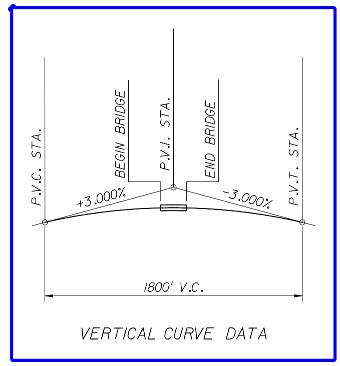
Overall bridge length............ $L_{bridge} \equiv 180 \cdot ft$

Bridge design span length...... $L_{span} := 90 \cdot ft$

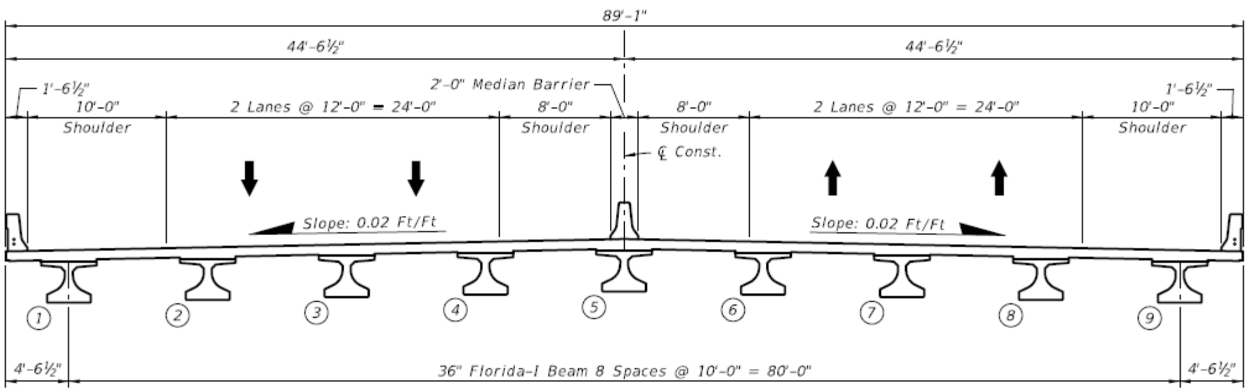
Beam grade.......................... $Beam_{Grade} := .15\%$

Height of superstructure........ $z_{sup} := 20.5ft$

Height of substructure.......... $z_{sub} := 8.25ft$



Typical Cross-secton



TYPICAL SECTION

Superstructure Beam Type..... $BeamType := \text{"FIB-36"}$

Number of beams................ $N_{beams} := 9$

Beam Spacing..................... $BeamSpacing := 10 \cdot ft$

Deck overhang at End Bent
and Pier.............................. $Overhang := 4ft + 6.5in = 4.542\,ft$

Average buildup.................. $h_{buildup} := 1in$

A2. Number of Lanes

Design Lanes [LRFD 3.6.1.1.1]

Current lane configurations show two striped lanes per roadway with a traffic median barrier separating the roadways. Using the roadway clear width between barriers, $Rdwy_{width}$, the number of design traffic lanes per roadway, N_{lanes} , can be calculated as:

Roadway clear width........... $Rdwy_{width} := 42 \cdot ft$

Number of design traffic lanes per roadway........................ $N_{lanes} := floor\left(\frac{Rdwy_{width}}{12 \cdot ft}\right) = 3$

Substructure Design

In order to maximize the design loads of the substructure components, e.g. pier cap negative moment, pier columns, footing loads, etc., HL-93 vehicle loads were placed on the deck. In some cases, the placement of the loads ignored the location of the median traffic barrier. This assumption is considered to be conservative.

Braking forces

The bridge is NOT expected to become one-directional in the future. Future widening is expected to occur to the outside if additional capacity is needed. Therefore, for braking force calculations, $N_{lanes} = 3$.

The designer utilized engineering judgement to ignore the location of the median barrier for live load placement for the substructure design and NOT ignore the median barrier for braking forces. The designer feels that the probability exists that the combination of lanes loaded on either side of the median barrier exists. However, this same approach was not used for the braking forces since these loaded lanes at either side of the median traffic barrier will NOT be braking in the same direction.

A3. Concrete, Reinforcing and Prestressing Steel Properties

Unit weight of concrete......... $\gamma_{conc} := 150 \cdot pcf$

Modulus of elasticity for reinforcing steel.................. $E_s := 29000 \cdot ksi$

Ultimate tensile strength for prestressing tendon.............. $f_{pu} := 270 \cdot ksi$

Modulus of elasticity for prestressing tendon.............. $E_p := 28500 \cdot ksi$

B. LRFD Criteria

The bridge components are designed in accordance with the following LRFD design criteria:

B1. Dynamic Load Allowance [LRFD 3.6.2]

An impact factor will be applied to the static load of the design truck or tandem, except for centrifugal and braking forces.

Impact factor for fatigue and fracture limit states.............. $IM_{fatigue} := 1 + \frac{15}{100} = 1.15$

Impact factor for all other limit states................................. $IM := 1 + \frac{33}{100} = 1.33$

B2. Resistance Factors [LRFD 5.5.4.2]

Flexure and tension of reinforced concrete............. $\phi := 0.9$

Flexure and tension of prestressed concrete............ $\phi' := 1.00$

Shear and torsion of normal weight concrete.................. $\phi_v := 0.90$

B3. Limit States [LRFD 1.3.2]

The LRFD defines a limit state as a condition beyond which the bridge or component ceases to satisfy the provisions for which it was designed. There are four limit states prescribed by LRFD. These are as follows:

STRENGTH LIMIT STATES

Load combinations which ensure that strength and stability, both local and global, are provided to resist the specified statistically significant load combinations that a bridge is expected to experience in its design life. Extensive distress and structural damage may occur under strength limit state, but overall structural integrity is expected to be maintained.

EXTREME EVENT LIMIT STATES

Load combinations which ensure the structural survival of a bridge during a major earthquake or flood, or when collided by a vessel, vehicle, or ice flow, possibly under scoured conditions. Extreme event limit states are considered to be unique occurrences whose return period may be significantly greater than the design life of the bridge.

SERVICE LIMIT STATES

Load combinations which place restrictions on stress, deformation, and crack width under regular service conditions.

FATIGUE LIMIT STATES

Load combinations which place restrictions on stress range as a result of a single design truck occurring at the number of expected stress range cycles. It is intended to limit crack growth under repetitive loads to prevent fracture during the design life of the bridge.

Table 3.4.1-1 - Load Combinations and Load Factors

Load Combination Limit State	*DC DD DW EH EV ES EL PS CR SH*	*LL IM CE BR PL LS*	*WA*	*WS*	*WL*	*FR*	*TU*	*TG*	*SE*	Use One of These at a Time			
										EQ	*IC*	*CT*	*CV*
Strength I (unless noted)	γ_p	1.75	1.00	—	—	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—
Strength II	γ_p	1.35	1.00	—	—	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—
Strength III	γ_p	—	1.00	1.40	—	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—
Strength IV	γ_p	—	1.00	—	—	1.00	0.50/1.20	—	—	—	—	—	—
Strength V	γ_p	1.35	1.00	0.40	1.0	1.00	0.50/1.20	γ_{TG}	γ_{SE}	—	—	—	—
Extreme Event I	γ_p	γEQ	1.00	—	—	1.00	—	—	—	1.00	—	—	—
Extreme Event II	γ_p	0.50	1.00	—	—	1.00	—	—	—	—	1.00	1.00	1.00
Service I	1.00	1.00	1.00	0.30	1.0	1.00	1.00/1.20	γ_{TG}	γ_{SE}	—	—	—	—
Service II	1.00	1.30	1.00	—	—	1.00	1.00/1.20	—	—	—	—	—	—
Service III	1.00	0.80	1.00	—	—	1.00	1.00/1.20	γ_{TG}	γ_{SE}	—	—	—	—
Service IV	1.00	—	1.00	0.70	—	1.00	1.00/1.20	—	1.0	—	—	—	—
Fatigue I—*LL, IM & CE* only	—	1.50	—	—	—	—	—	—	—	—	—	—	—
Fatigue I II—*LL, IM & CE* only	—	0.75	—	—	—	—	—	—	—	—	—	—	—

Revisions to **LRFD Table 3.4.1-1** above per **SDG**:

1. **SDG 2.1.1** states: In **LRFD Table 3.4.1-1**, under Load Combination: LL, IM, etc., Limit State: Extreme Event I, use $\gamma_{eq} := 0.0$.

2. Per **SDG 2.4.1B**:

Table 2.4.1-1Load Factors

LOAD COMBINATION LIMIT STATE	γ_{WS}	BASIC WIND SPEED, V (MPH)
STRENGTH III	1.40	Per Table 2.4.1-2
STRENGTH V	1.30	70
SERVICE I	1.00	70
SERVICE IV	0.60	Per Table 2.4.1-2

Table 3.4.1-2 - Load factors for permanent loads, γ_p

Type of Load, Foundation Type, and Method Used to Calculate Downdrag		Load Factor	
		Maximum	Minimum
DC: Component and Attachments		1.25	0.90
DC: Strength IV only		1.50	0.90
DD: Downdrag	Piles, α Tomlinson Method	1.4	0.25
	Piles, λ Method	1.05	0.30
	Drilled shafts, O'Neill and Reese (*1999*) Method	1.25	0.35
DW: Wearing Surfaces and Utilities		1.50	0.65
EH: Horizontal Earth Pressure			
• Active		1.50	0.90
• At-Rest		1.35	0.90
• *AEP* for anchored walls		1.35	N/A
EL: Locked-in Construction Stresses		1.00	1.00
EV: Vertical Earth Pressure			
• Overall Stability		1.00	N/A
• Retaining Walls and Abutments		1.35	1.00
• Rigid Buried Structure		1.30	0.90
• Rigid Frames		1.35	0.90
• Flexible Buried Structures other than Metal Box Culverts		1.95	0.90
• Flexible Metal Box Culverts **and Structural Plate Culverts with Deep Corrugations**		1.50	0.90
ES: Earth Surcharge		1.50	0.75

The load factor for wind in Strength Load Combination III in construction is 1.25 **[LRFD 3.4.2.1]**.

C. FDOT Criteria

C1. Chapter 1 - General Requirements

General [SDG 1.1]

The design life for bridge structures is 75 years.

Criteria for Deflection and Span-to-Depth Ratios [SDG 1.2]

Per SDG 1.2, either LRFD 2.5.2.6.3 or 2.5.2.6.2 should be met. Based on the superstructure depth; 2.5.2.6.3 is not met, so 2.5.2.6.2 should be met. The deflection limit is span/800 for vehicular load and span/300 on cantilever arms.

Environmental Classifications [SDG 1.3]

The environment can be classified as either "Slightly", "Moderately" or "Extremely" aggressive. Per 1.02 General Notes:

Environmental classification for superstructure................ $Environment_{super} \equiv \text{"Slightly"}$

Environmental classification for substructure.................. $Environment_{sub} \equiv \text{"Moderately"}$

Concrete and Environment [SDG 1.4]

The concrete cover for each bridge component is based on either the environmental classification or the length of bridge **[SDG 1.4]**.

Concrete cover for the deck.. $cover_{deck} := \begin{cases} 2\cdot in \text{ if } L_{bridge} < 100ft \\ 2.5\cdot in \text{ otherwise} \end{cases} = 2.5\cdot in$ [SDG 4.2.1]

Concrete cover for substructure not in contact with water.......................... $cover_{sub} := \begin{cases} 4\cdot in \text{ if } Environment_{sub} = \text{"Extremely"} \\ 3\cdot in \text{ otherwise} \end{cases} = 3\cdot in$

Concrete cover for substructure cast against earth or in contact with water......... $cover_{sub.earth} := \begin{cases} 4.5\cdot in \text{ if } Environment_{sub} = \text{"Extremely"} \\ 4\cdot in \text{ otherwise} \end{cases} = 4\cdot in$

Minimum 28-day compressive strength of concrete components........................

Class		Location
II (Bridge Deck)	CIP Bridge Deck Approach Slabs	$f_{c.slab} := 4.5 \cdot ksi$
IV	CIP Substructure	$f_{c.sub} := 5.5 \cdot ksi$
V (Special)	Concrete Piling	$f_{c.pile} := 6.0 \cdot ksi$
VI	Prestressed Beams	$f_{c.beam} := 8.5 \cdot ksi$

Correction factor for Florida lime rock coarse aggregate.....

$K_1 := 0.9$

Unit Weight of Florida lime rock concrete (kcf)..............

$w_{c.limerock} := .145 \frac{kip}{ft^3}$

Modulus of elasticity for slab...................................

$$E_{c.slab} := 33000 K_1 \cdot \left(\frac{w_{c.limerock}}{\frac{kip}{ft^3}} \right)^{1.5} \cdot \sqrt{f_{c.slab} \cdot ksi} = 3479 \cdot ksi$$

Modulus of elasticity for beam................................

$$E_{c.beam} := 33000 \cdot K_1 \cdot \left(\frac{w_{c.limerock}}{\frac{kip}{ft^3}} \right)^{1.5} \cdot \sqrt{f_{c.beam} \cdot ksi} = 4781 \cdot ksi$$

Modulus of elasticity for substructure.......................

$$E_{c.sub} := 33000 \cdot K_1 \cdot \left(\frac{w_{c.limerock}}{\frac{kip}{ft^3}} \right)^{1.5} \sqrt{f_{c.sub} \cdot ksi} = 3846 \cdot ksi$$

Modulus of elasticity for piles..................................

$$E_{c.pile} := 33000 \cdot K_1 \cdot \left(\frac{w_{c.limerock}}{\frac{kip}{ft^3}} \right)^{1.5} \cdot \sqrt{f_{c.pile} \cdot ksi} = 4017 \cdot ksi$$

C2. Chapter 2 - Loads and Load Factors

Dead loads [SDG 2.2]

Weight of future wearing surface..............................

$$\rho_{fws} := \begin{vmatrix} 15 \cdot psf \ \ \text{if } L_{bridge} < 100ft \\ 0 \cdot psf \ \ \text{otherwise} \end{vmatrix} = 0 \cdot psf$$

[SDG 4.2.1]

Weight of sacrificial milling surface, using $t_{mill} = 0.5 \cdot in$......

$$\rho_{mill} := t_{mill} \cdot \gamma_{conc} = 6.25 \cdot psf$$

[SDG 4.2.2.A]

Table 2.2-1 Miscellaneous Dead Loads

ITEM	UNIT	LOAD
General		
Concrete, Counterweight (Plain)	Lb/cf	145
Concrete, Structural	Lb/cf	150
Future Wearing Surface	Lb/sf	15[1]
Soil; Compacted	Lb/cf	115
Stay-in-Place Metal Forms	Lb/sf	20[2]
Traffic Railings		
32" F-Shape (Index 420)	Lb/ft	420
Median, 32" F-Shape (Index 421)	Lb/ft	485
42" Vertical Shape (Index 422)	Lb/ft	590
32" Vertical Shape (Index 423)	Lb/ft	385
42" F-Shape (Index 425)	Lb/ft	625
Corral Shape (Index 424)	Lb/ft	460
Thrie-Beam Retrofit (Index 471, 475 & 476)	Lb/ft	40
Thrie-Beam Retrofit (Index 472, 473 & 474)	Lb/ft	30
Vertical Face Retrofit with 8" curb height (Index 480-483)	Lb/ft	270
Traffic Railing/Sound Barrier (8'-0") (Index 5210)	Lb/ft	1010
Prestressed Beams[3]		
Florida-I 36 Beam (Index 20036)	Lb/ft	840

1. Future Wearing Surface allowance applies only to minor widenings of existing bridges originally designed for a Future Wearing Surface, regardless of length (see SDG 7.2 Widening Classifications and Definitions) or new short bridges (see SDG 4.2 Bridge Length Definitions).

2. Unit load of metal forms and concrete required to fill the form flutes. Apply load over the projected plan area of the metal forms.

3. Weight of buildup concrete for camber and cross slope not included.

Weight of traffic railing barrier................................ $w_{barrier.ea} := 420 \cdot plf$

Weight of traffic railing median barrier................................ $w_{median.bar} := 485 \cdot plf$

Weight of compacted soil...... $\gamma_{soil} := 115 \cdot pcf$

Weight of stay-in-place metal forms................................ $\rho_{forms} := 20 \cdot psf$

Barrier / Railing Distribution for Beam-Slab Bridges [SDG 2.8 & LRFD 4.6.2.2.1]

Dead load of barriers applied to the exterior and interior beams............ $w_{barrier} := \frac{w_{barrier.ea}}{N_{beams}} \cdot 2 = 0.093 \cdot klf$

For purposes of this design example, all barriers will be equally distributed amongst all the beams comprising the superstructure.

Include the dead load of the traffic barriers on the design load of the exterior beams...... $w_{barrier.exterior} := w_{barrier} + \frac{w_{median.bar}}{N_{beams}} = 0.147 \cdot klf$

Include the dead load of the traffic barriers on the design load of the interior beams....... $w_{barrier.interior} := w_{barrier} + \frac{w_{median.bar}}{N_{beams}} = 0.147 \cdot klf$

Seismic Provisions [SDG 2.3 & LRFD 4.7.4.3 & 4.7.4.4]

Seismic provisions for minimum bridge support length only.

Wind Loads [SDG 2.4]

Basic wind speed (mph)........ $V := 150$

Height, superstructure.......... $z_{sup} = 20.5\,ft$

Height, substructure............. $z_{sub} = 8.25\,ft$

Gust effect factor................ $G := 0.85$

Pressure coefficient, superstructure............................ $C_{p.sup} := 1.1$

Pressure coefficient, substructure............................ $C_{p.sub} := 1.6$

Velocity pressure exposure coefficient, superstructure.... $K_{z.sup} := \max\left[0.85, 2.01 \cdot \left(\frac{z_{sup}}{900ft}\right)^{.2105}\right] = 0.907$

Velocity pressure exposure coefficient, substructure......

$$K_{z.sub} := \max\left[0.85, 2.01\cdot\left(\frac{z_{sub}}{900ft}\right)^{.2105}\right] = 0.85$$

Wind pressure, super-structure, Strength III, Service IV.........................

$$P_{z.sup.StrIII.ServIV} := 2.56\cdot10^{-6}\cdot K_{z.sup}\cdot V^2\cdot G\cdot C_{p.sup} = 0.049$$

Wind pressure, super-structure, Strength V, Service I...........................

$$P_{z.sup.StrV.ServI} := 2.56\cdot10^{-6}\cdot K_{z.sup}\cdot 70^2\cdot G\cdot C_{p.sup} = 0.011$$

Wind pressure, sub-structure, Strength III, Service IV.........................

$$P_{z.sub.StrIII.ServIV} := 2.56\cdot10^{-6}\cdot K_{z.sub}\cdot V^2\cdot G\cdot C_{p.sub} = 0.067$$

Wind pressure, sub-structure, Strength V, Service I...........................

$$P_{z.sub.StrV.ServI} := 2.56\cdot10^{-6}\cdot K_{z.sub}\cdot 70^2\cdot G\cdot C_{p.sub} = 0.015$$

C3. Chapter 4 - Superstructure Concrete

General [SDG 4.1]

Yield strength of reinforcing steel...................................

$$f_y := 60\cdot ksi$$

Note: Epoxy coated reinforcing not allowed on FDOT projects.

Deck Slabs [SDG 4.2]

Bridge length definition.........

$$BridgeType := \begin{cases} \text{"Short"} & \text{if } L_{bridge} < 100ft \\ \text{"Long"} & \text{otherwise} \end{cases} = \text{"Long"}$$

Thickness of sacrificial milling surface...............................

$$t_{mill} \equiv \left(\begin{cases} 0\cdot in & \text{if } L_{bridge} < 100ft \\ 0.5\cdot in & \text{otherwise} \end{cases}\right) = 0.5\cdot in$$

Deck thickness...................

$$t_{slab} \equiv 8.0\cdot in$$

Deck Slab Design [SDG 4.2.4]

The empirical design method is not permitted per SDG 4.2.4.A. Therefore, the traditional design method will be used.

The minimum transverse top slab reinforcing at the median barrier and overhang may be determined using the table in SDG 4.2.4.B because a minimum 8" slab depth and less than 6' overhang is provided. A minimum area of steel of $0.40 \cdot in^2$ per foot should be provided in the top of the deck slab at the median barrier, and $0.80 \cdot in^2$ per foot should be provided at the F-shape barrier.

Pretensioned Beams [SDG 4.3] *(**Note**: Compression = +, Tension = -)*

Minimum compressive concrete strength at release is the greater of 4.0 ksi or 0.6 f'c..............................

$$f_{ci.beam.min} := \max\left(4ksi, 0.6 \cdot f_{c.beam}\right) = 5.1 \cdot ksi$$

Maximum compressive concrete strength at release is the lesser of 6.0 ksi or 0.8 f'c..........

$$f_{ci.beam.max} := \min\left(0.8 \cdot f_{c.beam}, 6.0ksi\right) = 6 \cdot ksi$$

Any value between the minimum and maximum may be selected for the design.

Compressive concrete strength at release....

$$f_{ci.beam} := 6 \cdot ksi$$

Corresponding modulus of elasticity.............................

$$E_{ci.beam} := 33000 \cdot K_1 \cdot \left(\frac{w_{c.limerock}}{\frac{kip}{ft^3}}\right)^{1.5} \cdot \sqrt{f_{ci.beam} \cdot ksi} = 4017 \cdot ksi$$

Limits for tension in top of beam at release (straight strand only) [SDG 4.3.1.C]

Outer 15 percent of design beam............................

$$f_{top.outer15} := -12 \cdot \sqrt{f_{ci.beam} \cdot psi} = -930 \cdot psi$$

Center 70 percent of design beam [LRFD 5.9.4.1.2-1]...................

$$f_{top.center70} := \min\left(-0.2ksi, 0.0948 \cdot \sqrt{f_{ci.beam} \cdot ksi}\right) = -200 \cdot psi$$

Time-dependent variables for creep and shrinkage calculations

Relative humidity............ $H := 75$

Age (days) of concrete when load is applied........ $T_0 := 1$

Age (days) of concrete when section becomes composite..................... $T_1 := 120$

Age (days) of concrete used to determine long term losses.................... $T_2 := 10000$

C4. Chapter 6 - Superstructure Components

Temperature Movement [SDG 6.3]

Table 2.7.1-1 Temperature Range by Superstructure Material

Superstructure Material	Temperature Range (Degrees Fahrenheit)			
	Mean	High	Low	Range
Concrete Only	70	105	35	70
Concrete Deck on Steel Girder	70	110	30	80
Steel Only	70	120	30	90

The temperature values for "Concrete Only" in the preceding table apply to this example.

Temperature mean......... $t_{mean} := 70 \cdot °F$

Temperature high........... $t_{high} := 105 \cdot °F$

Temperature low............ $t_{low} := 35 \cdot °F$

Temperature rise............ $\Delta t_{rise} := t_{high} - t_{mean} = 35 \cdot °F$

Temperature fall............. $\Delta t_{fall} := t_{mean} - t_{low} = 35 \cdot °F$

Coefficient of thermal expansion **[LRFD 5.4.2.2]** for normal weight concrete......... $\alpha_t := \dfrac{6 \cdot 10^{-6}}{°F}$

Expansion Joints [SDG 6.4]

Expansion Joint Type	Maximum Open Width "W" (measured in the direction of travel at deck surface)
Hot Poured or Poured Joint without Backer Rod	3/4-inch
Poured Joint with Backer Rod	3-inches
Armored Elastomeric Strip Seal (Single gap)	Per LRFD [14.5.3.2]
Modular Joint (Multiple modular gaps)	Per LRFD [14.5.3.2]
Finger Joint	Per LRFD [14.5.3.2]

For new construction, use only the joint types listed in the preceding table. A typical joint for most prestressed beam bridges is the poured joint with backer rod [DS Index 21110].

Maximum joint width...... $W_{max} := 3 \cdot in$

Proposed joint width at 70° F............................ $W := 1 \cdot in$

Minimum joint width....... $W_{min} := 0.5 \cdot W$

Movement [6.4.2]

For prestressed concrete structures, the movement is based on the greater of the following combinations:

Movement from the combination of temperature fall, creep, and shrinkage................. $\Delta x_{fall} = \Delta x_{temperature.fall} \cdots + \Delta x_{creep.shrinkage}$

*(**Note**: A temperature rise with creep and shrinkage is not investigated since they have opposite effects).*

Temperature Load Factor $\gamma_{TU} := 1.2$

Movement from factored effects of temperature..... $\Delta x_{rise} = \gamma_{TU} \cdot \Delta x_{temperature.rise}$

$\Delta x_{fall} = \gamma_{TU} \cdot \Delta x_{temperature.fall}$

*(**Note**: For concrete structures, the temperature rise and fall ranges are the same.*

C5. Miscellaneous

Beam Parameters

Distance from centerline pier to centerline bearing................. $K1 := 11 \cdot in$

*(**Note**: Sometimes the K value at the end bent and pier may differ.*

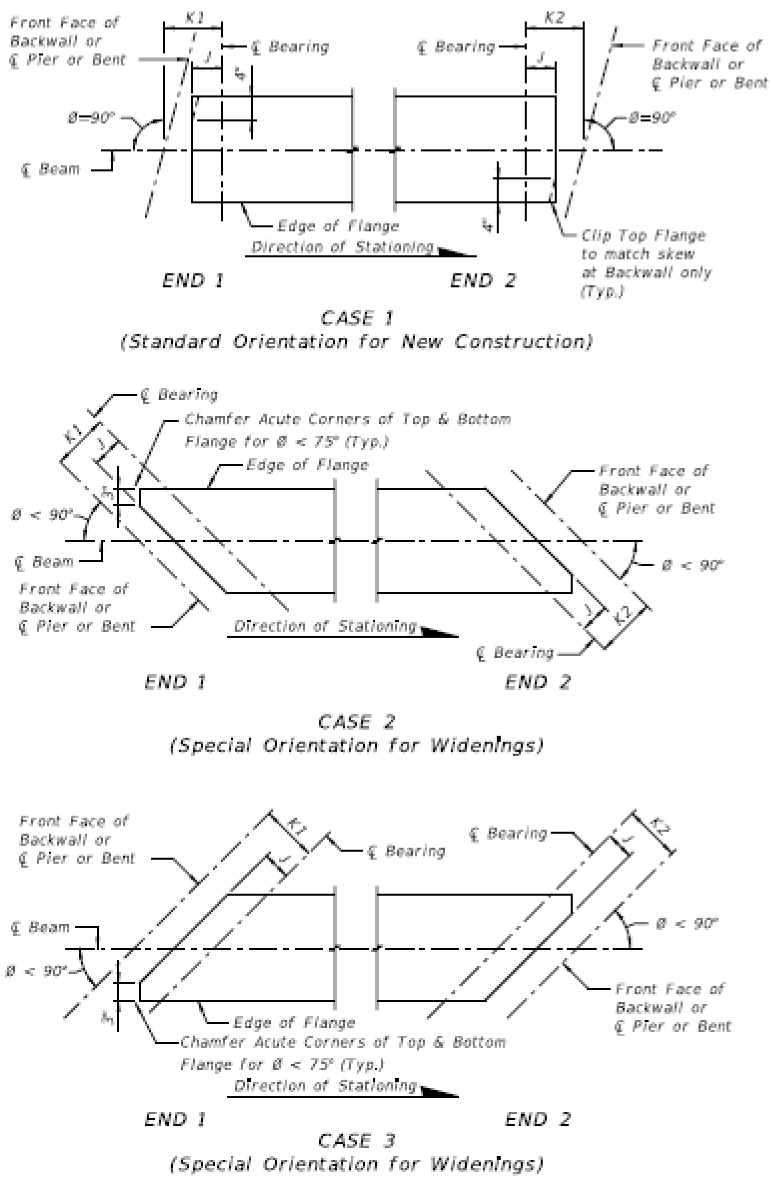
Distance from centerline end bent (FFBW) to centerline bearing............................... $K2 := 16 \cdot in$

Distance from end of beam to centerline of bearing.......... $J := 8 \cdot in$

Beam length......................... $L_{beam} := L_{span} - (K1 - J) - (K2 - J) = 89.083\,ft$

Beam design length...............

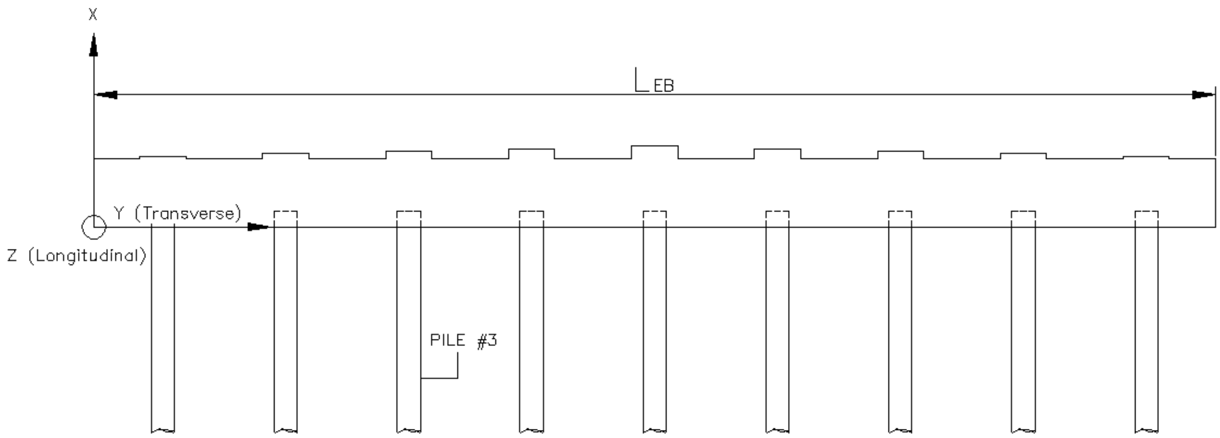
$L_{design} := L_{span} - K1 - K2 = 87.75\,ft$

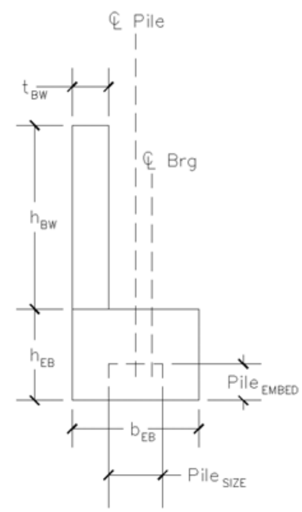


D. Substructure

D1. End Bent Geometry

*(**Note**: End bent back wall not shown)*





Depth of end bent cap........... $h_{EB} := 2.5ft$

Width of end bent cap........... $b_{EB} := 3.5 \cdot ft$

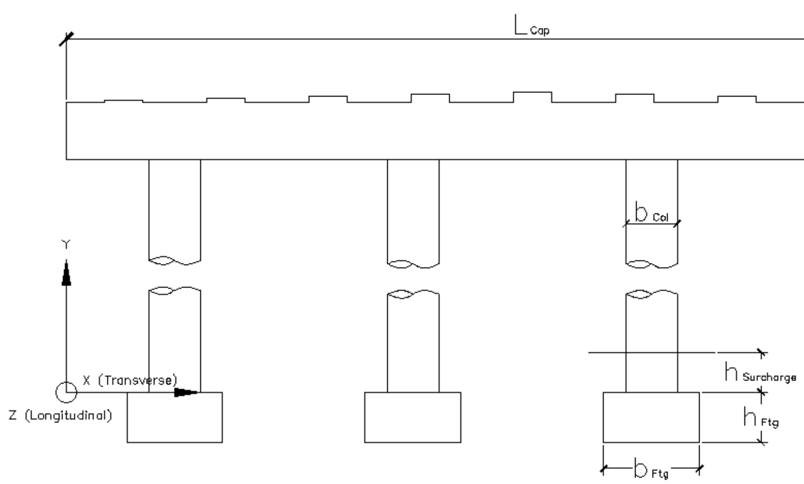
Length of end bent cap.......... $L_{EB} := 88 \cdot ft$

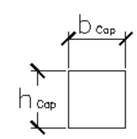
Height of back wall.............. $h_{BW} := 3.6 \cdot ft$

Back wall design width.......... $L_{BW} := 1 \cdot ft$

Thickness of back wall.......... $t_{BW} := 12 \cdot in$

D2. Pier Geometry





Depth of pier cap................. $h_{Cap} := 4.5 \cdot ft$

Width of pier cap................. $b_{Cap} := 4.5 \cdot ft$

Length of pier cap................ $L_{Cap} := 88 \cdot ft$

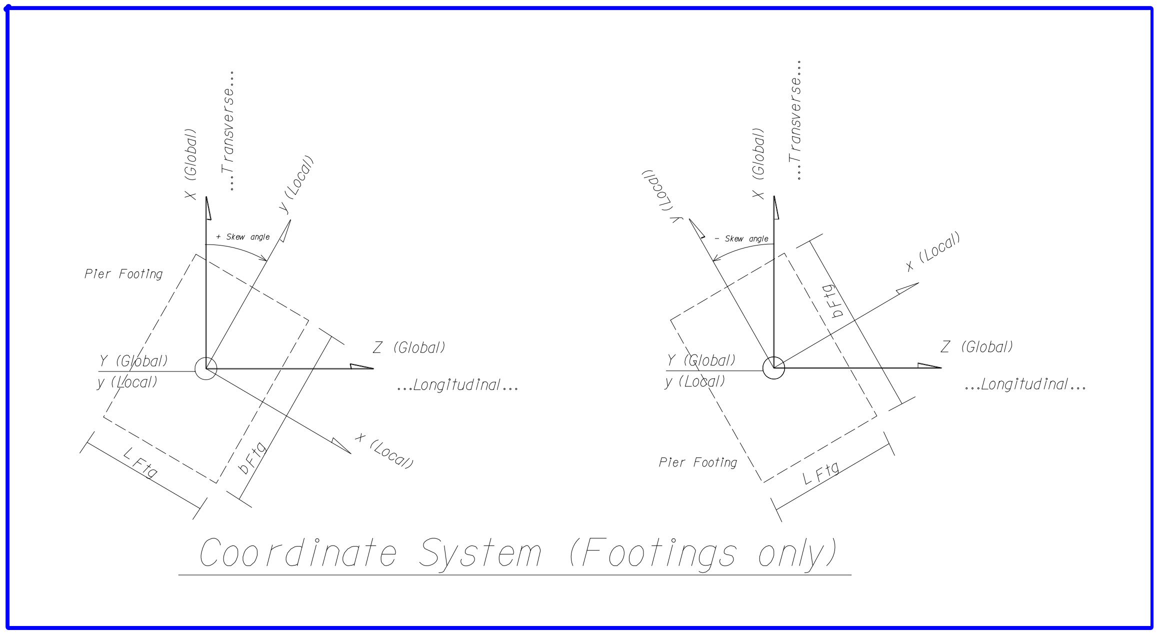
Height of pier column........... $h_{Col} := 14.0 \cdot ft$

Column diameter.................. $b_{Col} := 4.0 \cdot ft$

Number of columns............. $n_{Col} := 4$

Surcharge on top of footing... $h_{Surcharge} := 2.0 \cdot ft$

D3. Footing Geometry



Depth of footing.................. $h_{Ftg} := 4.0 \cdot ft$

Width of footing................... $b_{Ftg} := 7.5 \cdot ft$

Length of footing.................. $L_{Ftg} := 7.5 \cdot ft$

D4. Pile Geometry

Pile Embedment Depth.......... $Pile_{embed} := 12 \cdot in$

Pile Size............................. $Pile_{size} := 18 \cdot in$

D5. Approach Slab Geometry

Approach slab thickness....... $t_{ApprSlab} := 13.75 \cdot in$

Approach slab length............ $L_{ApprSlab} := 32 \cdot ft$

*(**Note**: The min. approach slab dimension due to the skew is* $\frac{30 \cdot ft}{\cos(Skew)} = 31.93\ ft$ *).*

D6. Soil Properties

Unit weight of soil................ $\gamma_{soil} = 115 \cdot pcf$

▶ Defined Units

Dead Loads

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\103DesignPar.xmcd(R)

Description

This section provides the dead loads for design of the bridge components.

Page Contents

A. Non-Composite Section Properties

A1. Summary of Properties for the Selected Beam Type

BeamTypeTog := BeamType

PCBeams :=

NON-COMPOSITE PROPERTIES			FIB-36	FIB-45	FIB-54	FIB-63	FIB-72	FIB-78
Moment of Inertia	[in^4]	I	127545	226625	360041	530560	740895	904567
Section Area	[in^2]	A_b	807	870	933	996	1059	1101
ytop	[in]	y_{top}	19.51	24.79	29.97	35.06	40.09	43.4
ybot	[in]	y_{bot}	16.49	20.21	24.03	27.94	31.91	34.6
Depth	[in]	h	36	45	54	63	72	78
Top flange width	[in]	b_{tf}	48	48	48	48	48	**48**
Top flange depth	[in]	h_{tf}	3.5	3.5	3.5	3.5	3.5	**3.5**
Width of web	[in]	b_{web}	7	7	7	7	7	**7**
Bottom flange width	[in]	b	38	38	38	38	38	**38**
Bottom flange depth	[in]	h_{bf}	7	7	7	7	7	**7**
Bottom flange taper	[in]	E	15.5	17	17	16	17	17

$$\text{output(beamprops, type)} := \left| \begin{array}{ll} \text{beamprops}^{\langle 1 \rangle} & \text{if type} = \text{"FIB-36"} \\ \text{beamprops}^{\langle 2 \rangle} & \text{if type} = \text{"FIB-45"} \\ \text{beamprops}^{\langle 3 \rangle} & \text{if type} = \text{"FIB-54"} \\ \text{beamprops}^{\langle 4 \rangle} & \text{if type} = \text{"FIB-63"} \\ \text{beamprops}^{\langle 5 \rangle} & \text{if type} = \text{"FIB-72"} \\ \text{beamprops}^{\langle 6 \rangle} & \text{if type} = \text{"FIB-78"} \\ \left(\text{beamprops}^{\langle 0 \rangle}\right)_{0,0} & \text{otherwise} \end{array} \right.$$

$$FIB36 := \begin{pmatrix} 5 & -36 \\ 5 & -29 \\ 20.5 & -20.5 \\ 20.5 & -8.5 \\ 17 & -5 \\ 0 & -3.5 \\ 0 & 0 \\ 48 & 0 \\ 48 & -3.5 \\ 31 & -5 \\ 27.5 & -8.5 \\ 27.5 & -20.5 \\ 43 & -29 \\ 43 & -36 \\ 5 & -36 \end{pmatrix} \cdot in \quad FIB45 := \begin{pmatrix} 5 & -45 \\ 5 & -38 \\ 20.5 & -21 \\ 20.5 & -8.5 \\ 17 & -5 \\ 0 & -3.5 \\ 0 & 0 \\ 48 & 0 \\ 48 & -3.5 \\ 31 & -5 \\ 27.5 & -8.5 \\ 27.5 & -21 \\ 43 & -38 \\ 43 & -45 \\ 5 & -45 \end{pmatrix} \cdot in \quad FIB54 := \begin{pmatrix} 5 & -54 \\ 5 & -47 \\ 20.5 & -30 \\ 20.5 & -8.5 \\ 17 & -5 \\ 0 & -3.5 \\ 0 & 0 \\ 48 & 0 \\ 48 & -3.5 \\ 31 & -5 \\ 27.5 & -8.5 \\ 27.5 & -30 \\ 43 & -47 \\ 43 & -54 \\ 5 & -54 \end{pmatrix} \cdot in \quad FIB63 := \begin{pmatrix} 5 & -63 \\ 5 & -56 \\ 20.5 & -40 \\ 20.5 & -8.5 \\ 17 & -5 \\ 0 & -3.5 \\ 0 & 0 \\ 48 & 0 \\ 48 & -3.5 \\ 31 & -5 \\ 27.5 & -8.5 \\ 27.5 & -40 \\ 43 & -56 \\ 43 & -63 \\ 5 & -63 \end{pmatrix} \cdot in$$

$$Beamtype := \left| \begin{array}{l} FIB36 \text{ if } BeamType = \text{"FIB-36"} \\ FIB45 \text{ if } BeamType = \text{"FIB-45"} \\ FIB54 \text{ if } BeamType = \text{"FIB-54"} \\ FIB63 \text{ if } BeamType = \text{"FIB-63"} \\ FIB72 \text{ if } BeamType = \text{"FIB-72"} \\ FIB78 \text{ if } BeamType = \text{"FIB-78"} \end{array} \right.$$

output(PCBeams, BeamTypeTog) =

	0
0	$1.275 \cdot 10^5$
1	807
2	19.51
3	16.49
4	36
5	48
6	3.5
7	7
8	38
9	7
10	15.5
11	$2.865 \cdot 10^4$

$I_{nc} := output(PCBeams, BeamTypeTog)_0 \cdot in^4$

$A_{nc} := output(PCBeams, BeamTypeTog)_1 \cdot in^2$

$yt_{nc} := output(PCBeams, BeamTypeTog)_2 \cdot in$

$yb_{nc} := output(PCBeams, BeamTypeTog)_3 \cdot in$

$h_{nc} := output(PCBeams, BeamTypeTog)_4 \cdot in$

$b_{tf} := output(PCBeams, BeamTypeTog)_5 \cdot in$

$h_{tf} := output(PCBeams, BeamTypeTog)_6 \cdot in$

$b_w := output(PCBeams, BeamTypeTog)_7 \cdot in$

$b_{bf} := output(PCBeams, BeamTypeTog)_8 \cdot in$

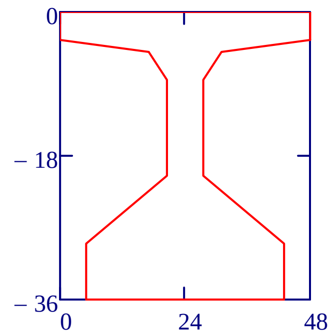
$h_{bf} := output(PCBeams, BeamTypeTog)_9 \cdot in$

$Jx := output(PCBeams, BeamTypeTog)_{11} \cdot in^4$

The non-composite beam properties are given and can be obtained from the FDOT Instructions for Design Standards, Index 20010 Series.

NON-COMPOSITE PROPERTIES			FIB-36
Moment of Inertia	[in^4]	I_{nc}	127545
Section Area	[in^2]	A_{nc}	807
ytop	[in]	yt_{nc}	19.51
ybot	[in]	yb_{nc}	16.49
Depth	[in]	h_{nc}	36
Top flange width	[in]	b_{tf}	48
Top flange depth	[in]	h_{tf}	3.5
Width of web	[in]	b_w	7
Bottom flange width	[in]	b_{bf}	38
Bottom flange depth	[in]	h_{bf}	7
Bottom flange taper	[in]	E	15.5
Section Modulus top	[in^3]	S_{tnc}	6537
Section Modulus bottom	[in^3]	S_{bnc}	7735

BeamType = "FIB-36"



A2. Effective Flange Width [LRFD 4.6.2.6]

Interior beams

The effective flange width: $b_{eff.interior} := BeamSpacing = 10\,ft$

Exterior beams

For exterior beams, the effective flange width:

$$b_{eff.exterior} = \frac{BeamSpacing}{2} + Overhang + \Delta w$$ where:

Cross-sectional area of the barrier..... $A_b := 2.77ft^2$

$$\Delta w := \frac{A_b}{2 \cdot t_{slab}} = 2.078\,ft$$

Effective flange width: $b_{eff.exterior} := \left(\frac{BeamSpacing}{2} + Overhang + \Delta w\right) = 11.619\,ft$

Transformed Properties

To develop composite section properties, the effective flange width of the slab should be transformed to the concrete properties of the beam.

Modular ratio between the deck and beam. $n := \dfrac{E_{c.slab}}{E_{c.beam}} = 0.728$

Transformed slab width for interior beams $b_{tr.interior} := n \cdot \left(b_{eff.interior}\right) = 87.313 \cdot in$

Transformed slab width for exterior beams $b_{tr.exterior} := n \cdot \left(b_{eff.exterior}\right) = 101.45 \cdot in$

B. Composite Section Properties

B1. Interior beams

Height of the composite section.....................

$$h := h_{nc} + h_{buildup} + t_{slab} = 45 \cdot in$$

Area of the composite section........................

$$A_{slab} := b_{tr.interior} \cdot t_{slab} = 698.5 \cdot in^2$$

$$A_{fillet} := b_{tf} \cdot h_{buildup} = 48 \cdot in^2$$

$$A_{Interior} := A_{nc} + A_{fillet} + A_{slab} = 1553.5 \cdot in^2$$

Distance from centroid of beam to extreme fiber in tension

$$y_b := \frac{A_{nc} \cdot yb_{nc} + A_{fillet} \cdot \left(h_{nc} + \frac{h_{buildup}}{2}\right) + A_{slab} \cdot \left(h_{nc} + h_{buildup} + \frac{t_{slab}}{2}\right)}{A_{Interior}} = 28.1 \cdot in$$

Distance from centroid of beam to extreme fiber in compression....................................

$$y_t := h - y_b = 16.9 \cdot in$$

Moment of Inertia.......................................

$$I_{slab} := \frac{1}{12} \cdot \left(b_{tr.interior}\right) \cdot t_{slab}^{\,3} + A_{slab} \cdot \left(h - \frac{t_{slab}}{2} - y_b\right)^2 = 119446 \cdot in^4$$

$$I_{fillet} := \frac{\left(b_{tf}\right) \cdot h_{buildup}^{\,3}}{12} + A_{fillet} \cdot \left(h_{nc} + \frac{h_{buildup}}{2} - y_b\right)^2 = 3368 \cdot in^4$$

$$I_{Interior} := I_{nc} + A_{nc} \cdot \left(y_b - yb_{nc}\right)^2 + I_{slab} + I_{fillet} = 359675 \cdot in^4$$

Section Modulus (top, top of beam, bottom)...

$$S_t := \frac{I_{Interior}}{y_t} = 21319 \cdot in^3$$

$$S_{tb} := \frac{I_{Interior}}{h_{nc} - y_b} = 45695 \cdot in^3$$

$$S_b := \frac{I_{Interior}}{y_b} = 12787 \cdot in^3$$

B2. Exterior beams

Calculations are similar to interior beams.

Height of the composite section.................... $h := h_{nc} + h_{buildup} + t_{slab} = 45 \cdot in$

Area of the composite section....................... $A_{slab} := b_{tr.exterior} \cdot t_{slab} = 811.602 \cdot in^2$

$A_{fillet} := b_{tf} \cdot h_{buildup} = 48 \cdot in^2$

$A_{Exterior} := A_{nc} + A_{fillet} + A_{slab} = 1666.6 \cdot in^2$

Distance from centroid of beam to extreme fiber in tension

$$y'_b := \frac{A_{nc} \cdot yb_{nc} + A_{fillet} \cdot \left(h_{nc} + \frac{h_{buildup}}{2}\right) + A_{slab} \cdot \left(h_{nc} + h_{buildup} + \frac{t_{slab}}{2}\right)}{A_{Exterior}} = 29.002 \cdot in$$

Distance from centroid of beam to extreme fiber in compression.................................. $y'_t := h - y'_b = 15.998 \cdot in$

Moment of Inertia.....................................

$$I_{slab} := \frac{1}{12} \cdot \left(b_{tr.exterior}\right) \cdot t_{slab}^{\,3} + A_{slab} \cdot \left(h - \frac{t_{slab}}{2} - y'_b\right)^2 = 121157 \cdot in^4$$

$$I_{fillet} := \frac{\left(b_{tf}\right) \cdot h_{buildup}^{\,3}}{12} + A_{fillet} \cdot \left(h_{nc} + \frac{h_{buildup}}{2} - y'_b\right)^2 = 2702 \cdot in^4$$

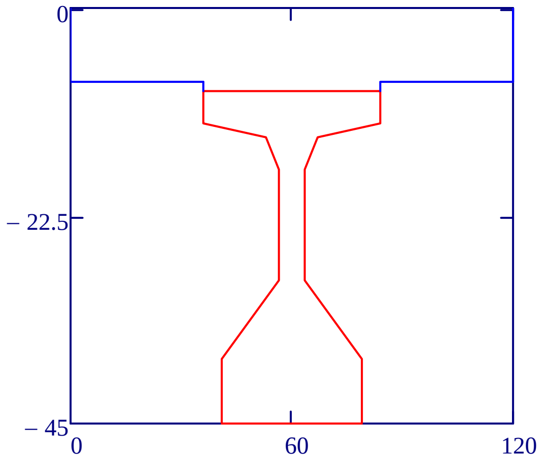
$$I_{Exterior} := I_{nc} + A_{nc} \cdot \left(y'_b - yb_{nc}\right)^2 + I_{slab} + I_{fillet} = 377744 \cdot in^4$$

Section Modulus (top, top of beam, bottom)... $S_t := \frac{I_{Exterior}}{y'_t} = 23612.2 \cdot in^3$

$$S_{tb} := \frac{I_{Exterior}}{h_{nc} - y'_b} = 53980.3 \cdot in^3$$

$$S_b := \frac{I_{Exterior}}{y'_b} = 13024.7 \cdot in^3$$

B3. Summary of Properties



COMPOSITE SECTION PROPERTIES			INTERIOR	EXTERIOR
Effective slab width	[in]	$b_{eff.interior/exterior}$	120.0	139.4
Transformed slab width	[in]	$b_{tr.interior/exterior}$	87.3	101.5
Height of composite section	[in]	h	45.0	45.0
Effective slab area	[in^2]	A_{slab}	698.5	811.6
Area of composite section	[in^2]	$A_{Interior/Exterior}$	1553.5	1666.6
Neutral axis to bottom fiber	[in]	y_b	28.1	29.0
Neutral axis to top fiber	[in]	y_t	16.9	16.0
Inertia of composite section	[in^4]	$I_{Interior/Exterior}$	359675.0	377743.8
Section modulus top of slab	[in^3]	S_t	21318.8	23612.2
Section modulus top of beam	[in^3]	S_{tb}	45694.6	53980.3
Section modulus bottom of beam	[in^3]	S_b	12786.8	13024.7

C. Dead Loads

Calculate the moments and shears as a function of "x", where "x" represents any point along the length of the beam from 0 feet to L_{design}. The values for the moment and shear at key design check points are given...

where $Support := 0 \cdot ft$

$ShearChk := 3.2 \cdot ft$

$Debond1 := 10 \cdot ft$ *{Check beam for debonding, if not debonding, enter 0 ft.)*

$Debond2 := 20 \cdot ft$ *(Check beam for debonding, if not debonding, enter 0 ft.)*

$Midspan := 0.5 \cdot L_{design} = 43.875\,ft$

For convenience in Mathcad, place these points in a matrix.......................................

$$x := \begin{pmatrix} Support \\ ShearChk \\ Debond1 \\ Debond2 \\ Midspan \end{pmatrix} = \begin{pmatrix} 0 \\ 3.2 \\ 10 \\ 20 \\ 43.875 \end{pmatrix} ft \qquad pt := 0 .. 4$$

C1. Interior Beams

Design Moments and Shears for DC Dead Loads

Weight of beam

$$w_{BeamInt} := A_{nc} \cdot \gamma_{conc} = 0.841 \cdot klf$$

- Moment - self-weight of beam at Release.. $M_{RelBeamInt}(x) := \dfrac{w_{BeamInt} \cdot L_{beam}}{2} \cdot x - \dfrac{w_{BeamInt} \cdot x^2}{2}$
- Moment - self-weight of beam................. $M_{BeamInt}(x) := \dfrac{w_{BeamInt} \cdot L_{design}}{2} \cdot x - \dfrac{w_{BeamInt} \cdot x^2}{2}$
- Shear - self-weight of beam .... $V_{BeamInt}(x) := \dfrac{w_{BeamInt} \cdot L_{design}}{2} - w_{BeamInt} \cdot x$

Weight of deck slab, includes haunch and milling surface

$$w_{SlabInt} := \left[\left(t_{slab} + t_{mill}\right) \cdot BeamSpacing + h_{buildup} \cdot b_{tf}\right] \cdot \gamma_{conc} = 1.112 \cdot klf$$

- Moment - self-weight of deck slab, includes haunch and milling surface .... $M_{SlabInt}(x) := \dfrac{w_{SlabInt} \cdot L_{design}}{2} \cdot x - \dfrac{w_{SlabInt} \cdot x^2}{2}$
- Shear - self-weight of deck slab, includes haunch and milling surface..................... $V_{SlabInt}(x) := \dfrac{w_{SlabInt} \cdot L_{design}}{2} - w_{SlabInt} \cdot x$

Weight of stay-in-place forms

$$w_{FormsInt} := \left(BeamSpacing - b_{tf}\right)\cdot\rho_{forms} = 0.12\cdot klf$$

- Moment - stay-in-place forms..................

$$M_{FormsInt}(x) := \frac{w_{FormsInt}\cdot L_{design}}{2}\cdot x - \frac{w_{FormsInt}\cdot x^2}{2}$$

- Shear - stay-in-place forms. ....

$$V_{FormsInt}(x) := \frac{w_{FormsInt}\cdot L_{design}}{2} - w_{FormsInt}\cdot x$$

Weight of traffic railing barriers

$$w_{barrier.interior} = 0.147\cdot klf$$

- Moment - traffic railing barriers..............

$$M_{TrbInt}(x) := \frac{w_{barrier.interior}\cdot L_{design}}{2}\cdot x - \frac{w_{barrier.interior}\cdot x^2}{2}$$

- Shear - traffic railing barriers..................

$$V_{TrbInt}(x) := \frac{w_{barrier.interior}\cdot L_{design}}{2} - w_{barrier.interior}\cdot x$$

DC Load total

$$w_{DC.BeamInt} := w_{BeamInt} + w_{SlabInt} + w_{FormsInt} + w_{barrier.interior} = 2.22\cdot klf$$

DC Load Moment

$$M_{DC.BeamInt}(x) := M_{BeamInt}(x) + M_{SlabInt}(x) + M_{FormsInt}(x) + M_{TrbInt}(x)$$

DC Load Shear

$$V_{DC.BeamInt}(x) := V_{BeamInt}(x) + V_{SlabInt}(x) + V_{FormsInt}(x) + V_{TrbInt}(x)$$

DC Load Rotation

$$\theta_{DC.BeamInt} := \frac{\left(w_{DC.BeamInt} - w_{barrier.interior}\right)\cdot L_{design}^{3}}{24\cdot E_{c.beam}\cdot I_{nc}} + \frac{w_{barrier.interior}\cdot L_{design}^{3}}{24\cdot E_{c.beam}\cdot I_{Interior}} = 0.81\cdot deg$$

Design Moments and Shears for DW Dead Loads

Weight of future wearing surface

$$w_{FwsInt} := BeamSpacing\cdot\rho_{fws} = 0\cdot klf$$

- Moment - weight of future wearing surface..

$$M_{FwsInt}(x) := \frac{w_{FwsInt}\cdot L_{design}}{2}\cdot x - \frac{w_{FwsInt}\cdot x^2}{2}$$

- Shear - weight of future wearing surface . $V_{FwsInt}(x) := \frac{w_{FwsInt} \cdot L_{design}}{2} - w_{FwsInt} \cdot x$

<u>Weight of utility loads</u> $w_{UtilityInt} := 0 \cdot klf$

- Moment - utility loads............................ $M_{UtilityInt}(x) := \frac{w_{UtilityInt} \cdot L_{design}}{2} \cdot x - \frac{w_{UtilityInt} \cdot x^2}{2}$

- Shear - utility loads................................ $V_{UtilityInt}(x) := \frac{w_{UtilityInt} \cdot L_{design}}{2} - w_{UtilityInt} \cdot x$

<u>DW Load total</u>

$$w_{DW.BeamInt} := w_{FwsInt} + w_{UtilityInt} = 0 \cdot klf$$

<u>DW Load Moment</u>

$$M_{DW.BeamInt}(x) := M_{FwsInt}(x) + M_{UtilityInt}(x)$$

<u>DW Load Shear</u>

$$V_{DW.BeamInt}(x) := V_{FwsInt}(x) + V_{UtilityInt}(x)$$

<u>DW Load Rotation</u>

$$\theta_{DW.BeamInt} := \frac{w_{DW.BeamInt} \cdot L_{design}^3}{24 \cdot E_{c.beam} \cdot I_{Interior}} = 0 \cdot deg$$

C2. Exterior Beams

Design Moments and Shears for DC Dead Loads

<u>Weight of beam</u> $w_{BeamExt} := A_{nc} \cdot \gamma_{conc} = 0.841 \cdot klf$

- Moment - self-weight of beam at Release.. $M_{RelBeamExt}(x) := \frac{w_{BeamExt} \cdot L_{beam}}{2} \cdot x - \frac{w_{BeamExt} \cdot x^2}{2}$

- Moment - self-weight of beam................. $M_{BeamExt}(x) := \frac{w_{BeamExt} \cdot L_{design}}{2} \cdot x - \frac{w_{BeamExt} \cdot x^2}{2}$

- Shear - self-weight of beam .... $V_{BeamExt}(x) := \frac{w_{BeamExt} \cdot L_{design}}{2} - w_{BeamExt} \cdot x$

Weight of deck slab, includes haunch and milling surface

$$w_{SlabExt} := \left[\left(t_{slab} + t_{mill}\right)\cdot\left(\text{Overhang} + \frac{\text{BeamSpacing}}{2}\right) + h_{buildup}\cdot b_{tf}\right]\cdot\gamma_{conc} = 1.064\cdot\text{klf}$$

- Moment - self-weight of deck slab, includes haunch and milling surface ....

$$M_{SlabExt}(x) := \frac{w_{SlabExt}\cdot L_{design}}{2}\cdot x - \frac{w_{SlabExt}\cdot x^2}{2}$$

- Shear - self-weight of deck slab, includes haunch and milling surface.....................

$$V_{SlabExt}(x) := \frac{w_{SlabExt}\cdot L_{design}}{2} - w_{SlabExt}\cdot x$$

Weight of stay-in-place forms

$$w_{FormsExt} := \left(\frac{\text{BeamSpacing} - b_{tf}}{2}\right)\cdot\rho_{forms} = 0.06\cdot\text{klf}$$

- Moment - stay-in-place forms.................

$$M_{FormsExt}(x) := \frac{w_{FormsExt}\cdot L_{design}}{2}\cdot x - \frac{w_{FormsExt}\cdot x^2}{2}$$

- Shear - stay-in-place forms. ....

$$V_{FormsExt}(x) := \frac{w_{FormsExt}\cdot L_{design}}{2} - w_{FormsExt}\cdot x$$

Weight of traffic railing barriers

$$w_{barrier.exterior} = 0.147\cdot\text{klf}$$

- Moment - traffic railing barriers..............

$$M_{TrbExt}(x) := \frac{w_{barrier.exterior}\cdot L_{design}}{2}\cdot x - \frac{w_{barrier.exterior}\cdot x^2}{2}$$

- Shear - traffic railing barriers..................

$$V_{TrbExt}(x) := \frac{w_{barrier.exterior}\cdot L_{design}}{2} - w_{barrier.exterior}\cdot x$$

DC Load total

$$w_{DC.BeamExt} := w_{BeamExt} + w_{SlabExt} + w_{FormsExt} + w_{barrier.exterior} = 2.112\cdot\text{klf}$$

DC Load Moment

$$M_{DC.BeamExt}(x) := M_{BeamExt}(x) + M_{SlabExt}(x) + M_{FormsExt}(x) + M_{TrbExt}(x)$$

DC Load Shear

$$V_{DC.BeamExt}(x) := V_{BeamExt}(x) + V_{SlabExt}(x) + V_{FormsExt}(x) + V_{TrbExt}(x)$$

DC Load Rotation

$$\theta_{DC.BeamExt} := \frac{\left(w_{DC.BeamExt} - w_{barrier.exterior}\right)\cdot L_{design}^{3}}{24\cdot E_{c.beam}\cdot I_{nc}} + \frac{w_{barrier.exterior}\cdot L_{design}^{3}}{24\cdot E_{c.beam}\cdot I_{Exterior}} = 0.767\cdot deg$$

Design Moments and Shears for DW Dead Loads

Weight of future wearing surface

$$w_{FwsExt} := \left(Overhang - 1.5417\cdot ft + \frac{BeamSpacing}{2}\right)\cdot \rho_{fws} = 0\cdot klf$$

- Moment - weight of future wearing surface..

$$M_{FwsExt}(x) := \frac{w_{FwsExt}\cdot L_{design}}{2}\cdot x - \frac{w_{FwsExt}\cdot x^{2}}{2}$$

- Shear - weight of future wearing surface .

$$V_{FwsExt}(x) := \frac{w_{FwsExt}\cdot L_{design}}{2} - w_{FwsExt}\cdot x$$

Weight of utility loads

$$w_{UtilityExt} := 0\cdot klf$$

- Moment - utility loads............................

$$M_{UtilityExt}(x) := \frac{w_{UtilityExt}\cdot L_{design}}{2}\cdot x - \frac{w_{UtilityExt}\cdot x^{2}}{2}$$

- Shear - utility loads................................

$$V_{UtilityExt}(x) := \frac{w_{UtilityExt}\cdot L_{design}}{2} - w_{UtilityExt}\cdot x$$

DW Load total

$$w_{DW.BeamExt} := w_{FwsExt} + w_{UtilityExt} = 0\cdot klf$$

DW Load Moment

$$M_{DW.BeamExt}(x) := M_{FwsExt}(x) + M_{UtilityExt}(x)$$

DW Load Shear

$$V_{DW.BeamExt}(x) := V_{FwsExt}(x) + V_{UtilityExt}(x)$$

DW Load Rotation

$$\theta_{DW.BeamExt} := \frac{w_{DW.BeamExt}\cdot L_{design}^{3}}{24\cdot E_{c.beam}\cdot I_{Exterior}} = 0\cdot deg$$

C3. Summary

	DESIGN MOMENTS (ft-kip)				
	Support	ShrChk	Debond1	Debond2	Midspan
Load/Location, x (ft)=	0.0	3.2	10.0	20.0	43.9
INTERIOR BEAM					
Beam at Release	0.0	115.5	332.4	580.7	833.7
Beam	0.0	113.7	326.8	569.5	809.1
Slab	0.0	150.5	432.5	753.7	1070.8
Forms	0.0	16.2	46.7	81.3	115.5
Barrier	0.0	19.9	57.2	99.7	141.7
TOTAL DC	**0.0**	**300.4**	**863.2**	**1504.3**	**2137.1**
FWS	0.0	0.0	0.0	0.0	0.0
Utilities	0.0	0.0	0.0	0.0	0.0
TOTAL DW	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**
EXTERIOR BEAM					
Beam at Release	0.0	115.5	332.4	580.7	833.7
Beam	0.0	113.7	326.8	569.5	809.1
Slab	0.0	143.9	413.6	720.7	1023.9
Forms	0.0	8.1	23.3	40.7	57.8
Barrier	0.0	19.9	57.2	99.7	141.7
TOTAL DC	**0.0**	**285.7**	**820.9**	**1430.6**	**2032.5**
FWS	0.0	0.0	0.0	0.0	0.0
Utilities	0.0	0.0	0.0	0.0	0.0
TOTAL DW	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**

	CORRESPONDING SHEARS (kip)				
	Support	ShrChk	Debond1	Debond2	Midspan
Load/Location, x (ft)=	0.0	3.2	10.0	20.0	43.9
INTERIOR BEAM					
Beam	36.9	34.2	28.5	20.1	0.0
Slab	48.8	45.3	37.7	26.6	0.0
Forms	5.3	4.9	4.1	2.9	0.0
Barrier	6.5	6.0	5.0	3.5	0.0
TOTAL DC	**97.4**	**90.3**	**75.2**	**53.0**	**0.0**
FWS	0.0	0.0	0.0	0.0	0.0
Utilities	0.0	0.0	0.0	0.0	0.0
TOTAL DW	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**
EXTERIOR BEAM					
Beam	36.9	34.2	28.5	20.1	0.0
Slab	46.7	43.3	36.0	25.4	0.0
Forms	2.6	2.4	2.0	1.4	0.0
Barrier	6.5	6.0	5.0	3.5	0.0
TOTAL DC	**92.6**	**85.9**	**71.5**	**50.4**	**0.0**
FWS	0.0	0.0	0.0	0.0	0.0
Utilities	0.0	0.0	0.0	0.0	0.0
TOTAL DW	**0.0**	**0.0**	**0.0**	**0.0**	**0.0**

Write Out Data

Live Load Distribution Factors

Reference

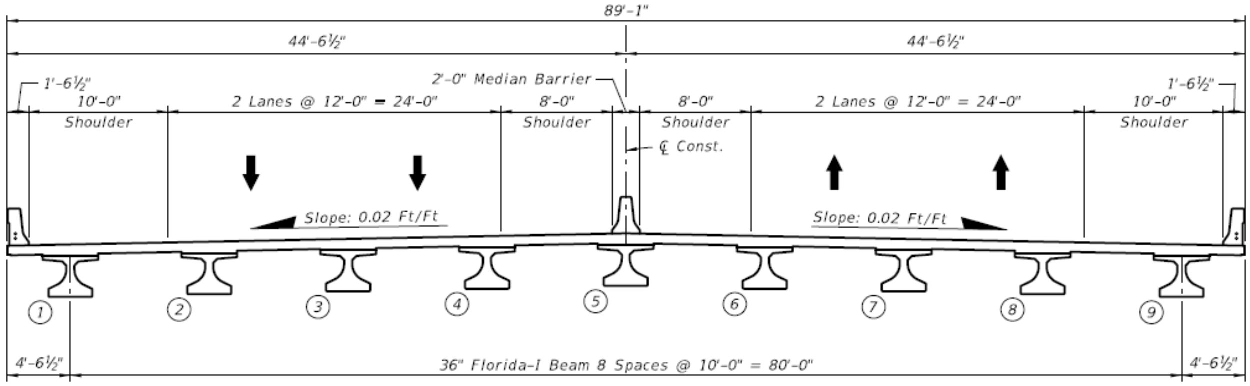
Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\201DeadLoads.xmcd(R)

Description

This document calculates the live load distribution factors as per the LRFD.

Page Contents

A. Input Variables



A1. Bridge Geometry

Overall bridge length............ $L_{bridge} = 180\ ft$

Bridge design span length...... $L_{span} = 90\ ft$

Beam design length.............. $L_{design} = 87.750\ ft$

Skew angle......................... $Skew = -20 \cdot deg$

Superstructure Beam Type.... BeamType = "FIB-36"

Number of beams................ $N_{beams} = 9$

Beam Spacing..................... $BeamSpacing = 10\ ft$

Deck overhang.................... $Overhang = 4.5417\ ft$

Roadway clear width........... $Rdwy_{width} = 42\ ft$

Number of design traffic lanes................................. $N_{lanes} = 3$

Height of composite section... $h = 45.0 \cdot in$

Distance from neutral axis to bottom fiber of non-composite section............................... $yb_{nc} = 16.5 \cdot in$

Thickness of deck slab......... $t_{slab} = 8 \cdot in$

Modular ratio between beam and deck............................ $n^{-1} = 1.374$

Moment of inertia for non-composite section.......... $I_{nc} = 127545.0 \cdot in^4$

Area of non-composite section............................... $A_{nc} = 807.0 \cdot in^2$

B. Beam-Slab Bridges - Application [LRFD 4.6.2.2.1]

Live load on the deck must be distributed to the precast, prestressed beams. AASHTO provides factors for the distribution of live load into the beams. The factors can be used if the following criteria is met:

- Width of deck is constant
- Number of beams is not less than four
- Beams are parallel and have approximately the same stiffness
- The overhang minus the barrier width does not exceed 3.0 feet
- Curvature in plan is less than the limit specified in Article 4.6.1.2.4

If these conditions are not met, a refined method of analysis is required and diaphragms shall be provided.

Distance between center of gravity of non-composite beam and deck...................... $e_g := \left(h - yb_{nc}\right) - \dfrac{t_{slab}}{2} = 24.51 \cdot in$

Longitudinal stiffness parameter.................... $K_g := n^{-1} \cdot \left(I_{nc} + A_{nc} \cdot e_g^{\ 2}\right) = 841584 \cdot in^4$

C. Moment Distribution Factors

C1. Moment: Interior Beams [LRFD 4.6.2.2.2b]

- **One design lane**

Distribution factor for moment in interior beams when one design lane is loaded

$$g_{m.Interior} = 0.06 + \left(\frac{S}{14}\right)^{0.4} \cdot \left(\frac{S}{L}\right)^{0.3} \cdot \left(\frac{K_g}{12.0 \cdot L \cdot t_s^{\,3}}\right)^{0.1}$$

Using variables defined in this example,

$$g_{m.Interior1} := 0.06 + \left(\frac{BeamSpacing}{14 \cdot ft}\right)^{0.4} \cdot \left(\frac{BeamSpacing}{L_{design}}\right)^{0.3} \cdot \left(\frac{K_g}{12.0 \cdot \frac{in}{ft} \cdot L_{design} \cdot t_{slab}^{\,3}}\right)^{0.1} = 0.536$$

- **Two or more design lanes**

Distribution factor for moment in interior beams when two or more design lanes are loaded

$$g_{m.Interior} = 0.075 + \left(\frac{S}{9.5}\right)^{0.6} \cdot \left(\frac{S}{L}\right)^{0.2} \cdot \left(\frac{K_g}{12.0 \cdot L \cdot t_s^{\,3}}\right)^{0.1}$$

Using variables defined in this example,

$$g_{m.Interior2} := 0.075 + \left(\frac{BeamSpacing}{9.5 \cdot ft}\right)^{0.6} \cdot \left(\frac{BeamSpacing}{L_{design}}\right)^{0.2} \cdot \left(\frac{K_g}{12.0 \cdot \frac{in}{ft} \cdot L_{design} \cdot t_{slab}^{\,3}}\right)^{0.1} = 0.773$$

- **Range of Applicability**

The greater distribution factor is selected for moment design of the beams.

$$g_{m.Interior} := \max\left(g_{m.Interior1}, g_{m.Interior2}\right) = 0.773$$

Verify the distribution factor satisfies LRFD criteria for "Range of Applicability".

$$g_{m.Interior} := \left| \begin{array}{l} S \leftarrow (BeamSpacing \geq 3.5 \cdot ft) \cdot (BeamSpacing \leq 16.0 \cdot ft) \\ t_s \leftarrow (t_{slab} \geq 4.5 \cdot in) \cdot (t_{slab} \leq 12 \cdot in) \\ L \leftarrow (L_{design} \geq 20 \cdot ft) \cdot (L_{design} \leq 240 \cdot ft) \\ N_b \leftarrow (N_{beams} \geq 4) \\ K_g \leftarrow (K_g \geq 10000 \cdot in^4) \cdot (K_g \leq 7000000 \cdot in^4) \\ g_{m.Interior} \;\; \text{if} \;\; (S \cdot t_s \cdot L \cdot N_b \cdot K_g) \\ \text{"NG, does not satisfy Range of Applicability"} \quad \text{otherwise} \end{array} \right. = 0.773$$

C2. Moment: Exterior Beams [LRFD 4.6.2.2.2d]

- **One design lane**

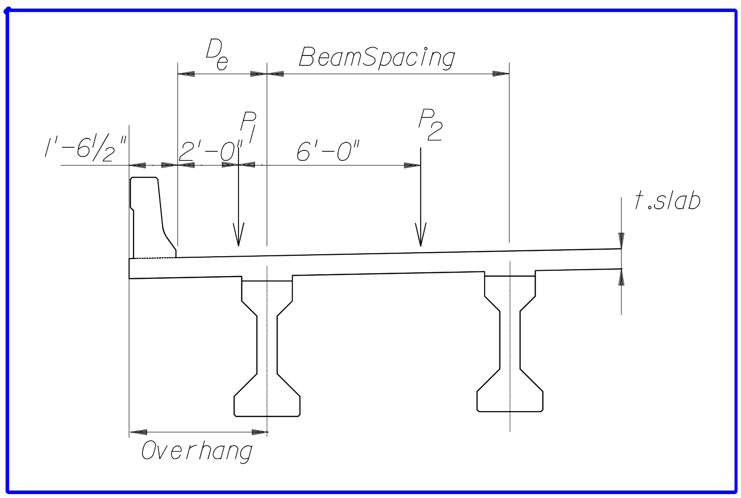
Distribution factor for moment in exterior beams when one design lane is loaded

$$P_1 = \frac{D_e + S - 2 \cdot ft}{S}$$

$$P_2 = \frac{D_e + S - 8 \cdot ft}{S}$$

$$D_e := Overhang - 1.5417 \cdot ft = 3\,ft$$

$$S := BeamSpacing = 10\,ft$$



The distribution factor for one design lane loaded is based on the lever rule, which includes a 0.5 factor for converting the truck load to wheel loads and a 1.2 factor for multiple truck presence.

$$g_{m.Exterior1} := \text{if}\left[(2 \cdot ft + 6 \cdot ft) < (D_e + S), \frac{(2 \cdot S + 2D_e - 10 \cdot ft)}{S} \cdot 0.5, \frac{(S + D_e - 2 \cdot ft)}{S} \cdot 0.5\right] \cdot 1.2 = 0.96$$

- **Two or more design lanes**

Distribution factor for moment in exterior beams when two or more design lanes are loaded

$$g_{m.Exterior} = g_{m.Interior} \cdot \left(0.77 + \frac{d_e}{9.1}\right)$$

Using variables defined in this example,

Distance from centerline of web for exterior beam to barrier

$$d_e := Overhang - 1.5417 \cdot ft = 3\,ft$$

$$g_{m.Exterior2} := g_{m.Interior2} \cdot \left(0.77 + \frac{d_e}{9.1 \cdot ft}\right) = 0.85$$

- **Range of Applicability**

The greater distribution factor is selected for moment design of the beams.

$$g_{m.Exterior} := \max\left(g_{m.Exterior1}, g_{m.Exterior2}\right) = 0.96$$

Verify the distribution factor satisfies LRFD criteria for "Range of Applicability".

$$g_{m.Exterior} := \left| \begin{array}{ll} d_e \leftarrow \left(d_e \le 5.5 \cdot ft\right) \cdot \left(d_e \ge -1 \cdot ft\right) & \\ g_{m.Exterior} & \text{if } d_e \\ \text{"NG, does not satisfy Range of Applicability"} & \text{otherwise} \end{array} \right. = 0.96$$

C3. Moment: Skew Modification Factor [LRFD 4.6.2.2.2e]

A skew modification factor for moments **may** be used if the supports are skewed and the difference between skew angles of two adjacent supports does not exceed 10 degrees.

$$g_{m.Skew} = 1 - 0.25 \cdot \left[\left(\frac{K_g}{12.0 \cdot L \cdot t_s^{\,3}}\right)^{0.25}\right] \cdot \left(\frac{S}{L}\right)^{0.5} \cdot \tan(\theta)^{1.5}$$

Using variables defined in this example,

$$c_1 := 0.25 \cdot \left[\left(\frac{K_g}{12.0 \cdot \frac{in}{ft} L_{design} \cdot t_{slab}^{\,3}}\right)^{0.25}\right] \cdot \left(\frac{BeamSpacing}{L_{design}}\right)^{0.5} = 0.094$$

$$g_{m.Skew} := 1 - c_1 \cdot \tan\left(\left|Skew\right|\right)^{1.5} = 0.979$$

Verify the distribution factor satisfies LRFD criteria for "Range of Applicability".

$$g_{m.Skew} := \left| \begin{array}{l} \theta \leftarrow (|Skew| \geq 30 \cdot deg) \cdot (|Skew| \leq 60 \cdot deg) \\ S \leftarrow (BeamSpacing \geq 3.5 \cdot ft) \cdot (BeamSpacing \leq 16.0 \cdot ft) \\ L \leftarrow (L_{design} \geq 20 \cdot ft) \cdot (L_{design} \leq 240 \cdot ft) \\ N_b \leftarrow (N_{beams} \geq 4) \\ g_{m.Skew} \ \text{ if } (\theta \cdot S \cdot L \cdot N_b) \\ \text{"NG, does not satisfy Range of Applicability"} \quad \text{otherwise} \end{array} \right.$$

$$g_{m.Skew} := \text{if}(|Skew| < 30deg, 1, g_{m.Skew}) = 1$$

C4. Distribution Factors for Design Moments

Moment Distribution Factors		
	Interior	**Exterior**
1 Lane	0.536	0.960
2+ Lanes	0.773	0.850
Skew	1.000	1.000
DESIGN	**0.773**	**0.960**

D. Shear Distribution Factors

D1. Shear: Interior Beams [LRFD 4.6.2.2.3a]

- **One design lane**

Distribution factor for shear in interior beams when one design lane is loaded

$$g_v = 0.36 + \frac{S}{25}$$

Using variables defined in this example,

$$g_{v.Interior1} := 0.36 + \frac{BeamSpacing}{25 \cdot ft} = 0.76$$

- **Two or more design lanes**

Distribution factor for shear in interior beams when two or more design lanes are loaded

$$g_v = 0.2 + \frac{S}{12} - \left(\frac{S}{35}\right)^{2.0}$$

Using variables defined in this example,

$$g_{v.Interior2} := 0.2 + \frac{BeamSpacing}{12 \cdot ft} - \left(\frac{BeamSpacing}{35 \cdot ft}\right)^{2.0} = 0.952$$

- **Range of Applicability**

The greater distribution factor is selected for shear design of the beams

$$g_{v.Interior} := \max\left(g_{v.Interior1}, g_{v.Interior2}\right) = 0.952$$

Verify the distribution factor satisfies LRFD criteria for "Range of Applicability".

$$g_{v.Interior} := \left| \begin{array}{l} S \leftarrow (BeamSpacing \geq 3.5 \cdot ft) \cdot (BeamSpacing \leq 16.0 \cdot ft) \\ t_s \leftarrow \left(t_{slab} \geq 4.5 \cdot in\right) \cdot \left(t_{slab} \leq 12 \cdot in\right) \\ L \leftarrow \left(L_{design} \geq 20 \cdot ft\right) \cdot \left(L_{design} \leq 240 \cdot ft\right) \\ N_b \leftarrow \left(N_{beams} \geq 4\right) \\ g_{v.Interior} \quad \text{if } \left(S \cdot t_s \cdot L \cdot N_b\right) \\ \text{"NG, does not satisfy Range of Applicability"} \quad \text{otherwise} \end{array} \right. = 0.952$$

D2. Shear: Exterior Beams [LRFD 4.6.2.2.3b]

- **One design lane**

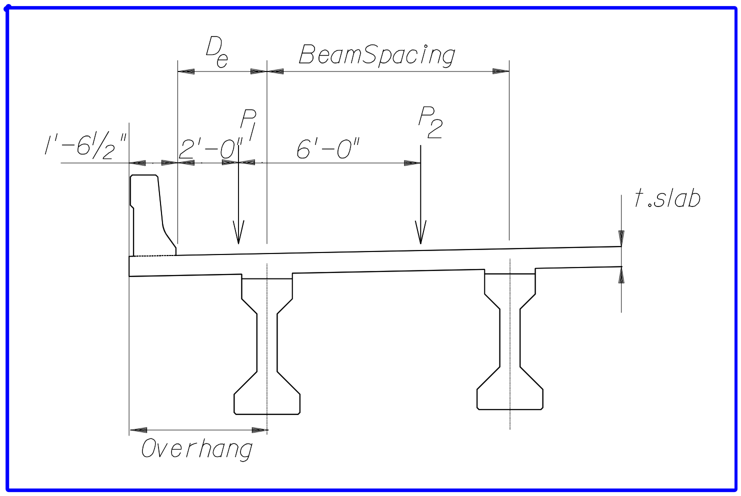
Distribution factor for shear in exterior beams when one design lane is loaded

$$P_1 = \frac{D_e + S - 2\cdot ft}{S}$$

$$P_2 = \frac{D_e + S - 8\cdot ft}{S}$$

$D_e = 3\,ft$

$S = 10\,ft$



The distribution factor for one design lane loaded is based on the lever rule, which includes a 0.5 factor for converting the truck load to wheel loads and a 1.2 factor for multiple truck presence.

$$g_{v.Exterior1} := \text{if}\left[(2\cdot ft + 6\cdot ft) < (D_e + S), \frac{(2\cdot S + 2D_e - 10\cdot ft)}{S}\cdot 0.5, \frac{(S + D_e - 2\cdot ft)}{S}\cdot 0.5\right]\cdot 1.2 = 0.96$$

- **Two or more design lanes**

Distribution factor for shear in exterior beams when two or more design lanes are loaded

$$g_{v.Exterior} = g_{v.Interior}\cdot\left(0.6 + \frac{d_e}{10}\right)$$

Using variables defined in this example,

$d_e = 3.000\,ft$

$$g_{v.Exterior2} := g_{v.Interior2}\cdot\left(0.6 + \frac{d_e}{10\cdot ft}\right) = 0.857$$

- **Range of Applicability**

The greater distribution factor is selected for shear design of the beams

$$g_{v.Exterior} := \max(g_{v.Exterior1}, g_{v.Exterior2}) = 0.96$$

Verify the distribution factor satisfies LRFD criteria for "Range of Applicability".

$$g_{v.Exterior} := \left| \begin{array}{l} d_e \leftarrow \left(d_e \le 5.5\cdot ft\right)\cdot\left(d_e \ge -1\cdot ft\right) \\ g_{v.Exterior} \ \ \text{if } d_e \\ \text{"NG, does not satisfy Range of Applicability"} \ \ \text{otherwise} \end{array} \right. = 0.96$$

D3. Shear: Skewed Modification Factor [LRFD 4.6.2.2.3c]

Skew modification factor for shear **shall** be applied to the exterior beam at the obtuse corner (θ > 90º) and to all beams in a multibeam bridge.

$$g_{v.Skew} = 1 + 0.20\cdot\left(\frac{12.0\cdot L\cdot t_s^{\,3}}{K_g}\right)^{0.3}\cdot\tan(\theta)$$

Using variables defined in this example,

$$g_{v.Skew} := 1 + 0.20\cdot\left(\frac{12.0\cdot\frac{in}{ft}\cdot L_{design}\cdot t_{slab}^{\,3}}{K_g}\right)^{0.3}\cdot\tan(|Skew|) = 1.064$$

Verify the distribution factor satisfies LRFD criteria for "Range of Applicability".

$$g_{v.Skew} := \left| \begin{array}{l} \theta \leftarrow (|Skew| \ge 0\cdot deg)\cdot(|Skew| \le 60\cdot deg) \\ S \leftarrow (BeamSpacing \ge 3.5\cdot ft)\cdot(BeamSpacing \le 16.0\cdot ft) \\ L \leftarrow \left(L_{design} \ge 20\cdot ft\right)\cdot\left(L_{design} \le 240\cdot ft\right) \\ N_b \leftarrow \left(N_{beams} \ge 4\right) \\ g_{v.Skew} \ \ \text{if } \left(\theta\cdot S\cdot L\cdot N_b\right) \\ \text{"NG, does not satisfy Range of Applicability"} \ \ \text{otherwise} \end{array} \right. = 1.064$$

If uplift is a design issue, the skew factor for all beams is unconservative. However, uplift is not a design issue for prestressed concrete beam bridges designed as simple spans.

D4. Distribution Factors for Design Shears

Shear Distribution Factors		
	Interior	**Exterior**
1 Lane	0.760	0.960
2+ Lanes	0.952	0.857
Skew	1.064	1.064
DESIGN	**1.012**	**1.021**

Live Load Analysis

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\202LLDistFactors.xmcd(R)

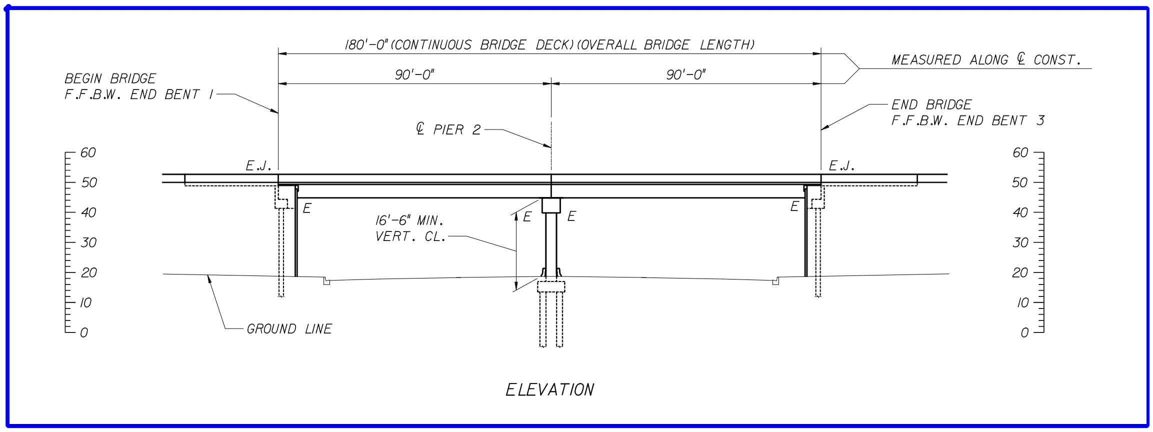
Description

This section provides examples of the LRFD HL-93 live load analysis necessary for the superstructure design.

Page Contents

A. Input Variables

A1. Bridge Geometry



Overall bridge length............ $L_{bridge} = 180\ ft$

Bridge design span length...... $L_{span} = 90\ ft$

A2. Beam Parameters

Beam length......................... $L_{beam} = 89.083\ ft$

Beam design length............... $L_{design} = 87.75\ ft$

Modulus of elasticity for beam.................................. $E_{c.beam} = 4781 \cdot ksi$

Moment of inertia for the interior beam........................ $I_{Interior} = 359675 \cdot in^4$

Moment of inertia of the exterior beam....................... $I_{Exterior} = 377744 \cdot in^4$

A3. Dynamic Load Allowance [LRFD 3.6.2]

Impact factor for limit states, except fatigue and fracture.... $IM = 1.33$

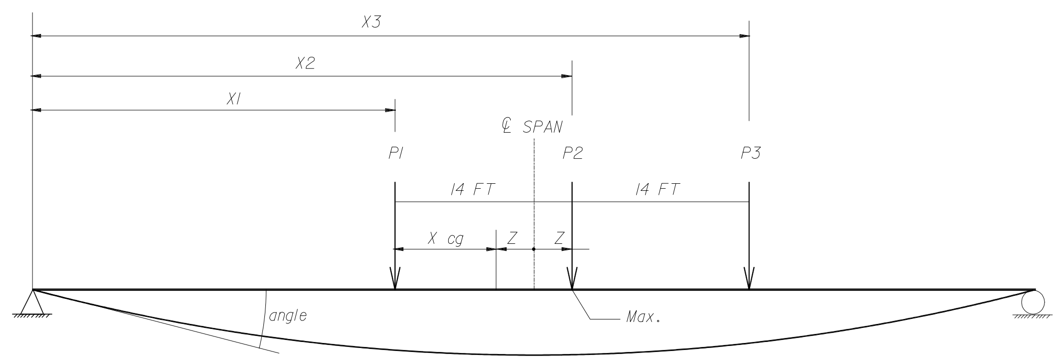
B. Maximum Live Load Moment, Reaction and Rotation

This section shows how to calculate the maximum live load moment, reaction (shear), and rotation. The formulas for rotation were obtained from Roark's Formulas for Stress and Strain by Warren C. Young, 6th Edition, McGraw-Hill.

B1. Maximum Live Load Rotation - One HL-93 vehicle

The rotations are calculated for one vehicle over the interior and exterior beams. The composite beam sections are used to calculate the stiffness ($E_{c.beam} \cdot I$) of the beams.

The maximum live load rotation in a simple span is calculated by positioning the axle loads of an HL-93 design truck in the following locations:



Maximum Live Load Moment and Rotation

Axle loads........................... $P1 := 32 \cdot kip$

$P2 := 32 \cdot kip$

$P3 := 8 \cdot kip$

Lane load............................ $w_L := 0.64 \cdot \frac{kip}{ft}$

Center of gravity for axle loads................................. $x_{cg} := \frac{P1 \cdot (0 \cdot ft) + P2 \cdot (14 \cdot ft) + P3 \cdot (28 \cdot ft)}{P1 + P2 + P3} = 9.333\ ft$

Distance from center of gravity for axle loads to centerline of span .... $z := \frac{14 \cdot ft - x_{cg}}{2} = 2.333\ ft$

Distance from left support to axle loads..........................

$$X_1 := \frac{L_{design}}{2} - z - x_{cg} = 32.208\,\mathrm{ft}$$

$$X_2 := X_1 + 14\cdot\mathrm{ft} = 46.208\,\mathrm{ft}$$

$$X_3 := X_1 + 28\cdot\mathrm{ft} = 60.208\,\mathrm{ft}$$

Interior Beam

Rotation induced by each axle load..................................

$$\Theta1 := \frac{P1\cdot X_1}{6\cdot\left(E_{c.beam}\cdot I_{Interior}\right)\cdot L_{design}}\cdot\left(2\cdot L_{design} - X_1\right)\cdot\left(L_{design} - X_1\right) = 0.075\cdot\mathrm{deg}$$

$$\Theta2 := \frac{P2\cdot X_2}{6\cdot\left(E_{c.beam}\cdot I_{Interior}\right)\cdot L_{design}}\cdot\left(2\cdot L_{design} - X_2\right)\cdot\left(L_{design} - X_2\right) = 0.072\cdot\mathrm{deg}$$

$$\Theta3 := \frac{P3\cdot X_3}{6\cdot\left(E_{c.beam}\cdot I_{Interior}\right)\cdot L_{design}}\cdot\left(2\cdot L_{design} - X_3\right)\cdot\left(L_{design} - X_3\right) = 0.014\cdot\mathrm{deg}$$

Rotation induced by HL-93 truck.................................

$$\Theta_{truck} := (\Theta1 + \Theta2 + \Theta3) = 0.161\cdot\mathrm{deg}$$

Rotation induced by lane load..................................

$$\Theta_{lane} := \frac{w_L\cdot L_{design}^{\;3}}{24\cdot\left(E_{c.beam}\cdot I_{Interior}\right)} = 0.086\cdot\mathrm{deg}$$

Rotation induced by HL-93 truck and lane load..............

$$\Theta_{LL.Interior} := \Theta_{truck} + \Theta_{lane} = 0.248\cdot\mathrm{deg}$$

Exterior Beam

Rotations induced by each axle load..................................

$$\Theta1 := \frac{P1\cdot X_1}{6\cdot\left(E_{c.beam}\cdot I_{Exterior}\right)\cdot L_{design}}\cdot\left(2\cdot L_{design} - X_1\right)\cdot\left(L_{design} - X_1\right) = 0.071\cdot\mathrm{deg}$$

$$\Theta2 := \frac{P2\cdot X_2}{6\cdot\left(E_{c.beam}\cdot I_{Exterior}\right)\cdot L_{design}}\cdot\left(2\cdot L_{design} - X_2\right)\cdot\left(L_{design} - X_2\right) = 0.069\cdot\mathrm{deg}$$

$$\Theta3 := \frac{P3\cdot X_3}{6\cdot\left(E_{c.beam}\cdot I_{Exterior}\right)\cdot L_{design}}\cdot\left(2\cdot L_{design} - X_3\right)\cdot\left(L_{design} - X_3\right) = 0.013\cdot\mathrm{deg}$$

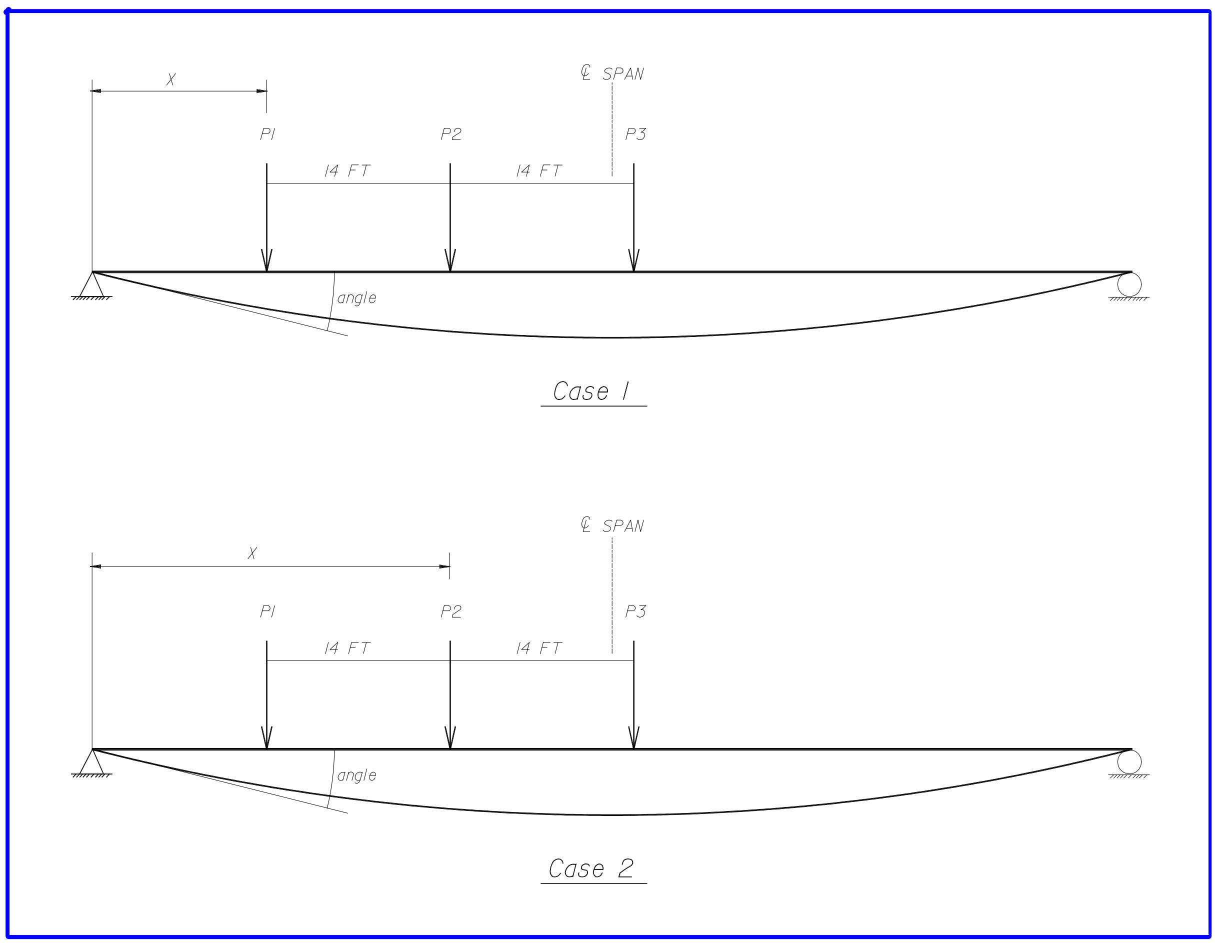
Rotation induced by HL-93 truck.................................. $\Theta_{truck} := (\Theta 1 + \Theta 2 + \Theta 3) = 0.153 \cdot deg$

Rotation induced by lane load. $\Theta_{lane} := \dfrac{w_L \cdot L_{design}^{\ 3}}{24 \cdot \left(E_{c.beam} \cdot I_{Exterior}\right)} = 0.082 \cdot deg$

Rotation induced by HL-93 truck and lane load.............. $\Theta_{LL.Exterior} := \Theta_{truck} + \Theta_{lane} = 0.236 \cdot deg$

B2. Live Load Moments and Shears - One HL-93 truck

The live load moments and shears in a simple span is calculated by positioning the axle loads of an HL-93 design truck in the following locations:



Case 1 HL-93 truck moment and shear:

$$M_{truck1}(x) := P1 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot x + P2 \cdot \frac{(L_{design} - x - 14 \cdot ft)}{L_{design}} \cdot x + P3 \cdot \frac{(L_{design} - x - 28 \cdot ft)}{L_{design}} \cdot x$$

$$V_{truck1}(x) := P1 \cdot \frac{(L_{design} - x)}{L_{design}} + P2 \cdot \frac{(L_{design} - x - 14 \cdot ft)}{L_{design}} + P3 \cdot \frac{(L_{design} - x - 28 \cdot ft)}{L_{design}}$$

Case 2 HL-93 truck moment and shear:

$$M_{truck2}(x) := P1 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot (x - 14 \cdot ft) + P2 \cdot \frac{(L_{design} - x)}{L_{design}} \cdot x + P3 \cdot \frac{(L_{design} - x - 14 \cdot ft)}{L_{design}} \cdot x$$

$$V_{truck2}(x) := P1 \cdot \frac{-(x - 14 \cdot ft)}{L_{design}} + P2 \cdot \frac{(L_{design} - x)}{L_{design}} + P3 \cdot \frac{(L_{design} - x - 14 \cdot ft)}{L_{design}}$$

Maximum moment and shear induced by the HL-93 truck...

$$M_{truck}(x) := \max(M_{truck1}(x), M_{truck2}(x))$$ *(**Note**: Choose maximum value)*

$$V_{truck}(x) := \max(V_{truck1}(x), V_{truck2}(x))$$

Moment and shear induced by the lane load.........................

$$M_{lane}(x) := \frac{w_L \cdot L_{design}}{2} \cdot x - \frac{w_L \cdot x^2}{2}$$

$$V_{lane}(x) := \frac{w_L \cdot L_{design}}{2} - w_L \cdot x$$

Live load moment and shear for HL-93 truck load (including impact) and lane load.............

$$M_{LLI}(x) := M_{truck}(x) \cdot IM + M_{lane}(x)$$

$$V_{LLI}(x) := V_{truck}(x) \cdot IM + V_{lane}(x)$$

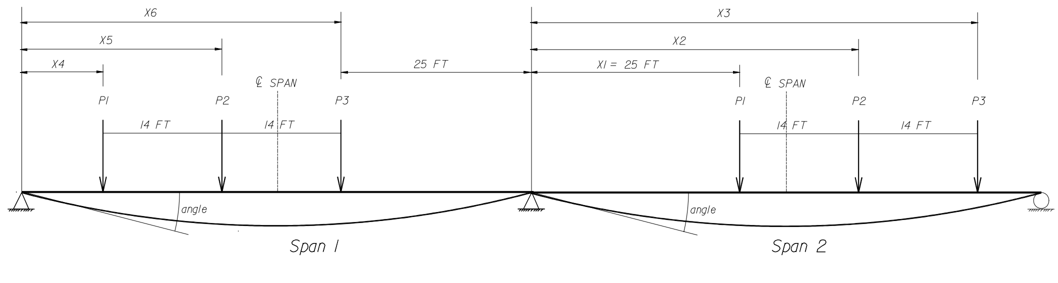
Live load reaction (without impact) ....

$$R_{LL}(x) := V_{truck}(x) + V_{lane}(x)$$

$R_{LL}(\text{Support}) = 92.4 \cdot kip$

B3. Maximum Live Load Reaction at Intermediate Pier - Two HL-93 vehicles

While two HL-93 vehicles controls in this design, the tandem and single truck with lane load needs to be investigated for other design span lengths. The maximum live load reaction at an intermediate pier is calculated by positioning the axle loads of an HL-93 design truck in the following locations:



Maximum Live Load Reaction at Intermediate Pier

Distance from left support of corresponding span to axle loads..................................

$X_1 := 25 \cdot ft$

$X_2 := X_1 + 14 \cdot ft = 39\,ft$

$X_3 := X_1 + 28 \cdot ft = 53\,ft$

$X_4 := L_{design} - 28 \cdot ft - 25 \cdot ft = 34.75\,ft$

$X_5 := X_4 + 14 \cdot ft = 48.75\,ft$

$X_6 := X_4 + 28 \cdot ft = 62.75\,ft$

Reaction induced by each axle load...................................

$$R_1 := \frac{P1}{L_{design}} \cdot \left[\left(L_{design} - X_1\right) + X_4\right] = 35.6 \cdot kip$$

$$R_2 := \frac{P2}{L_{design}} \cdot \left[\left(L_{design} - X_2\right) + X_5\right] = 35.6 \cdot kip$$

$$R_3 := \frac{P3}{L_{design}} \cdot \left[\left(L_{design} - X_3\right) + X_6\right] = 8.9 \cdot kip$$

Reaction induced by HL-93 trucks................................

$$R_{trucks} := \left(R_1 + R_2 + R_3\right) = 80 \cdot kip$$

Reaction induced by lane load on both spans................

$$R_{lanes} := \frac{w_L \cdot L_{span}}{2} \cdot (2) = 57.6 \cdot kip$$

Reaction induced by HL-93 truck and lane load..............

$$R_{LLs} := 90\% \cdot \left(R_{trucks} + R_{lanes}\right) = 123.8 \cdot kip$$

Reaction induced by HL-93 truck (including impact factor) and lane load.......................

$$R_{LLIs} := 90\% \cdot \left(R_{trucks} \cdot IM + R_{lanes}\right) = 147.6 \cdot kip$$

B4. Summary

DESIGN LIVE LOAD	Support	ShrChk	Debond1	Debond2	Midspan
Load/Location, x (ft)=	**0.0**	**3.2**	**10.0**	**20.0**	**43.9**
MOMENTS: INTERIOR BEAM					
Live load + DLA	0.0	349.2	995.4	1708.6	2344.3
Distribution Factor	0.773	0.773	0.773	0.773	0.773
Design Live Load + DLA Moment	**0.0**	**270.1**	**769.8**	**1321.3**	**1812.9**
MOMENTS: EXTERIOR BEAM					
Live load + DLA	0.0	349.2	995.4	1708.6	2344.3
Distribution Factor	0.960	0.960	0.960	0.960	0.960
Design Live Load + DLA Moment	**0.0**	**335.3**	**955.6**	**1640.2**	**2250.6**
SHEARS: INTERIOR BEAM					
Live load + DLA	113.7	108.1	96.3	79.0	37.7
Distribution Factor	1.012	1.012	1.012	1.012	1.012
Design Live Load + DLA Shear	**115.1**	**109.4**	**97.5**	**80.0**	**38.2**
SHEARS: EXTERIOR BEAM					
Live load + DLA	113.7	108.1	96.3	79.0	37.7
Distribution Factor	1.021	1.021	1.021	1.021	1.021
Design Live Load + DLA Shear	**116.1**	**110.4**	**98.4**	**80.7**	**38.5**

LL ROTATIONS (BRG PADS)	**Interior Beam**	**Exterior Beam**
Live load w/o DLA	0.00432	0.00411
Distribution Factor	0.773	0.960
Design Live Load Rotation	**0.00334**	**0.00395**

LL REACTIONS (BRG PADS)	**Interior Beam**	**Exterior Beam**
Live load w/o DLA	92.4	92.4
Distribution Factor	1.012	1.021
Design Live Load Reactions	**93.6**	**94.4**

1 HL-93 REACTION	**w/o DLA**	**w/ DLA**
Pier/End Bent (1 Truck)	**92.4**	**113.7**
Pier (2 Trucks)	**123.8**	**147.6**

Redefine Variables

Prestressed Beam Design - Part I

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\203LiveLoads.xmcd(R)

Description

This section provides the design of the prestressed concrete beam - interior beam design.

Page Contents

LRFD Criteria

STRENGTH I - Basic load combination relating to the normal vehicular use of the bridge without wind.

$WA = 0$ For superstructure design, water load and stream pressure are not applicable.

$FR = 0$ No friction forces.

$TU = 0$ No uniform temperature load effects due to simple spans. Movements are unrestrained.

CR, SH These effects are accounted during the design of the prestressed strands with a factor of 1.0 for all Limit States $1.0 \cdot (CR + SH)$.

$$Strength1 = 1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL$$

STRENGTH II - Load combination relating to the use of the bridge by Owner-specified special design vehicles, evaluation permit vehicles, or both without wind.

"The FL120 permit vehicle is not evaluated in this design example"

SERVICE I - Load combination relating to the normal operational use of the bridge with a 55 MPH wind and all loads taken at their nominal values.

$BR, WL = 0$ For prestressed beam design, braking forces and wind on live load are negligible.

$$Service1 = 1.0 \cdot DC + 1.0 \cdot DW + 1.0 \cdot LL + 1.0 \cdot (CR + SH)$$

"Applicable for maximum compressive stresses in beam ONLY. For tension, see Service III."

SERVICE III - Load combination for longitudinal analysis relating only to tension in prestressed concrete structures with the objective of crack control.

$$Service3 = 1.0 \cdot DC + 1.0 \cdot DW + 0.8 \cdot LL + 1.0 \cdot (CR + SH)$$

"Applicable for maximum tension at midspan ONLY. For compression, see Service I."

A. Input Variables

A1. Bridge Geometry

Overall bridge length....... $L_{bridge} = 180\ ft$

Design span length......... $L_{span} = 90\ ft$

Skew angle................... $Skew = -20 \cdot deg$

A2. Section Properties

NON-COMPOSITE PROPERTIES			FIB-36
Moment of Inertia	[in^4]	I_{nc}	127545
Section Area	[in^2]	A_{nc}	807
ytop	[in]	yt_{nc}	19.51
ybot	[in]	yb_{nc}	16.49
Depth	[in]	h_{nc}	36
Top flange width	[in]	b_{tf}	48
Top flange depth	[in]	h_{tf}	3.5
Width of web	[in]	b_w	7
Bottom flange width	[in]	b_{bf}	38
Bottom flange depth	[in]	h_{bf}	7
Bottom flange taper	[in]	E	15.5
Section Modulus top	[in^3]	S_{tnc}	6537
Section Modulus bottom	[in^3]	S_{bnc}	7735

COMPOSITE SECTION PROPERTIES			INTERIOR	EXTERIOR
Effective slab width	[in]	$b_{eff.interior/exterior}$	120.0	139.4
Transformed slab width	[in]	$b_{tr.interior/exterior}$	87.3	101.5
Height of composite section	[in]	h	45.0	45.0
Effective slab area	[in^2]	A_{slab}	698.5	811.6
Area of composite section	[in^2]	$A_{Interior/Exterior}$	1553.5	1666.6
Neutral axis to bottom fiber	[in]	y_b	28.1	29.0
Neutral axis to top fiber	[in]	y_t	16.9	16.0
Inertia of composite section	[in^4]	$I_{Interior/Exterior}$	359675.0	377743.8
Section modulus top of slab	[in^3]	S_t	21318.8	23612.2
Section modulus top of beam	[in^3]	S_{tb}	45694.6	53980.3
Section modulus bottom of beam	[in^3]	S_b	12786.8	13024.7

A3. Superstructure Loads at Midspan

DC Moment of Beam at Release............... $M_{RelBeam} := M_{RelBeamInt}(Midspan) = 833.7 \cdot kip \cdot ft$

DC Moment of Beam.............................. $M_{Beam} := M_{BeamInt}(Midspan) = 809.1 \cdot kip \cdot ft$

DC Moment of Slab............................... $M_{Slab} := M_{SlabInt}(Midspan) = 1070.8 \cdot kip \cdot ft$

DC Moment of stay-in-place forms.......... $M_{Forms} := M_{FormsInt}(Midspan) = 115.5 \cdot kip \cdot ft$

DC Moment of traffic railing barriers........ $M_{Trb} := M_{TrbInt}(Midspan) = 141.7 \cdot kip \cdot ft$

DW Moment of future wearing surface.... $M_{Fws} := M_{FwsInt}(Midspan) = 0 \cdot kip \cdot ft$

DW Moment of Utilities.......................... $M_{Utility} := M_{UtilityInt}(Midspan) = 0 \cdot kip \cdot ft$

Live Load Moment................................ $M_{LLI} := M_{LLI.Interior}(Midspan) = 1812.9 \cdot kip \cdot ft$

$Service1 = 1.0 \cdot DC + 1.0 \cdot DW + 1.0 \cdot LL$

- Service I Limit State.........................

$$M_{Srv1} := 1.0 \cdot \left(M_{Beam} + M_{Slab} + M_{Forms} + M_{Trb}\right) + \left[1.0 \cdot \left(M_{Fws} + M_{Utility}\right) + 1.0 \cdot \left(M_{LLI}\right)\right] = 3950 \cdot kip \cdot ft$$

$Service3 = 1.0 \cdot DC + 1.0 \cdot DW + 0.8 \cdot LL$

- Service III Limit State.......................

$$M_{Srv3} := 1.0 \cdot \left(M_{Beam} + M_{Slab} + M_{Forms} + M_{Trb}\right) + \left[1.0 \cdot \left(M_{Fws} + M_{Utility}\right) + 0.8 \cdot \left(M_{LLI}\right)\right] = 3587.4 \cdot kip \cdot ft$$

$Strength1 = 1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL$

- Strength I Limit State........................

$$M_r := 1.25 \cdot \left(M_{Beam} + M_{Slab} + M_{Forms} + M_{Trb}\right) + \left[1.50 \cdot \left(M_{Fws} + M_{Utility}\right) + 1.75 \cdot \left(M_{LLI}\right)\right] = 5844 \cdot kip \cdot ft$$

A4. Superstructure Loads at Debonding Locations

DC Moment of Beam at Release -
$Debond1 = 10\,ft$ Location........................ $M_{RelBeamD1} := M_{RelBeamInt}(Debond1) = 332.4 \cdot kip \cdot ft$

DC Moment of Beam at Release -
$Debond2 = 20\,ft$ Location........................ $M_{RelBeamD2} := M_{RelBeamInt}(Debond2) = 580.7 \cdot kip \cdot ft$

A5. Superstructure Loads at the Other Locations

At Support location

DC Shear & Moment........................ $V_{DC.BeamInt}(Support) = 97.4 \cdot kip$ $M_{DC.BeamInt}(Support) = 0\,ft \cdot kip$

DW Shear & Moment .... $V_{DW.BeamInt}(Support) = 0 \cdot kip$ $M_{DW.BeamInt}(Support) = 0\,ft \cdot kip$

LL Shear & Moment.. .... $V_{LLI.Interior}(Support) = 115.1 \cdot kip$ $M_{LLI.Interior}(Support) = 0\,ft \cdot kip$

$Strength1 = 1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL$

- Strength I Limit State.......................

$$V_{u.Support} := 1.25 \cdot V_{DC.BeamInt}(Support) + \left[1.50 \cdot \left(V_{DW.BeamInt}(Support)\right) + 1.75 \cdot \left(V_{LLI.Interior}(Support)\right)\right] = 323.1 \cdot ki$$

At Shear Check location

DC Shear & Moment........................ $V_{DC.BeamInt}(ShearChk) = 90.3 \cdot kip$ $M_{DC.BeamInt}(ShearChk) = 300.4\,ft \cdot kip$

DW Shear & Moment .... $V_{DW.BeamInt}(ShearChk) = 0 \cdot kip$ $M_{DW.BeamInt}(ShearChk) = 0\,ft \cdot kip$

LL Shear & Moment.. .... $V_{LLI.Interior}(ShearChk) = 109.4 \cdot kip$ $M_{LLI.Interior}(ShearChk) = 270.1\,ft \cdot kip$

$Strength1 = 1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL$

- Strength I Limit State.......................

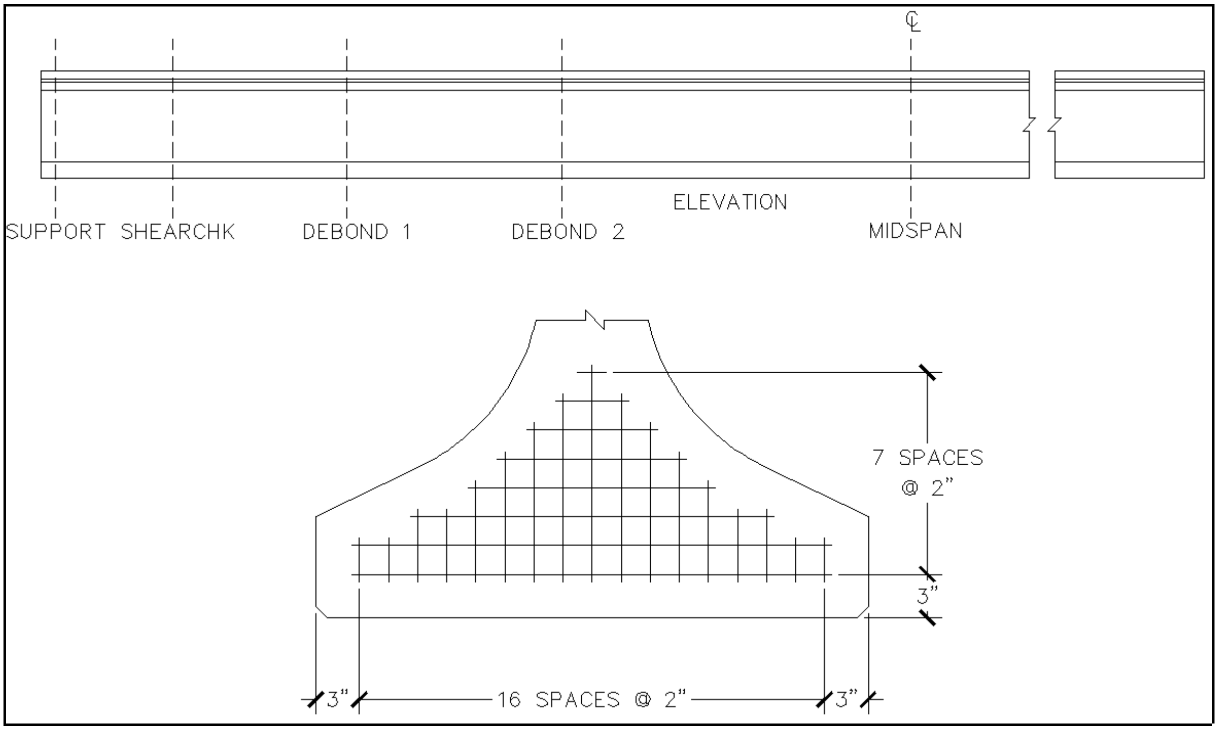
$$V_u := 1.25 \cdot V_{DC.BeamInt}(ShearChk) + 1.50 \cdot V_{DW.BeamInt}(ShearChk) + 1.75 \cdot V_{LLI.Interior}(ShearChk) = 304.4 \cdot kip$$

$$M_{shr} := 1.25 \cdot M_{DC.BeamInt}(ShearChk) + 1.50 \cdot M_{DW.BeamInt}(ShearChk) + 1.75 \cdot M_{LLI.Interior}(ShearChk) = 848.1\,ft \cdot ki$$

B. Interior Beam Midspan Moment Design

B1. Strand Pattern definition at Midspan

Using the following schematic, the proposed strand pattern at the midspan section can be defined.



STRAND PATTERN DEFINITIONS AND BEAM LOCATIONS

$Support = 0$ $\quad ShearChk = 3.2\,ft$ $\quad Debond1 = 10\,ft$ $\quad Debond2 = 20\,ft$ $\quad Midspan = 43.88\,ft$

Strand pattern at midspan

Strand type.................... $strand_{type} := \text{"LowLax"}$ (***Note***: *Options* ("LowLax" "StressRelieved"))

Strand size..................... $strand_{dia} := 0.6 \cdot in$ (***Note***: *Options* $(0.5 \cdot in \quad 0.5625 \cdot in \quad 0.6 \cdot in)$)

Strand area....................

$$StrandArea := \left(\begin{array}{|l} 0.153 \text{ if } strand_{dia} = 0.5 \cdot in \\ 0.192 \text{ if } strand_{dia} = 0.5625 \cdot in \\ 0.217 \text{ if } strand_{dia} = 0.6 \cdot in \\ 0.0 \text{ otherwise} \end{array} \right) \cdot in^2 = 0.217 \cdot in^2$$

Define the number of strands and eccentricity of strands from bottom of beam......

BeamType = "FIB-36"

MIDSPAN Strand Pattern Data			
Rows of strand from bottom of beam	Input (inches)	Number of strands per row	MIDSPAN
y9 =	19	n9 =	0
y8 =	17	n8 =	0
y7 =	15	n7 =	0
y6 =	13	n6 =	0
y5 =	11	n5 =	0
y4 =	9	n4 =	0
y3 =	7	n3 =	5
y2 =	5	n2 =	17
y1 =	3	n1 =	17
Strand c.g. =	4.38	Total strands =	39

Area of prestressing steel........................ $A_{ps.midspan} := \left(strands_{total} \cdot StrandArea\right) = 8.5 \cdot in^2$

Transformed section properties

SDG 4.3.1-C6 states: *When calculating the service limit state capacity for pretensioned concrete flat slabs and girders, use the transformed section properties as follows: at strand transfer; for calculation of prestress losses; for live load application.*

Modular ratio between the prestressing strand and beam. .... $n_p := \dfrac{E_p}{E_{c.beam}} = 5.961$

Non-composite area transformed............ $A_{nc.tr} := A_{nc} + \left(n_p - 1\right) \cdot A_{ps.midspan} = 849 \cdot in^2$

Non-composite neutral axis transformed... $yb_{nc.tr} := \dfrac{yb_{nc} \cdot A_{nc} + strand_{cg} \cdot in \cdot \left[\left(n_p - 1\right) \cdot A_{ps.midspan}\right]}{A_{nc.tr}} = 15.9 \cdot in$

Non-composite inertia transformed... $I_{nc.tr} := I_{nc} + \left(yb_{nc.tr} - strand_{cg} \cdot in\right)^2 \cdot \left[\left(n_p - 1\right) \cdot A_{ps.midspan}\right] = 133104.1 \cdot in^4$

Non-composite section modulus top......... $S_{topnc.tr} := \dfrac{I_{nc.tr}}{h_{nc} - yb_{nc.tr}} = 6619.2 \cdot in^3$

Non-composite section modulus bottom.... $S_{botnc.tr} := \dfrac{I_{nc.tr}}{yb_{nc.tr}} = 8375.9 \cdot in^3$

Modular ratio between the mild reinforcing and transformed concrete beam..............

$$n_m := \frac{E_s}{E_{c.beam}} = 6.066$$

Assumed area of reinforcement in deck slab per foot width of deck slab..............

$$A_{deck.rebar} := 0.62 \cdot \frac{in^2}{ft}$$

*(**Note**: Assuming #5 at 12" spacing, top and bottom longitudinally).*

Distance from bottom of beam to rebar....

$$y_{bar} := h - t_{mill} - \frac{t_{slab}}{2} = 40.5 \cdot in$$

Total reinforcing steel within effective width of deck slab................................

$$A_{bar} := b_{eff.interior} \cdot A_{deck.rebar} = 6.2 \cdot in^2$$

Composite area transformed...................

$$A_{tr} := A_{Interior} + (n_p - 1) \cdot A_{ps.midspan} + (n_m - 1) \cdot A_{bar} = 1626.9 \cdot in^2$$

Composite neutral axis transformed.......

$$y_{b.tr} := \frac{y_b \cdot A_{Interior} + strand_{cg} \cdot in \cdot (n_p - 1) \cdot A_{ps.midspan} + y_{bar} \cdot (n_m - 1) \cdot A_{bar}}{A_{tr}} = 27.8 \cdot in$$

Composite inertia transformed.......

$$I_{tr} := I_{Interior} + (y_{b.tr} - strand_{cg} \cdot in)^2 \cdot (n_p - 1) \cdot A_{ps.midspan} + (y_{b.tr} - y_{bar})^2 \cdot (n_m - 1) \cdot A_{bar} = 387708 \cdot in^4$$

Composite section modulus top of slab..

$$S_{slab.tr} := \frac{I_{tr}}{h - y_{b.tr}} = 22482 \cdot in^3$$

Composite section modulus top of beam..

$$S_{top.tr} := \frac{I_{tr}}{h - y_{b.tr} - t_{slab} - t_{mill} - h_{buildup}} = 50057.7 \cdot in^3$$

Composite section modulus bottom of beam..

$$S_{bot.tr} := \frac{I_{tr}}{y_{b.tr}} = 13969.1 \cdot in^3$$

Eccentricity of strands at midspan for composite section................................

$$e_{cg.tr} := y_{b.tr} - strand_{cg} \cdot in = 23.4 \cdot in$$

B2. Prestressing Losses [LRFD 5.9.5]

For prestressing members, the total loss, Δf_{pT}, is expressed as:

$$\Delta f_{pT} = \Delta f_{pLT}$$

where... long-term loss shrinkage and creep for concrete, and relaxation of the steel....... Δf_{pLT}

Loss due to elastic shortening is not included in the total loss equation due to the use of transformed section properties.

Initial Stresses in Strands

Specified yield strength of the prestressing steel [**LRFD 5.4.4.1**]............................. $f_{py} := \begin{cases} (0.85 \cdot f_{pu}) & \text{if } strand_{type} = \text{"StressRelieved"} \\ (0.90 \cdot f_{pu}) & \text{if } strand_{type} = \text{"LowLax"} \end{cases} = 243 \cdot ksi$

Jacking stress [**LRFD 5.9.3**]................ $f_{pj} := \begin{cases} (0.70 \cdot f_{pu}) & \text{if } strand_{type} = \text{"StressRelieved"} \\ (0.75 \cdot f_{pu}) & \text{if } strand_{type} = \text{"LowLax"} \end{cases} = 202.5 \cdot ksi$

Elastic Shortening

When calculating concrete stresses using transformed section properties, the effects of losses and gains due to elastic deformations are implicitly accounted for and Δf_{pES} should not be included in the prestressing force applied to the transformed section at transfer. However, the elastic shortening loss is needed for calculation of the stress in prestressing and relaxation of the prestressing strands. The loss due to elastic shortening in pretensioned members shall be taken as:

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} \cdot f_{cgp}$$

where...

Modulus of elasticity of concrete at transfer of prestress force...................................... $E_{ci.beam} = 4016.8 \cdot ksi$

Modulus elasticity of prestressing steel..... $E_p = 28500 \cdot ksi$

Eccentricity of strands at midspan for non-composite section........................... $e_{cg.nc.tr} := yb_{nc.tr} - strand_{cg} \cdot in = 11.5 \cdot in$

Corresponding total prestressing force...... $F_{ps} := A_{ps} \cdot f_{pj} = 1713.8 \cdot kip$

Concrete stresses at c.g. of the prestressing force at transfer and the self wt of the beam at maximum moment location.................

$$f_{cgp} := \frac{F_{ps}}{A_{nc.tr}} + \frac{F_{ps} \cdot e_{cg.nc.tr}^{2}}{I_{nc.tr}} - \frac{M_{RelBeam} \cdot e_{cg.nc.tr}}{I_{nc.tr}} = 2.9 \cdot ksi$$

Losses due to elastic shortening..............

$$\Delta f_{pES} := \frac{E_p}{E_{ci.beam}} \cdot f_{cgp} = 20.3 \cdot ksi$$

Prestressing stress at transfer.................

$$f_{pt} := \text{if}\left(f_{pj} - \Delta f_{pES} \geq 0.55 f_{py}, f_{pj} - \Delta f_{pES}, 0.55 f_{py}\right) = 182.2 \cdot ksi$$

<u>Time-Dependent Losses - </u> *<u>Approximate Estimate</u>* LRFD 5.9.5.3

Long-term prestress loss due to creep of concrete, shrinkage of concrete, and relaxation of steel:

$$\Delta f_{pLT} = 10.0 \cdot \frac{f_{pj} \cdot A_{ps}}{A_{nc.tr}} \cdot \gamma_h \cdot \gamma_{st} + 12.0 \cdot \gamma_h \cdot \gamma_{st} + \Delta f_{pR}$$

where...

$$\gamma_h := 1.7 - 0.01 \cdot H = 1$$

$$\gamma_{st} := \frac{5}{1 + \frac{f_{ci.beam}}{ksi}} = 0.7$$

$$\Delta f_{pR} := \left| \begin{array}{l} 2.4ksi \ \text{ if } \ strand_{type} = \text{"LowLax"} \\ (10.0ksi) \ \text{ if } \ strand_{type} = \text{"StressRelieved"} \end{array} \right. = 2.4 \cdot ksi$$

$$\Delta f_{pLT} := 10.0 \cdot \frac{f_{pj} \cdot A_{ps}}{A_{nc.tr}} \cdot \gamma_h \cdot \gamma_{st} + 12.0ksi \cdot \gamma_h \cdot \gamma_{st} + \Delta f_{pR} = 24.2 \cdot ksi$$

Total Prestress Loss - *Approximate Estimate*

The total loss, Δf_{pT_0}, is expressed as........ $\Delta f_{pT_0} := \Delta f_{pLT} = 24.2 \cdot ksi$

Percent loss of strand force.................... $Loss := \frac{\Delta f_{pT_0}}{f_{pj}} = 12 \cdot \%$

Time-Dependent Losses - *Refined Estimates* LRFD 5.9.5.4

Long-term prestress loss due to creep of concrete, shrinkage of concrete, and relaxation of steel:

$$\Delta f_{pLT} = \left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right)_{id} + \left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS}\right)_{df}$$

where...

Shrinkage$_{id}$

$$\Delta f_{pSR} = \varepsilon_{bid} \cdot E_p \cdot K_{id}$$

where...

$$\varphi_{b.fi} := 1.9 \cdot k_{vs.g} \cdot k_{hc.g} \cdot k_{f.g} \cdot k_{td.fi} \cdot T_0^{-0.118} = 1.3 \qquad \varphi_{d.fd} := 1.9 \cdot k_{vs.d} \cdot k_{hc.d} \cdot k_{f.d} \cdot k_{td.d.fd} \cdot T_0^{-0.118} = 1.7$$

$$\varphi_{b.di} := 1.9 \cdot k_{vs.g} \cdot k_{hc.g} \cdot k_{f.g} \cdot k_{td.di} \cdot T_0^{-0.118} = 1$$

$$\varphi_{b.fd} := 1.9 \cdot k_{vs.g} \cdot k_{hc.g} \cdot k_{f.g} \cdot k_{td.fd} \cdot T_1^{-0.118} = 0.7$$

Shrinkage strain

$$\varepsilon_{bid} := k_{vs.g} \cdot k_{hs} \cdot k_{f.g} \cdot k_{td.di} \cdot 0.00048 = 0.000248$$

Transformed section coefficient

$$K_{id} := \frac{1}{1 + \frac{E_p}{E_{ci.beam}} \cdot \frac{A_{ps}}{A_{nc.tr}} \cdot \left(1 + \frac{A_{nc.tr} \cdot e_{cg.nc.tr}^{\,2}}{I_{nc.tr}}\right) \cdot \left(1 + 0.7 \cdot \varphi_{b.fi}\right)} = 0.8$$

Losses due to shrinkage of girder between time of transfer and deck placement

$$\Delta f_{pSR} := \varepsilon_{bid} \cdot E_p \cdot K_{id} = 5.7 \cdot ksi$$

Creep$_{id}$

Losses due to creep of girder between time of transfer and deck placement

$$\Delta f_{pCR} := \frac{E_p}{E_{ci.beam}} \cdot f_{cgp} \cdot \varphi_{b.di} \cdot K_{id} = 16.1 \cdot ksi$$

Relaxation of Prestressing Strands$_{id}$

$$K_L := \begin{cases} 30 & \text{if } strand_{type} = \text{"LowLax"} \\ 7 & \text{otherwise} \end{cases} = 30$$

Per AASHTO LRFD 5.9.5.4.2c Δf_{pR1} may be assumed to equal 1.2 ksi for low-relaxation strands.

Losses due to relaxation of prestressing strands between time of transfer and deck placement

$$\Delta f_{pR1} := \begin{cases} \frac{f_{pt}}{K_L} \cdot \left(\frac{f_{pt}}{f_{py}} - 0.55\right) & \text{if } strand_{type} = \text{"StressRelieved"} \\ 1.2ksi & \text{if } strand_{type} = \text{"LowLax"} \end{cases} = 1.2 \cdot ksi$$

Total Time-Dependent Losses - *Between time of transfer and deck placement*

$\Delta f_{pLT.id} := \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1} = 23 \cdot ksi$

Shrinkage$_{df}$

$\Delta f_{pSD} = \varepsilon_{bdf} \cdot E_p \cdot K_{df}$

Shrinkage Strain

$\varepsilon_{bdf} := k_{vs.g} \cdot k_{hs} \cdot k_{f.g} \cdot k_{td.fd} \cdot 0.00048 = 0.000324$

Transformed section coefficient

$$K_{df} := \frac{1}{1 + \frac{E_p}{E_{ci.beam}} \cdot \frac{A_{ps}}{A_{tr}} \cdot \left(1 + \frac{A_{tr} \cdot e_{cg.tr}^{\ 2}}{I_{tr}}\right) \cdot \left(1 + 0.7 \cdot \varphi_{b.fi}\right)} = 0.812$$

Losses due to shrinkage of girder between deck placement and final time

$\Delta f_{pSD} := \varepsilon_{bdf} \cdot E_p \cdot K_{df} = 7.5 \cdot ksi$

Creep$_{df}$

$$\Delta f_{pCD} = \frac{E_p}{E_{ci.beam}} \cdot f_{cgp} \cdot \left(\varphi_{fi} - \varphi_{di}\right) \cdot K_{df} + \frac{E_p}{E_{c.beam}} \cdot \Delta f_{cd} \cdot \varphi_{fd} \cdot K_{df}$$

Permanent load moments at midspan acting on non-composite section (except beam at transfer)...

$M_{nc} := M_{Slab} + M_{Forms}$

$M_{nc} = 1186.3 \cdot kip \cdot ft$

Permanent load moments at midspan acting on composite section.............................

$M := M_{Trb} + M_{Fws} + M_{Utility}$

$M = 141.7 \cdot kip \cdot ft$

Loss in the strands

$P_{loss} := A_{ps} \cdot \Delta f_{pLT.id} = 194.7 \cdot kip$

Change in concrete stress at centroid of prestressing strands due to long-term losses between transfer and deck placement

$$\Delta f_{cd} := \frac{-P_{loss}}{A_{nc.tr}} - \frac{P_{loss} \cdot \left(e_{cg.nc.tr}\right)^2}{I_{nc.tr}} - \frac{M_{nc} \cdot e_{cg.nc.tr}}{I_{nc.tr}} - \frac{M \cdot e_{cg.tr}}{I_{tr}} = -1.8 \cdot ksi$$

Losses due to creep between deck placement and final time

$$\Delta f_{pCD} := \frac{E_p}{E_{ci.beam}} \cdot f_{cgp} \cdot \left(\varphi_{b.fi} - \varphi_{b.di}\right) \cdot K_{df} + \frac{E_p}{E_{c.beam}} \cdot \Delta f_{cd} \cdot \varphi_{b.fd} \cdot K_{df} = -1.3 \cdot ksi$$

Relaxation$_{df}$

Losses due to relaxation of prestressing strands in composite section between time of deck placement and final time

$$\Delta f_{pR2} := \Delta f_{pR1} = 1.2 \cdot ksi$$

Shrinkage of Decking$_{df}$

$$\Delta f_{pSS} = \frac{E_p}{E_{c.beam}} \cdot \Delta f_{cdf} \cdot K_{df} \cdot \left(1 + 0.7 \cdot \varphi_{b.fd}\right)$$ where...

Shrinkage Strain

$$\varepsilon_{ddf} := k_{vs.d} \cdot k_{hs} \cdot k_{f.d} \cdot k_{td.fd} \cdot 0.00048 = 0.000413$$

Eccentricity of deck with respect to gross composite section

$$e_d := y_t - \frac{t_{slab}}{2} = 12.9 \cdot in$$

Change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete

$$\Delta f_{cdf} := \frac{\varepsilon_{ddf} \cdot b_{eff.interior} \cdot t_{slab} \cdot E_{c.slab}}{\left(1 + 0.7 \cdot \varphi_{d.fd}\right)} \cdot \left(\frac{1}{A_{tr}} - \frac{e_{cg.tr} \cdot e_d}{I_{tr}}\right) = -0.1 \cdot ksi$$

Losses due to shrinkage of deck composite section

$$\Delta f_{pSS} := \frac{E_p}{E_{c.beam}} \cdot \Delta f_{cdf} \cdot K_{df} \cdot \left(1 + 0.7 \cdot \varphi_{b.fd}\right) = -0.8 \cdot ksi$$

Total Time-Dependent Losses - *Between deck placement and final time*

$$\Delta f_{pLT.df} := \Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS} = 8.2 \cdot ksi$$

Total Time-Dependent Losses

$$\Delta f_{pLT} := \Delta f_{pLT.id} + \Delta f_{pLT.df} = 31.2 \cdot ksi$$

Total Prestress Loss - *Refined Estimes*

The total loss, Δf_{pT_1}, is expressed as.....

$$\Delta f_{pT_1} := \Delta f_{pLT} = 31.2 \cdot ksi$$

Percent loss of strand force....................

$$Loss := \frac{\Delta f_{pT_1}}{f_{pj}} = 15.4 \cdot \%$$

Per SDG 4.3.1.C.6, use the Approximate Estimate of Time-Dependent Losses for precast, pretensioned, normal weight concrete members designed as simply supported beams (typical condition). For all other members use the Refined Estimates of Time-Dependent Losses.

Conditions := "Approximate Estimate"

(If refined method is preferred type in Refined Estimate" under conditions.)

Total Prestress Loss

Stress Summary Table		
Approximate Estimate		
Prestressing Stress Prior to Transfer	f_{pj}	202.50 ksi
Prestressing Stress at Transfer	f_{pt}	182.22 ksi
Loss Due to Relaxation	Δf_{pR}	2.40 ksi
Total Prestressing Loss	Δf_{pT}	24.24 ksi
Loss		11.97 %

B3. Stress Limits (*Compression* = +, *Tension* = -)

Initial Stresses [SDG 4.3]

Limit of tension in top of beam at release (straight strand only)

Outer 15 percent of design beam........ $f_{top.outer15} = -0.93 \cdot ksi$

Center 70 percent of design beam...... $f_{top.center70} = -0.2 \cdot ksi$

Compressive concrete strength at release.. $f_{ci.beam} = 6 \cdot ksi$

Limit of compressive concrete strength at release... $0.6 \cdot f_{ci.beam} = 3.6 \cdot ksi$

The actual stress in strand after losses at transfer have occured............................ $f_{pe} := f_{pj} = 202.5 \cdot ksi$

Calculate the stress at the top and bottom of beam at release:

Total force of strands............................ $F_{pe} := f_{pe} \cdot A_{ps} = 1713.8 \cdot kip$

Stress at top of beam at center 70%.........
$$\sigma_{pjTop70} := \frac{M_{RelBeam}}{S_{topnc.tr}} + \left(\frac{F_{pe}}{A_{nc.tr}} - \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{topnc.tr}}\right) = 0.551 \cdot ksi$$

Stress at bottom of beam at center 70%... $\sigma_{pjBotBeam} := \frac{-M_{RelBeam}}{S_{botnc.tr}} + \left(\frac{F_{pe}}{A_{nc.tr}} + \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{botnc.tr}}\right) = 3.179 \cdot ksi$

$$Top70Release := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjTop70} \le 0 \cdot ksi \wedge \sigma_{pjTop70} \ge f_{top.center70} \\ \text{"OK"} & \text{if } \sigma_{pjTop70} > 0 \cdot ksi \wedge \sigma_{pjTop70} \le 0.6f_{ci.beam} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $f_{top.center70} = -0.2 \cdot ksi$

where $0.6f_{ci.beam} = 3.6 \cdot ksi$

$Top70Release = \text{"OK"}$

$$BotRelease := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjBotBeam} \le 0.6f_{ci.beam} \\ \text{"NG"} & \text{if } \sigma_{pjBotBeam} \le 0 \cdot ksi \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $0.6f_{ci.beam} = 3.6 \cdot ksi$

$BotRelease = \text{"OK"}$

*(**Note**: Some Mathcad equation explanations-*

- *The check for the top beam stresses checks to see if tension is present,* $\sigma_{pjTop70} \le 0 \cdot ksi$*, and then applies the proper allowable. A separate line is used for the compression and tension allowables. The last line,* "NG" otherwise*, is a catch-all statement such that if the actual stress is not within the allowables, it is considered "NG".)*
- *For the bottom beam, the first line,* $\sigma_{pjBotBeam} \le 0.6f_{ci.beam}$*, checks that the allowable compression is not exceeded. The second line assures that no tension is present, if there is then the variable will be set to "NG." The catch-all statement,* "NG" otherwise*, will be ignored since the first line was satisfied. If the stress were to exceed the allowable, neither of the first two lines will be satisfied therefore the last line would produce the answer of "NG."*

<u>Final Stresses [LRFD Table 5.9.4.2.1-1 & 5.9.4.2.2-1]</u>

(1) Sum of effective prestress and permanent loads

Limit of compression in slab............. $f_{allow1.TopSlab} := 0.45 \cdot f_{c.slab} = 2.025 \cdot ksi$

Limit of compression in top of beam.. $f_{allow1.TopBeam} := 0.45 \cdot f_{c.beam} = 3.825 \cdot ksi$

(2) Sum of effective prestress, permanent loads and transient loads

*(**Note**: The engineer is reminded that this check needs to be made also for stresses during shipping and handling. For purposes of this design example, this calculation is omitted).*

Limit of compression in slab............. $f_{allow2.TopSlab} := 0.60 \cdot f_{c.slab} = 2.7 \cdot ksi$

Limit of compression in top of beam.. $f_{allow2.TopBeam} := 0.60 \cdot f_{c.beam} = 5.1 \cdot ksi$

(3) Tension at bottom of beam only

Limit of tension in bottom of beam.....

$$f_{allow3.BotBeam} := \begin{cases} \left(-0.0948\sqrt{f_{c.beam} \cdot ksi}\right) & \text{if } Environment_{super} = \text{"Extremely"} \\ \left(-0.19\sqrt{f_{c.beam} \cdot ksi}\right) & \text{otherwise} \end{cases} = -0.554 \cdot ksi$$

*(**Note**: For not worse than moderate corrosion conditions.)* $Environment_{super} = \text{"Slightly"}$

B4. Service I and III Limit States

At service, check the stresses of the beam for compression and tension. In addition, the forces in the strands after losses need to be checked.

The actual stress in strand after all losses have occured.. $f_{pe} := f_{pj} - \Delta f_{pT} = 178.3 \cdot ksi$

Allowable stress in strand after all losses have occured.. $f_{pe.Allow} := 0.80 \cdot f_{py} = 194.4 \cdot ksi$

$$LRFD_{5.9.3} := \begin{cases} \text{"OK, stress at service after losses satisfied"} & \text{if } f_{pe} \le f_{pe.Allow} \\ \text{"NG, stress at service after losses not satisfied"} & \text{otherwise} \end{cases}$$

$LRFD_{5.9.3} = \text{"OK, stress at service after losses satisfied"}$

Calculate the stress due to prestress at the top and bottom of beam at midspan:

Total force of strands............................ $F_{pe} := f_{pe} \cdot A_{ps} = 1508.6 \cdot kip$

Stress at top of beam............................. $\sigma_{peTopBeam} := \frac{F_{pe}}{A_{nc.tr}} - \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{topnc.tr}} = -0.846 \cdot ksi$

Stress at bottom of beam......................... $\sigma_{peBotBeam} := \frac{F_{pe}}{A_{nc.tr}} + \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{botnc.tr}} = 3.849 \cdot ksi$

Service I Limit State

The compressive stresses in the top of the beam will be checked for the following conditions:

(1) Sum of effective prestress and permanent loads

(2) Sum of effective prestress and permanent loads and transient loads

*(**Note**: Transient loads can include loads during shipping and handling. For purposes of this design example, these loads are omitted).*

(1) Sum of effective prestress and permanent loads. The stress due to permanent loads can be calculated as follows:

Stress in top of slab.........................
$$\sigma1_{TopSlab} := \frac{M_{Trb} + M_{Fws} + M_{Utility}}{S_{slab.tr}} = 0.076 \cdot ksi$$

Stress in top of beam.......................

$$\sigma1_{TopBeam} := \frac{M_{Beam} + M_{Slab} + M_{Forms}}{S_{topnc.tr}} + \left(\frac{M_{Trb} + M_{Fws} + M_{Utility}}{S_{top.tr}} + \sigma_{peTopBeam} \right) = 2.806 \cdot ksi$$

Check top slab stresses.....................
$$TopSlab1 := if\left(\sigma1_{TopSlab} \le f_{allow1.TopSlab}, \text{"OK"}, \text{"NG"}\right)$$

where $f_{allow1.TopSlab} = 2.03 \cdot ksi$

$TopSlab1 = \text{"OK"}$

Check top beam stresses...................
$$TopBeam1 := if\left(\sigma1_{TopBeam} \le f_{allow1.TopBeam}, \text{"OK"}, \text{"NG"}\right)$$

where $f_{allow1.TopBeam} = 3.83 \cdot ksi$

$TopBeam1 = \text{"OK"}$

(2) Sum of effective prestress, permanent loads and transient loads

Stress in top of slab.........................
$$\sigma2_{TopSlab} := \sigma1_{TopSlab} + \frac{M_{LLI}}{S_{slab.tr}} = 1.043 \cdot ksi$$

Stress in top of beam.......................
$$\sigma2_{TopBeam} := \sigma1_{TopBeam} + \frac{M_{LLI}}{S_{top.tr}} = 3.24 \cdot ksi$$

Check top slab stresses.....................
$$TopSlab2 := if\left(\sigma2_{TopSlab} \le f_{allow2.TopSlab}, \text{"OK"}, \text{"NG"}\right)$$

where $f_{allow2.TopSlab} = 2.7 \cdot ksi$

$TopSlab2 = \text{"OK"}$

Check top beam stresses...................
$$TopBeam2 := if\left(\sigma2_{TopBeam} \le f_{allow2.TopBeam}, \text{"OK"}, \text{"NG"}\right)$$

where $f_{allow2.TopBeam} = 5.1 \cdot ksi$

$TopBeam2 = \text{"OK"}$

Service III Limit State total stresses

(3) Tension at bottom of beam only

Stress in bottom of beam..................

$$\sigma3_{BotBeam} := \frac{-M_{Beam} - M_{Slab} - M_{Forms}}{S_{botnc.tr}} + \frac{-M_{Trb} - M_{Fws} - M_{Utility}}{S_{bot.tr}} + \sigma_{peBotBeam} + 0.8\cdot\frac{-M_{LLI}}{S_{bot.tr}} = -0.377\cdot k$$

Check bottom beam stresses............ $BotBeam3 := if\left(\sigma3_{BotBeam} \geq f_{allow3.BotBeam}, "OK", "NG"\right)$

where $f_{allow3.BotBeam} = -0.55\cdot ksi$ $BotBeam3 = "OK"$

B5. Strength I Limit State moment capacity [LRFD 5.7.3]

Strength I Limit State design moment.......

$M_r = 5844.0\, ft\cdot kip$

Factored resistance

$M_r = \phi\cdot M_n$

Nominal flexural resistance

$$M_n = A_{ps}\cdot f_{ps}\cdot\left(d_p - \frac{a}{2}\right) + A_s\cdot f_y\cdot\left(d_s - \frac{a}{2}\right) - A'_s\cdot f'_s\cdot\left(d'_s - \frac{a}{2}\right) + 0.85\cdot f'_c\cdot(b - b_w)\cdot h_f\cdot\left(\frac{a}{2} - \frac{h_f}{2}\right)$$

For a rectangular, section without compression reinforcement,

$$M_n = A_{ps}\cdot f_{ps}\cdot\left(d_p - \frac{a}{2}\right) + A_s\cdot f_y\cdot\left(d_s - \frac{a}{2}\right)$$

where $a = \beta_1\cdot c$

and

$$c = \frac{A_{ps}\cdot f_{pu} + A_s\cdot f_s}{0.85\cdot f'_c\cdot\beta_1\cdot b + k\cdot A_{ps}\cdot\frac{f_{pu}}{d_p}}$$

In order to determine the average stress in the prestressing steel to be used for moment capacity, a factor "k" needs to be computed.

Value for "k"... $k := 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) = 0.28$

Stress block factor................................ $\beta_1 := \min\left[\max\left[0.85 - 0.05\cdot\left(\frac{f_{c.beam} - 4000\cdot psi}{1000\cdot psi}\right), 0.65\right], 0.85\right] = 0.65$

Distance from the compression fiber to cg of prestress.. $d_p := h - strand_{cg}\cdot in = 40.6\cdot in$

Area of reinforcing mild steel.................. $A_s := 0\cdot in^2$

*(**Note**: For strength calculations, deck reinforcement is conservatively ignored.)*

Distance from compression fiber to reinforcing mild steel.............................. $d_s := 0\cdot in$

Assume $f_s = f_y$ [LRFD 5.7.2.1] $f_s := f_y = 60\cdot ksi$

Distance between the neutral axis and compressive face................................. $$c := \frac{A_{ps}\cdot f_{pu} + A_s\cdot f_s}{0.85\cdot f_{c.beam}\cdot \beta_1\cdot b_{tr.interior} + k\cdot A_{ps}\cdot \frac{f_{pu}}{d_p}} = 5.4\cdot in$$

Depth of equivalent stress block.............. $a := \beta_1\cdot c = 3.5\cdot in$

Average stress in prestressing steel.......... $f_{ps} := f_{pu}\cdot\left(1 - k\cdot\frac{c}{d_p}\right) = 260\cdot ksi$

Resistance factor for tension and flexure of prestressed members **[LRFD 5.5.4.2]**...... $\phi' = 1.00$

Moment capacity provided..................... $$M_{r.prov} := \phi'\cdot\left[A_{ps}\cdot f_{ps}\cdot\left(d_p - \frac{a}{2}\right) + A_s\cdot f_s\cdot\left(d_s - \frac{a}{2}\right)\right] = 7128\cdot kip\cdot ft$$

Check moment capacity provided exceeds required... $$Moment_{Capacity} := \left(\begin{array}{ll} \text{"OK"} & \text{if } M_{r.prov} \ge M_r \\ \text{"NG"} & \text{otherwise} \end{array} \right)$$

where $M_r = 5844\cdot ft\cdot kip$

$Moment_{Capacity} = \text{"OK"}$

Check assumption that $f_s = f_y$................ $$Check\_f_s := if\left(A_s = 0, \text{"OK"}, if\left(\frac{c}{d_s} < 0.6, \text{"OK"}, \text{"Not OK"}\right)\right)$$

$Check\_f_s = \text{"OK"}$

B6. Limits for Reinforcement [LRFD 5.7.3.3]

Minimum Reinforcement

The minimum reinforcement requirements ensure the factored moment capacity provided is at least equal to the lesser of the cracking moment and 1.33 times the factored moment required by the applicable strength load combinations.

Modulus of Rupture............................... $f_r := -0.24 \cdot \sqrt{f_{c.beam} \cdot ksi} = -0.7 \cdot ksi$ [SDG 1.4.1.B]

Total unfactored dead load moment on noncomposite section............................. $M_{d.nc} := M_{Beam} + M_{nc} = 1995.4 \cdot kip \cdot ft$

Flexural cracking variability factor (1.2 for precast segmental structures, 1.6 otherwise)....................................... $\gamma_1 := 1.6$

Prestress variability factor (1.1 for bonded tendons, 1.0 for unbonded tendons)......... $\gamma_2 := 1.1$

Ratio of specified minimum yield strength to ultimate tensile strength of reinforcement (0.67 for A615, Grade 60 reinforcement, 0.75 for A706, Grade 60 reinforcement, 1.00 for prestressed concrete structures).. $\gamma_3 := 1.00$

Cracking moment............. $M_{cr} := \gamma_3 \cdot \left[\left(\gamma_1 \cdot f_r + \gamma_2 \cdot \sigma_{peBotBeam} \right) \cdot S_{bot.tr} \right] + -M_{d.nc} \cdot \left(\frac{S_{bot.tr}}{S_{botnc.tr}} - 1 \right) = 2294 \cdot kip \cdot ft$

Required flexural resistance............. $M_{r.reqd} := \min\left(M_{cr}, 133 \cdot \% \cdot M_r\right) = 2293.5 \cdot kip \cdot ft$

Check the capacity provided, $M_{r.prov} = 7127.9 \cdot ft \cdot kip$, exceeds minimum requirements, $M_{r.reqd} = 2293.5 \cdot ft \cdot kip$

$$LRFD_{5.7.3.3.2} := \begin{cases} \text{"OK, minimum reinforcement for positive moment is satisfied"} & \text{if } M_{r.prov} \geq M_{r.reqd} \\ \text{"NG, reinforcement for positive moment is less than minimum"} & \text{otherwise} \end{cases}$$

$LRFD_{5.7.3.3.2} =$ "OK, minimum reinforcement for positive moment is satisfied"

C. Interior Beam Debonding Requirements

C1. Strand Pattern definition at Support

Define the number of strands and eccentricity of strands from bottom of beam at Support = 0

SUPPORT Strand Pattern Data

Rows of strand from bottom of beam	Input (inches)	Number of strands per row	MIDSPAN	Number of strands per row	SUPPORT	COMMENTS
y9 =	19	n9 =	0	n9 =	0	
y8 =	17	n8 =	0	n8 =	0	
y7 =	15	n7 =	0	n7 =	0	
y6 =	13	n6 =	0	n6 =	0	
y5 =	11	n5 =	0	n5 =	0	
y4 =	9	n4 =	0	n4 =	0	
y3 =	7	n3 =	5	n3 =	5	
y2 =	5	n2 =	17	n2 =	15	
y1 =	3	n1 =	17	n1 =	11	
Strand c.g. =	**4.61**		**39**	**Total strands =**	**31**	

Area of prestressing steel........................ $A_{ps.Support} := \left(strands_{total} \cdot StrandArea\right) = 6.7 \cdot in^2$

Non-composite area transformed............ $A_{nc.tr} := A_{nc} + \left(n_p - 1\right) \cdot A_{ps.Support} = 840.4 \cdot in^2$

Non-composite neutral axis transformed... $yb_{nc.tr} := \dfrac{yb_{nc} \cdot A_{nc} + strand_{cg} \cdot in \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Support}\right]}{A_{nc.tr}} = 16 \cdot in$

Non-composite inertia transformed... $I_{nc.tr} := I_{nc} + \left(yb_{nc.tr} - strand_{cg} \cdot in\right)^2 \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Support}\right] = 131886 \cdot in^4$

Non-composite section modulus top......... $S_{topnc.tr} := \dfrac{I_{nc.tr}}{h_{nc} - yb_{nc.tr}} = 6600.4 \cdot in^3$

Non-composite section modulus bottom.... $S_{botnc.tr} := \dfrac{I_{nc.tr}}{yb_{nc.tr}} = 8233.5 \cdot in^3$

C2. Stresses at support at release

The actual stress in strand after losses at transfer have occured............................ $f_{pe} := f_{pj} = 202.5 \cdot ksi$

Calculate the stress due to prestress at the top and bottom of beam at release:

Total force of strands............................ $F_{pe.s} := f_{pe} \cdot A_{ps.Support} = 1362.2 \cdot kip$

Eccentricity of strands at support........... $e_{cg.nc.tr.s} := yb_{nc.tr} - strand_{cg} \cdot in = 11.4 \cdot in$

Stress at top of beam at support.............. $\sigma_{pjTopEnd} := \left(\frac{F_{pe.s}}{A_{nc.tr}} - \frac{F_{pe.s} \cdot e_{cg.nc.tr.s}}{S_{topnc.tr}} \right) = -0.733 \cdot ksi$

Stress at bottom of beam at support... $\sigma_{pjBotEnd} := \left(\frac{F_{pe.s}}{A_{nc.tr}} + \frac{F_{pe.s} \cdot e_{cg.nc.tr.s}}{S_{botnc.tr}} \right) = 3.508 \cdot ksi$

$$TopRelease := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjTopEnd} \le 0 \cdot ksi \wedge \sigma_{pjTopEnd} \ge f_{top.outer15} \\ \text{"OK"} & \text{if } \sigma_{pjTopEnd} > 0 \cdot ksi \wedge \sigma_{pjTopEnd} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $f_{top.outer15} = -0.93 \cdot ksi$ where $0.6 f_{ci.beam} = 3.6 \cdot ksi$

$TopRelease = \text{"OK"}$

$$BotRelease := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjBotEnd} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{if } \sigma_{pjBotEnd} \le f_{top.center70} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ where $f_{top.center70} = -0.2 \cdot ksi$

$BotRelease = \text{"OK"}$

C3. Strand Pattern definition at Debond1

Define the number of strands and eccentricity of strands from bottom of beam at $Debond1 = 10\,ft$

DEBOND1 Strand Pattern Data

Rows of strand from bottom of beam	Input (inches)	Number of strands per row	MIDSPAN	SUPPORT	Number of strands per row	DEBOND1	COMMENTS
y9 =	19	n9 =	0	0	n9 =	0	
y8 =	17	n8 =	0	0	n8 =	0	
y7 =	15	n7 =	0	0	n7 =	0	
y6 =	13	n6 =	0	0	n6 =	0	
y5 =	11	n5 =	0	0	n5 =	0	
y4 =	9	n4 =	0	0	n4 =	0	
y3 =	7	n3 =	5	5	n3 =	5	
y2 =	5	n2 =	17	15	n2 =	17	
y1 =	3	n1 =	17	11	n1 =	13	
Strand c.g. =	4.54		39	31	Total strands =	35	

Area of prestressing steel........................ $A_{ps.Debond1} := \left(strands_{total} \cdot StrandArea\right) = 7.6 \cdot in^2$

Non-composite area transformed............ $A_{nc.tr} := A_{nc} + \left(n_p - 1\right) \cdot A_{ps.Debond1} = 844.7 \cdot in^2$

Non-composite neutral axis transformed... $yb_{nc.tr} := \dfrac{yb_{nc} \cdot A_{nc} + strand_{cg} \cdot in \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Debond1}\right]}{A_{nc.tr}} = 16 \cdot in$

Non-composite inertia transformed.. $I_{nc.tr} := I_{nc} + \left(yb_{nc.tr} - strand_{cg} \cdot in\right)^2 \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Debond1}\right] = 132454 \cdot in^4$

Non-composite section modulus top......... $S_{topnc.tr} := \dfrac{I_{nc.tr}}{h_{nc} - yb_{nc.tr}} = 6608.5 \cdot in^3$

Non-composite section modulus bottom.... $S_{botnc.tr} := \dfrac{I_{nc.tr}}{yb_{nc.tr}} = 8300.7 \cdot in^3$

C4. Stresses at Debond1 at Release

The actual stress in strand after losses at transfer have occured............................ $f_{pe} := f_{pj} = 202.5 \cdot ksi$

Calculate the stress due to prestress at the top and bottom of beam at release:

Total force of strands............................ $F_{pe} := f_{pe} \cdot A_{ps.Debond1} = 1538 \cdot kip$

Eccentricity of strands at Debond1.......... $e_{cg.nc.tr} := yb_{nc.tr} - strand_{cg} \cdot in = 11.4 \cdot in$

Stress at top of beam at Debond1........... $\sigma_{pjTop15} := \dfrac{M_{RelBeamD1}}{S_{topnc.tr}} + \left(\dfrac{F_{pe}}{A_{nc.tr}} - \dfrac{F_{pe} \cdot e_{cg.nc.tr}}{S_{topnc.tr}}\right) = -0.232 \cdot ksi$

Stress at bottom of beam at Debond1...... $\sigma_{pjBotBeam} := \dfrac{-M_{RelBeamD1}}{S_{botnc.tr}} + \left(\dfrac{F_{pe}}{A_{nc.tr}} + \dfrac{F_{pe} \cdot e_{cg.nc.tr}}{S_{botnc.tr}}\right) = 3.455 \cdot ksi$

Top stress limit............................ $f_{top.limit} := \text{if}\left(Debond1 \le 0.15 \cdot L_{beam}, f_{top.outer15}, f_{top.center70}\right) = -0.93 \cdot ksi$

$$TopRelease := \left| \begin{array}{ll} \text{"OK"} & \text{if } \sigma_{pjTop15} \le 0 \cdot ksi \wedge \sigma_{pjTop15} \ge f_{top.limit} \\ \text{"OK"} & \text{if } \sigma_{pjTop15} > 0 \cdot ksi \wedge \sigma_{pjTop15} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{otherwise} \end{array} \right.$$

where $f_{top.limit} = -0.93 \cdot ksi$ where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ $TopRelease = \text{"OK"}$

$$BotRelease := \left| \begin{array}{ll} \text{"OK"} & \text{if } \sigma_{pjBotBeam} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{if } \sigma_{pjBotBeam} \le f_{top.center70} \\ \text{"NG"} & \text{otherwise} \end{array} \right.$$

where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ where $f_{top.center70} = -0.2 \cdot ksi$ $BotRelease = \text{"OK"}$

C5. Strand Pattern definition at Debond2

Define the number of strands and eccentricity of strands from bottom of beam at $Debond2 = 20\,ft$

DEBOND2 Strand Pattern Data

Rows of strand from bottom of beam	Input (inches)	Number of strands per	MIDSPAN	SUPPORT	DEBOND1	Number of strands per row	DEBOND2	COMMENTS
y9 =	19	n9 =	0	0	0	n9 =	0	
y8 =	17	n8 =	0	0	0	n8 =	0	
y7 =	15	n7 =	0	0	0	n7 =	0	
y6 =	13	n6 =	0	0	0	n6 =	0	
y5 =	11	n5 =	0	0	0	n5 =	0	
y4 =	9	n4 =	0	0	0	n4 =	0	
y3 =	7	n3 =	5	5	5	n3 =	5	
y2 =	5	n2 =	17	15	17	n2 =	17	
y1 =	3	n1 =	17	11	13	n1 =	17	
Strand c.g. =	4.38		39	31	35	Total strands =	39	All strands are active beyond this point

Area of prestressing steel....................... $A_{ps.Debond2} := \left(strands_{total} \cdot StrandArea\right) = 8.5 \cdot in^2$

Non-composite area transformed............ $A_{nc.tr} := A_{nc} + \left(n_p - 1\right) \cdot A_{ps.Debond2} = 849 \cdot in^2$

Non-composite neutral axis transformed... $yb_{nc.tr} := \dfrac{yb_{nc} \cdot A_{nc} + strand_{cg} \cdot in \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Debond2}\right]}{A_{nc.tr}} = 15.9 \cdot in$

Non-composite inertia transformed.... $I_{nc.tr} := I_{nc} + \left(yb_{nc.tr} - strand_{cg} \cdot in\right)^2 \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Debond2}\right] = 133104 \cdot in^4$

Non-composite section modulus top......... $S_{topnc.tr} := \dfrac{I_{nc.tr}}{h_{nc} - yb_{nc.tr}} = 6619 \cdot in^3$

Non-composite section modulus bottom.... $S_{botnc.tr} := \dfrac{I_{nc.tr}}{yb_{nc.tr}} = 8376 \cdot in^3$

C6. Stresses at Debond2 at Release

The actual stress in strand after losses at transfer have occured............................ $f_{pe} := f_{pj} = 202.5 \cdot ksi$

Calculate the stress due to prestress at the top and bottom of beam at release:

Total force of strands............................ $F_{pe} := f_{pe} \cdot A_{ps.Debond2} = 1713.8 \cdot kip$

Eccentricity of strands at Debond2.......... $e_{cg.nc.tr} := yb_{nc.tr} - strand_{cg} \cdot in = 11.51 \cdot in$

Stress at top of beam at Debond2............ $\sigma_{pjTop15} := \dfrac{M_{RelBeamD2}}{S_{topnc.tr}} + \left(\dfrac{F_{pe}}{A_{nc.tr}} - \dfrac{F_{pe} \cdot e_{cg.nc.tr}}{S_{topnc.tr}} \right) = 0.092 \cdot ksi$

Stress at bottom of beam at Debond2....... $\sigma_{pjBotBeam} := \dfrac{-M_{RelBeamD2}}{S_{botnc.tr}} + \left(\dfrac{F_{pe}}{A_{nc.tr}} + \dfrac{F_{pe} \cdot e_{cg.nc.tr}}{S_{botnc.tr}} \right) = 3.54 \cdot ksi$

$\sigma_{pjBotBeam} = 3.54 \cdot ksi$

Top stress limit............................... $f_{top.limit} := \text{if}\left(Debond2 \le 0.15 \cdot L_{beam}, f_{top.outer15}, f_{top.center70}\right) = -0.2 \cdot ksi$

$$TopRelease := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjTop15} \le 0 \cdot ksi \wedge \sigma_{pjTop15} \ge f_{top.limit} \\ \text{"OK"} & \text{if } \sigma_{pjTop15} > 0 \cdot ksi \wedge \sigma_{pjTop15} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $f_{top.limit} = -0.2 \cdot ksi$ where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ $TopRelease = \text{"OK"}$

$$BotRelease := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjBotBeam} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{if } \sigma_{pjBotBeam} \le f_{top.center70} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $f_{top.center70} = -0.2 \cdot ksi$ where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ $BotRelease = \text{"OK"}$

D. Shear Design

D1. Determine Nominal Shear Resistance [LRFD 5.8.3.3]

The nominal shear resistance, Vn, shall be determined as the lesser of:

$$V_n = V_c + V_s + V_p$$

$$V_n = 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p$$

The shear resistance of a concrete member may be separated into a component, V_c , that relies on tensile stresses in the concrete, a component, V_s , that relies on tensile stresses in the transverse reinforcement, and a component, V_p , that is the vertical component of the prestressing force.

Nominal shear resistance of concrete section.. $V_c = 0.0316 \cdot \beta \cdot \sqrt{f_c} \cdot b_v \cdot d_v$

Nominal shear resistance of shear reinforcement section............................. $V_s = \frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{s}$

Nominal shear resistance from prestressing for straight strands (non-draped)............. $V_p := 0 \cdot kip$

Effective shear depth............................. $d_v := \max\left(d_p - \frac{a}{2}, 0.9 \cdot d_p, 0.72 \cdot h\right) = 3.24 \cdot ft$

*(**Note**: This location is the same location as previously estimated for* $ShearChk = 3.2\,ft$ *.)*

D2.1 β and θ Parameters Method 1 [LRFD Appendix B-5]

Tables are given in LRFD to determine β from the longitudinal strain and $\frac{v}{f'_c}$ parameter, so these values need to be calculated.

Longitudinal strain for sections with prestressing and transverse reinforcement. $\varepsilon_x = \frac{\frac{M_u}{d_v} + 0.5 \cdot V_u \cdot \cot(\theta) - A_{ps} \cdot f_{po}}{2 \cdot \left(E_s \cdot A_s + E_p \cdot A_{ps}\right)}$

Effective width............................... $b_v := b_w$ where $b_v = 7 \cdot in$

Effective shear depth....................... $d_v = 3.2 \cdot ft$

Factor indicating ability of diagonally cracked concrete to transmit tension.. β

(***Note***: *Values of* $\beta = 2$ *and* $\theta = 45 \cdot deg$ *cannot be assumed since beam is prestressed.)*

Angle of inclination for diagonal compressive stresses........................ θ

LRFD Table B5.2-1 presents values of θ and β for sections with transverse reinforcement . LRFD Appendix B5 states that data given by the table may be used over a range of values. Linear interpolation may be used, but is not recommended for hand calculations.

$\frac{v_u}{f'_c}$	$\varepsilon_x \times 1{,}000$								
	≤-0.20	≤-0.10	≤-0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00
≤0.075	22.3 6.32	20.4 4.75	21.0 4.10	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23
≤0.100	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18
≤0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13
≤0.150	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08
≤0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96
≤0.200	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79
≤0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64
≤0.250	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50

The initial value of ε_x should not be taken greater than 0.001.

The longitudinal strain and $\frac{v}{f'_c}$ parameter are calculated for the appropriate critical sections.

The shear stress on the concrete shall be determined as [LRFD 5.8.2.9-1]:

$$v = \frac{V_u - \phi \cdot V_p}{\phi \cdot b_v \cdot d_v}$$

Factored shear force at the critical section

$$V_u = 304.4 \cdot kip$$

Shear stress on the section...................... $v := \frac{V_u - \phi_v \cdot V_p}{\phi_v \cdot b_v \cdot d_v} = 1.243 \cdot ksi$

Parameter for locked in difference in strain between prestressing tendon and concrete. $f_{po} := 0.7 \cdot f_{pu} = 189 \cdot ksi$

The prestressing strand force becomes effective with the transfer length.............

$L_{transfer} := 60 \cdot strand_{dia} = 3\,ft$

Since the transfer length, $L_{transfer} = 3\,ft$, is less than the shear check location, $ShearChk = 3.2\,ft$, from the end of the beam, the full force of the strands are effective.

Factored moment on section...................

$M_u := \max(M_{shr}, V_u \cdot d_v) = 986.1 \cdot kip \cdot ft$

For the longitudinal strain calculations, an initial assumption for θ must be made.......

$\theta := 23.3 \cdot deg$

Area of concrete on the tension side of the member...

$$A_c := A_{nc} - b_{tf} \cdot (h_{tf} + 1.5in) - b_w \cdot (h_{nc} - 0.5 \cdot h - h_{tf}) - A_{ps.Support} - A_s = 490.3 \cdot in^2$$

Longitudinal strain................................

$$\varepsilon_{x1} := \min\left[\frac{\frac{M_u}{d_v} + 0.5 \cdot V_u \cdot \cot(\theta) - A_{ps.Support} \cdot f_{po}}{2 \cdot (E_s \cdot A_s + E_p \cdot A_{ps.Support})} \cdot (1000), 1\right] = -1.6$$

$$\varepsilon_x := \text{if}\left[\varepsilon_{x1} < 0, \max\left[-0.2, \frac{\frac{|M_u|}{d_v} + 0.5 \cdot |V_u| \cdot \cot(\theta) - A_{ps.Support} \cdot f_{po}}{2 \cdot (E_{c.beam} \cdot A_c + E_s \cdot A_s + E_p \cdot A_{ps.Support})} \cdot (1000)\right], \varepsilon_{x1}\right] = -0.121$$

$\frac{v}{f'_c}$ parameter......................................

$\frac{v}{f_{c.beam}} = 0.146$

Based on **LRFD Table B5.2-1**, the values of θ and β can be approximately taken as:

Angle of inclination of compression stresses..

$\theta = 23.3 \cdot deg$

Factor relating to longitudinal strain on the shear capacity of concrete.....................

$\beta := 2.79$

Nominal shear resistance of concrete section..

$V_c := 0.0316 \cdot \beta \cdot \sqrt{f_{c.beam} \cdot ksi} \cdot b_v \cdot d_v = 69.9 \cdot kip$

D2.2 β and θ Parameters Method 2 [LRFD 5.8.3.4.2]

The strain in nonprestressed long tension reinforcement.......................................

$$\varepsilon_{s1} := \min\left(\frac{\frac{|M_u|}{d_v} + |V_u - V_p| - A_{ps.Support} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps.Support}}, 0.006\right) = -0.003456$$

$$\varepsilon_s := \text{if}\left(\varepsilon_{s1} < 0, \max\left(-0.0004, \frac{\frac{|M_u|}{d_v} + |V_u - V_p| - A_{ps.Support} \cdot f_{po}}{E_{c.beam} \cdot A_c + E_s \cdot A_s + E_p \cdot A_{ps.Support}}\right), \varepsilon_{s1}\right) = -0.000261$$

Angle of inclination of compression stresses

$$\beta_1 := \frac{4.8}{1 + 750 \cdot |\varepsilon_s|} = 4.013$$

Factor relating to longitudinal strain on the shear capacity of concrete......................

$$\theta_1 := 29 + 3500 \cdot \varepsilon_s = 28.1$$

Nominal shear resistance of concrete section...

$$V_{c.1} := 0.0316 \cdot \beta_1 \cdot \sqrt{f_{c.beam} \cdot ksi} \cdot b_v \cdot d_v = 100.6 \cdot kip$$

D2.3 β and θ Parameters Method 3 [LRFD 5.8.3.4.3]

Nominal shear resistance provided by concrete when inclined cracking results from combined shear and moment

$$V_{ci} = 0.02 \cdot \sqrt{f'_c} \cdot b_v \cdot d_v + V_d + \frac{V_i \cdot M_{cr}}{M_{max}} \geq 0.06 \cdot \sqrt{f'_c} \cdot b_v \cdot d_v$$

Nominal shear resistance provided by concrete when inclined cracking results from excessive principal tensions in web

$$V_{cw} = \left(0.06 \cdot \sqrt{f'_c} + 0.30 \cdot f_{pc}\right) \cdot b_v \cdot d_v + V_p$$

Modulus of rupture for LRFD Section 5.8.3.4.3..

$$f_{r.5.8.3.4.3} := -0.2 \cdot \sqrt{f_{c.beam} \cdot ksi} = -0.583 \cdot ksi$$

Shear force due to unfactored dead load (DC and DW)

$$V_d := V_{DC.BeamInt}(ShearChk) + V_{DW.BeamInt}(ShearChk) = 90.3 \cdot ki$$

Moment due to unfactored dead load on noncomposite member

$$M_d := M_{BeamInt}(ShearChk) + M_{SlabInt}(ShearChk) + M_{FormsInt}(ShearChk) = 280.5 \cdot kip \cdot ft$$

Radius of gyration

$$r := \sqrt{\frac{I_{nc}}{A_{nc}}} = 12.57 \cdot in$$

Compressive stress in concrete after allowance for all prestress losses at centroid of cross section

$$f_{pc} := \frac{-F_{pe.s}}{A_{nc}} \left[1 - \frac{e_{cg.nc.tr.s} \cdot (y_b - yb_{nc})}{r^2}\right] + \frac{M_d \cdot (y_b - yb_{nc})}{I_{nc}} = 0.037 \cdot ksi$$

Compressive stress due to effective prestress forces only at the extreme fiber where tensile stress is caused by externally applied loads

$$f_{cpe} := \frac{-F_{pe.s}}{A_{nc}} \cdot \left(1 + \frac{e_{cg.nc.tr.s} \cdot yb_{nc}}{r^2}\right) = -3.697 \cdot ksi$$

Moment causing flexural cracking at section due to externally applied loads

$$M_{cre} := \left| S_b \cdot \left(f_{r.5.8.3.4.3} + f_{cpe} + \frac{M_d}{S_{bnc}}\right)\right| = 4173 \cdot kip \cdot ft$$

Maximum factored moment at section due to externally applied loads

$$M_{max} := 1.75 \cdot M_{LLI.Interior}(ShearChk) = 472.6 \cdot kip \cdot ft$$

Factored shear force at section due to externally applied loads

$$V_i := 1.75 \cdot V_{LLI.Interior}(ShearChk) = 191.5 \cdot kip$$

Nominal shear resistance provided by concrete when inclined cracking results from combined shear and moment

$$V_{ci} := \max\left(0.02 \cdot \sqrt{f_{c.beam} \cdot ksi} \cdot b_v \cdot d_v + V_d + \frac{V_i \cdot M_{cre}}{M_{max}}, 0.06 \cdot \sqrt{f_{c.beam} \cdot ksi} \cdot b_v \cdot d_v\right) = 1797.2 \cdot kip$$

Nominal shear resistance provided by concrete when inclined cracking result from excessive principal tensions in web

$$V_{cw} := \left[\left(0.06 \cdot \sqrt{\frac{f_{c.beam}}{ksi}} \cdot ksi - 0.30 \cdot f_{pc}\right) \cdot b_v \cdot d_v + V_p\right] = 44.6 \cdot kip$$

Nominal shear resistance of concrete section...

$V_{c.2} := \min(V_{ci}, V_{cw}) = 44.6 \cdot kip$

$$\cot\theta := \left[\begin{array}{l} \min\left[1 + 3 \cdot \left(\frac{|f_{pc}|}{\sqrt{f_{c.beam} \cdot ksi}} \right), 1.8\right] \text{ if } V_{ci} > V_{cw} \\ 1.0 \text{ otherwise} \end{array} \right] = 1.038$$

$$\theta_2 := \operatorname{atan}\left(\frac{1}{\cot\theta}\right) = 43.9 \cdot deg$$

$\beta_2 :=$ "NA"

	θ	β	V_c
Method 1	23.30 deg	2.79	69.94 kip
Method 2	28.09 deg	4.01	100.61 kip
Method 3	43.93 deg	NA	44.59 kip

Stirrups

Size of stirrup bar ("4" "5" "6")...

bar := "5"

Area of shear reinforcement...................

$A_v = 0.620 \cdot in^2$

Diameter of shear reinforcement.............

$dia = 0.625 \cdot in$

Nominal shear strength provided by shear reinforcement

$$V_n = V_c + V_p + V_s$$

where...

$$V_n := \min\left(\frac{V_u}{\phi_v}, 0.25 \cdot f_{c.beam} \cdot b_v \cdot d_v + V_p\right) = 338.2 \cdot kip$$

and...

$$V_s := V_n - V_c - V_p = 268.3 \cdot kip$$

Spacing of stirrups

Minimum transverse reinforcement..........

$$s_{min} := \frac{A_v \cdot f_y}{0.0316 \cdot b_v \cdot \sqrt{f_{c.beam} \cdot ksi}} = 57.7 \cdot in$$

Transverse reinforcement required...........

$$s_{req} := \text{if}\left(V_s \le 0, s_{min}, \frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{V_s}\right) = 12.5 \cdot in$$

Minimum transverse reinforcement required...

$s := \min(s_{min}, s_{req}) = 12.5 \cdot in$

Maximum transverse reinforcement spacing

$$s_{max} := \text{if}\left[\frac{V_u - \phi_v \cdot V_p}{\phi_v \cdot (b_v \cdot d_v)} < 0.125 \cdot f_{c.beam}, \min(0.8 \cdot d_v, 24 \cdot in), \min(0.4 \cdot d_v, 12 \cdot in)\right] = 12 \cdot i$$

Spacing of transverse reinforcement cannot exceed the following spacing........ $\text{spacing} := \text{if}(s_{max} > s, s, s_{max}) = 12 \cdot in$

D3. Longitudinal Reinforcement [LRFD 5.8.3.5]

For sections not subjected to torsion, longitudinal reinforcement shall be proportioned so that at each section the tensile capacity of the reinforcement on the flexural tension side of the member, taking into account any lack of full development of that reinforcement, shall be proportioned to satisfy:

General equation for force in longitudinal reinforcement

$$T = \frac{|M_u|}{d_v \cdot \phi_b} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5 \cdot V_s\right) \cdot \cot(\theta)$$

where... $V_s := \min\left(\frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{\text{spacing}}, \frac{V_u}{\phi_v}\right) = 279.8 \cdot kip$

and... $T := \frac{|M_u|}{d_v \cdot \phi'} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5 \cdot V_s\right) \cdot i \cot(\theta) = 7212.7 \cdot kip$

At the shear check location

Longitudinal reinforcement, previously computed for positive moment design.......

$A_{ps.Support} = 6.7 \cdot in^2$

Equivalent force provided by this steel...... $T_{psShearChk} := A_{ps.Support} \cdot f_{pe} = 1362.2 \cdot kip$

$LRFD_{5.8.3.5} :=$ | "Ok, positive moment longitudinal reinforcement is adequate" if $T_{psShearChk} \geq T$
| "NG, positive moment longitudinal reinforcement shall be provided" otherwise

$LRFD_{5.8.3.5} =$ "NG, positive moment longitudinal reinforcement shall be provided"

At the support location

General equation for force in longitudinal reinforcement

$$T = \frac{M_u}{d_v \cdot \phi_b} + \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right) \cdot \cot(\theta) \qquad \text{where } M_u = 0 \cdot ft \cdot kip$$

where...

$$V_s := \min\left(\frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{spacing}, \frac{V_u}{\phi_v}\right) = 279.8 \cdot kip$$

and...

$$T := \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right) \cdot \cot(\theta) = 460.6 \cdot kip$$

In determining the tensile force that the reinforcement is expected to resist at the inside edge of the bearing area, the values calculated at $d_v = 3.2\,ft$ from the face of the support may be used. Note that the force is greater due to the contribution of the moment at d_v. For this example, the actual values at the face of the support will be used.

Longitudinal reinforcement, previously computed for positive moment design.......

$$A_{ps.Support} = 6.7 \cdot in^2$$

The prestressing strand force is not all effective at the support area due to the transfer length required to go from zero force to maximum force. A factor will be applied that takes this into account.

Transfer length.................................... $L_{transfer} = 36 \cdot in$

Distance from center line of bearing to end of beam.. $J = 8 \cdot in$

Estimated length of bearing pad.............. $L_{pad} := 8 \cdot in$

Determine the force effective at the inside edge of the bearing area.

Factor to account for effective force........

$$Factor := \frac{J + \frac{L_{pad}}{2}}{L_{transfer}} = 0.333$$

Equivalent force provided by this steel......

$$T_{psSupport} := A_{ps.Support} \cdot f_{pe} \cdot Factor = 454.1 \cdot kip$$

$$LRFD_{5.8.3.5} := \begin{cases} \text{"Ok, positive moment longitudinal reinforcement is adequate"} & \text{if } T_{psSupport} \ge T \\ \text{"NG, positive moment longitudinal reinforcement shall be provided"} & \text{otherwise} \end{cases}$$

$LRFD_{5.8.3.5}$ = "NG, positive moment longitudinal reinforcement shall be provided"

If the equation is not satisfied, we can increase the shear steel contribution by specifying the actual stirrup spacing used at this location. Based on the Design Standard, stirrups are at the following spacing.

$spacing := 3 \cdot in$

re-computing......................................

$$V_s := \min\left(\frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{spacing}, \frac{V_u}{\phi_v}\right) = 338.2 \cdot kip$$

and..

$$T := \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right) \cdot \cot(\theta) = 392.7 \cdot kip$$

Equivalent force provided by this steel....

$$T_{psSupport} := A_{ps.Support} \cdot f_{pe} \cdot Factor = 454.1 \cdot kip$$

$$LRFD_{5.8.3.5} := \begin{array}{|l} \text{"Ok, positive moment longitudinal reinforcement is adequate"} \quad \text{if } T_{psSupport} \geq T \\ \text{"NG, positive moment longitudinal reinforcement shall be provided"} \quad \text{otherwise} \end{array}$$

$LRFD_{5.8.3.5}$ = "Ok, positive moment longitudinal reinforcement is adequate"

D4. Interface Shear Reinforcement [LRFD 5.8.4]

Assumed a roughened surface per LRFD 5.8.4.3:

$c := 0.28 \cdot ksi \quad \mu := 1.00 \quad K_1 := 0.3 \quad K_2 := 1.8ksi$

Distance between the centroid of the steel in the tension side of the beam to the center of the compression blocks in the deck

$$d_{interface.avg} := d_p - \frac{t_{slab}}{2}$$

$$V_{u.interface} := V_u$$

Interface shear force per unit length per LRFD C5.8.4.2-7

$$V_{uh} := \frac{V_{u.interface}}{d_{interface.avg}} = 8.3 \cdot \frac{kip}{in}$$

$$V_{nh.reqd} := \frac{V_{uh}}{\phi_v} = 9.2 \cdot \frac{kip}{in}$$

$$A_{cv} := b_{tf} \cdot \left(\frac{in}{in}\right) = 48 \cdot \frac{in^2}{in}$$

$$V_{nh.max} := \min\left(K_2 \cdot A_{cv}, K_1 \cdot f_{c.slab} \cdot A_{cv}\right) \cdot ft = 777.6 \cdot kip$$

$$CheckV_{nh.reqd} := \text{if}\left(V_{nh.reqd} \cdot ft < V_{nh.max}, \text{"OK"}, \text{"No Good"}\right) = \text{"OK"}$$

The minimum reinforcement requirement may be waived if $\frac{V_{uh}}{A_{cv}} < 0.21\ \text{ksi}$ assuming requirements of LRFD 5.8.4.4 are satisfied.

$$MinInterfaceReinfReqd := \text{if}\left(\frac{V_{uh}}{A_{cv}} < 0.21 \cdot \text{ksi}, \text{"No"}, \text{"Yes"}\right) = \text{"No"}$$

Minimum interface steel, if required

$$A_{vf.min} := \text{if}\left(MinInterfaceReinfReqd = \text{"Yes"}, \min\left(0.05 \cdot A_{cv} \cdot \frac{\text{ksi}}{f_y}, \frac{1.33 \cdot V_{nh.reqd} - c \cdot A_{cv}}{\mu \cdot f_y}\right), 0 \cdot \frac{\text{in}^2}{\text{ft}}\right) = 0 \cdot \frac{\text{in}^2}{\text{ft}}$$

Design interface steel per LRFD 5.8.4.1-3, if required

$$A_{vf.des} := \max\left(\frac{V_{nh.reqd} - c \cdot A_{cv}}{\mu \cdot f_y}, 0 \cdot \frac{\text{in}^2}{\text{ft}}\right) = 0 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$A_{vf.reqd} := \max\left(A_{vf.min}, A_{vf.des}\right) = 0 \cdot \frac{\text{in}^2}{\text{ft}}$$

Area of reinforcement passing through the interface between the deck and the girder. Minimum interface steel, if required

$$A_{v.prov.interface} := \frac{A_v}{spacing} = 2.48 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$TotalInterfaceSteelProvided := A_{v.prov.interface} = 2.5 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$TotalInterfaceSteelRequired := A_{vf.reqd} = 0 \cdot \frac{\text{in}^2}{\text{ft}}$$

$$CheckInterfaceSteel := \text{if}\left(\frac{TotalInterfaceSteelProvided}{TotalInterfaceSteelRequired + 0.001 \cdot \frac{\text{in}^2}{\text{ft}}} \geq 1, \text{"OK"}, \text{"Add Interface Steel"}\right) = \text{"OK"}$$

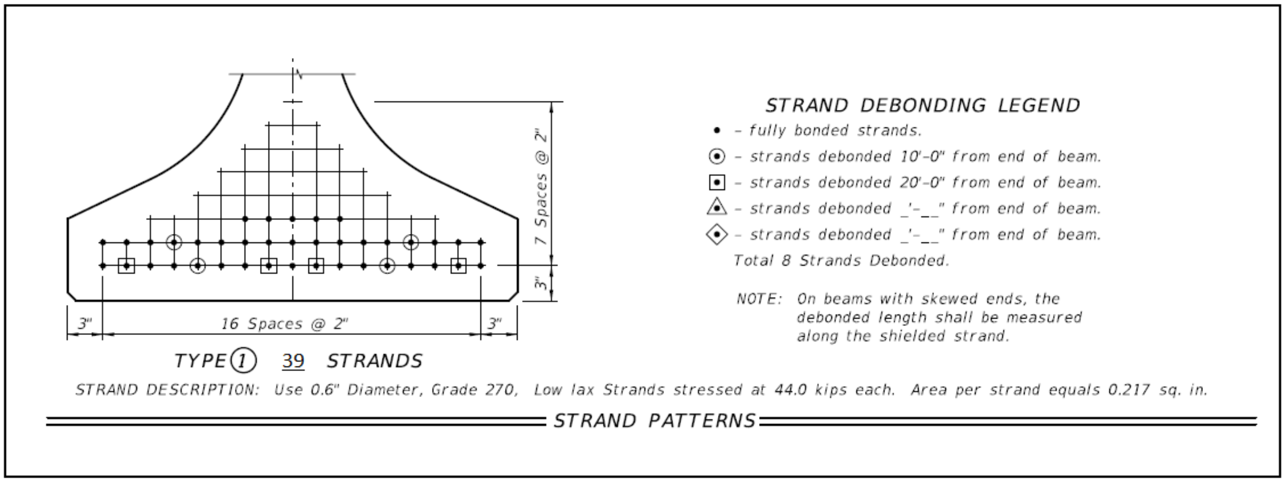
$$CR_{InterfaceSteel} := \frac{TotalInterfaceSteelProvided}{TotalInterfaceSteelRequired + 0.001 \cdot \frac{\text{in}^2}{\text{ft}}} = 2480$$

Note:
Typically shear steel is extended up into the deck slab.
These calculations are based on shear steel functioning as interface reinforcing.
The interface_factor can be used to adjust this assumption.

Several important design checks were not performed in this design example (to reduce the length of calculations). However, the engineer should assure that the following has been done at a minimum:

- Design for anchorage steel
- Design for camber
- Design check for beam transportation loads
- Design for fatigue checks when applicable

E. Summary



Prestressed Beam Design - Part II

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\204PSBeam1.xmcd(R)

Description

This section provides the design of the prestressed concrete beam - exterior beam design.

Page Contents

A. Input Variables

A1. Bridge Geometry

Overall bridge length....... $L_{bridge} = 180\ ft$

Design span length......... $L_{span} = 90\ ft$

Skew angle.................... $Skew = -20 \cdot deg$

A2. Section Properties

NON-COMPOSITE PROPERTIES			FIB-36
Moment of Inertia	[in^4]	I_{nc}	127545
Section Area	[in^2]	A_{nc}	807
ytop	[in]	yt_{nc}	19.51
ybot	[in]	yb_{nc}	16.49
Depth	[in]	h_{nc}	36
Top flange width	[in]	b_{tf}	48
Top flange depth	[in]	h_{tf}	3.5
Width of web	[in]	b_w	7
Bottom flange width	[in]	b_{bf}	38
Bottom flange depth	[in]	h_{bf}	7
Bottom flange taper	[in]	E	15.5
Section Modulus top	[in^3]	S_{tnc}	6537
Section Modulus bottom	[in^3]	S_{bnc}	7735

COMPOSITE SECTION PROPERTIES			INTERIOR	EXTERIOR
Effective slab width	[in]	$b_{eff.interior/exterior}$	120.0	139.4
Transformed slab width	[in]	$b_{tr.interior/exterior}$	87.3	101.5
Height of composite section	[in]	h	45.0	45.0
Effective slab area	[in^2]	A_{slab}	698.5	811.6
Area of composite section	[in^2]	$A_{Interior/Exterior}$	1553.5	1666.6
Neutral axis to bottom fiber	[in]	y_b	28.1	29.0
Neutral axis to top fiber	[in]	y_t	16.9	16.0
Inertia of composite section	[in^4]	$I_{Interior/Exterior}$	359675.0	377743.8
Section modulus top of slab	[in^3]	S_t	21318.8	23612.2
Section modulus top of beam	[in^3]	S_{tb}	45694.6	53980.3
Section modulus bottom of beam	[in^3]	S_b	12786.8	13024.7

A3. Superstructure Loads at Midspan

DC Moment of Beam at Release.............. $M_{RelBeam} := M_{RelBeamExt}(Midspan) = 833.7 \cdot kip \cdot ft$

DC Moment of Beam............................ $M_{Beam} := M_{BeamExt}(Midspan) = 809.1 \cdot kip \cdot ft$

DC Moment of Slab.............................. $M_{Slab} := M_{SlabExt}(Midspan) = 1023.9 \cdot kip \cdot ft$

DC Moment of stay-in-place forms.......... $M_{Forms} := M_{FormsExt}(Midspan) = 57.8 \cdot kip \cdot ft$

DC Moment of traffic railing barriers........ $M_{Trb} := M_{TrbExt}(Midspan) = 141.7 \cdot kip \cdot ft$

DW Moment of future wearing surface.... $M_{Fws} := M_{FwsExt}(Midspan) = 0 \cdot kip \cdot ft$

DW Moment of Utilities......................... $M_{Utility} := M_{UtilityExt}(Midspan) = 0 \cdot kip \cdot ft$

Live Load Moment................................ $M_{LLI} := M_{LLI.Exterior}(Midspan) = 2250.6 \cdot kip \cdot ft$

$Service1 = 1.0 \cdot DC + 1.0 \cdot DW + 1.0 \cdot LL$

- Service I Limit State.........................

$$M_{Srv1} := 1.0 \cdot (M_{Beam} + M_{Slab} + M_{Forms} + M_{Trb}) + 1.0 \cdot (M_{Fws} + M_{Utility}) + 1.0 \cdot (M_{LLI}) = 4283 \cdot kip \cdot ft$$

$Service3 = 1.0 \cdot DC + 1.0 \cdot DW + 0.8 \cdot LL$

- Service III Limit State.......................

$$M_{Srv3} := 1.0 \cdot (M_{Beam} + M_{Slab} + M_{Forms} + M_{Trb}) + 1.0 \cdot (M_{Fws} + M_{Utility}) + 0.8 \cdot (M_{LLI}) = 3832.9 \cdot kip \cdot ft$$

$Strength1 = 1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL$

- Strength I Limit State........................

$$M_r := 1.25 \cdot (M_{Beam} + M_{Slab} + M_{Forms} + M_{Trb}) + 1.50 \cdot (M_{Fws} + M_{Utility}) + 1.75 \cdot (M_{LLI}) = 6479.1 \cdot kip \cdot ft$$

A4. Superstructure Loads at Debonding Locations

DC Moment of Beam at Release -
$Debond1 = 10\,ft$ Location........................ $M_{RelBeamD1} := M_{RelBeamExt}(Debond1) = 332.4 \cdot kip \cdot ft$

DC Moment of Beam at Release -
$Debond2 = 20\,ft$ Location...................... $M_{RelBeamD2} := M_{RelBeamExt}(Debond2) = 580.7 \cdot kip \cdot ft$

A5. Superstructure Loads at the Other Locations

At Support location

DC Shear & Moment........................ $V_{DC.BeamExt}(Support) = 92.6 \cdot kip$ $M_{DC.BeamExt}(Support) = 0 \cdot kip \cdot ft$

DW Shear & Moment .... $V_{DW.BeamExt}(Support) = 0 \cdot kip$ $M_{DW.BeamExt}(Support) = 0 \cdot kip \cdot ft$

LL Shear & Moment.. .... $V_{LLI.Exterior}(Support) = 116.1 \cdot kip$ $M_{LLI.Exterior}(Support) = 0 \cdot kip \cdot ft$

$Strength1 = 1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL$

- Strength I Limit State.......................

$$V_{u.Support} := 1.25 \cdot V_{DC.BeamExt}(Support) + 1.50 \cdot \left(V_{DW.BeamExt}(Support)\right) + 1.75 \cdot \left(V_{LLI.Exterior}(Support)\right) = 318.9 \cdot k$$

At Shear Check location

DC Shear & Moment........................ $V_{DC.BeamExt}(ShearChk) = 85.9 \cdot kip$ $M_{DC.BeamExt}(ShearChk) = 285.7 \cdot kip \cdot ft$

DW Shear & Moment .... $V_{DW.BeamExt}(ShearChk) = 0 \cdot kip$ $M_{DW.BeamExt}(ShearChk) = 0 \cdot kip \cdot ft$

LL Shear & Moment.. .... $V_{LLI.Exterior}(ShearChk) = 110.4 \cdot kip$ $M_{LLI.Exterior}(ShearChk) = 335.3 \cdot kip \cdot ft$

$Strength1 = 1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL$

- Strength I Limit State.......................

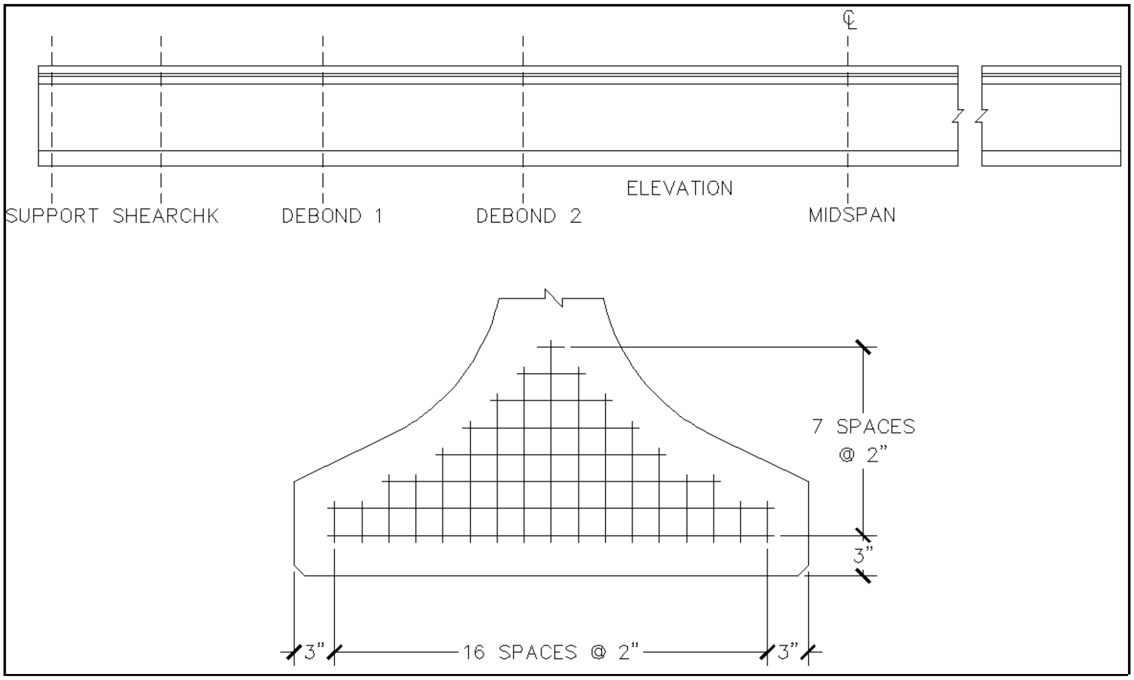
$$V_u := 1.25 \cdot V_{DC.BeamExt}(ShearChk) + 1.50 \cdot \left(V_{DW.BeamExt}(ShearChk)\right) + 1.75 \cdot \left(V_{LLI.Exterior}(ShearChk)\right) = 300.6 \cdot k$$

$$M_{shr} := 1.25 \cdot M_{DC.BeamExt}(ShearChk) + 1.50 \cdot \left(M_{DW.BeamExt}(ShearChk)\right) + 1.75 \cdot \left(M_{LLI.Exterior}(ShearChk)\right) = 944\,ft \cdot$$

B. Exterior Beam Midspan Moment Design

B1. Strand Pattern definition at Midspan

Using the following schematic, the proposed strand pattern at the midspan section can be defined.



STRAND PATTERN DEFINITIONS AND BEAM LOCATIONS

$\text{Support} = 0$ $\qquad \text{ShearChk} = 3.2\,\text{ft}$ $\qquad \text{Debond1} = 10\,\text{ft}$ $\qquad \text{Debond2} = 20\,\text{ft}$ $\qquad \text{Midspan} = 43.88\,\text{ft}$

Strand pattern at midspan

Strand type.................... $strand_{type} := \text{"LowLax"}$ (***Note***: *Options* ("LowLax" "StressRelieved")

Strand size..................... $strand_{dia} := 0.6 \cdot in$ (***Note***: *Options* $(0.5 \cdot in \quad 0.5625 \cdot in \quad 0.6 \cdot in)$

Strand area.................... $$StrandArea := \left(\begin{array}{l} 0.153 \text{ if } strand_{dia} = 0.5 \cdot in \\ 0.192 \text{ if } strand_{dia} = 0.5625 \cdot in \\ 0.217 \text{ if } strand_{dia} = 0.6 \cdot in \\ 0.0 \text{ otherwise} \end{array} \right) \cdot in^2 = 0.217 \cdot in^2$$

Define the number of strands and eccentricity of strands from bottom of beam......

$BeamType = "FIB\text{-}36"$

MIDSPAN Strand Pattern Data

Rows of strand from bottom of beam	Input (inches)	Number of strands per row	MIDSPAN
y9 =	19	**n9 =**	0
y8 =	17	**n8 =**	0
y7 =	15	**n7 =**	0
y6 =	13	**n6 =**	0
y5 =	11	**n5 =**	0
y4 =	9	**n4 =**	0
y3 =	7	**n3 =**	5
y2 =	5	**n2 =**	17
y1 =	3	**n1 =**	17
Strand c.g. =	4.38	**Total strands =**	39

Area of prestressing steel........................ $A_{ps.midspan} := \left(strands_{total} \cdot StrandArea\right)$

$A_{ps.midspan} = 8.5 \cdot in^2$

Transformed section properties

SDG 4.3.1-C6 states: *When calculating the Service limit state capacity for pretensioned concrete flat slabs and girders, use the transformed section properties as follows: at strand transfer; for calculation of prestress losses; for live load application.*

Modular ratio between the prestressing strand and beam. ....

$$n_p := \frac{E_p}{E_{c.beam}} = 5.961$$

Non-composite area transformed............

$$A_{nc.tr} := A_{nc} + \left(n_p - 1\right) \cdot A_{ps.midspan} = 849 \cdot in^2$$

Non-composite neutral axis transformed...

$$yb_{nc.tr} := \frac{yb_{nc} \cdot A_{nc} + strand_{cg} \cdot in \cdot \left[\left(n_p - 1\right) \cdot A_{ps.midspan}\right]}{A_{nc.tr}} = 15.9 \cdot in$$

Non-composite inertia transformed...

$$I_{nc.tr} := I_{nc} + \left(yb_{nc.tr} - strand_{cg} \cdot in\right)^2 \cdot \left[\left(n_p - 1\right) \cdot A_{ps.midspan}\right] = 133104.1 \cdot in^4$$

Non-composite section modulus top.........

$$S_{topnc.tr} := \frac{I_{nc.tr}}{h_{nc} - yb_{nc.tr}} = 6619.2 \cdot in^3$$

Non-composite section modulus bottom....

$$S_{botnc.tr} := \frac{I_{nc.tr}}{yb_{nc.tr}} = 8375.9 \cdot in^3$$

Modular ratio between the mild reinforcing and transformed concrete beam..............

$$n_m := \frac{E_s}{E_{c.beam}} = 6.066$$

Assumed area of reinforcement in deck slab per foot width of deck slab..............

$$A_{deck.rebar} := 0.62 \cdot \frac{in^2}{ft}$$

(***Note**: Assuming #5 at 12" spacing, top and bottom longitudinally).*

Distance from bottom of beam to rebar....

$$y_{bar} := h - t_{mill} - \frac{t_{slab}}{2} = 40.5 \cdot in$$

Total reinforcing steel within effective width of deck slab.................................

$$A_{bar} := b_{eff.exterior} \cdot A_{deck.rebar} = 7.2 \cdot in^2$$

Composite area transformed...................

$$A_{tr} := A_{Exterior} + (n_p - 1) \cdot A_{ps.midspan} + (n_m - 1) \cdot A_{bar} = 1745 \cdot in^2$$

Composite neutral axis transformed..........

$$y_{b.tr} := \frac{y'_b \cdot A_{Exterior} + strand_{cg} \cdot in \cdot (n_p - 1) \cdot A_{ps.midspan} + y_{bar} \cdot (n_m - 1) \cdot A_{bar}}{A_{tr}} = 28.7 \cdot in$$

Composite inertia transformed.................

$$I_{tr} := I_{Exterior} + (y_{b.tr} - strand_{cg} \cdot in)^2 \cdot \left[(n_p - 1) \cdot A_{ps.midspan}\right] + (y_{b.tr} - y_{bar})^2 \cdot (n_m - 1) \cdot A_{bar} = 407590 \cdot in^4$$

Composite section modulus top of slab..

$$S_{slab.tr} := \frac{I_{tr}}{h - y_{b.tr}} = 24929.6 \cdot in^3$$

Composite section modulus top of beam..

$$S_{top.tr} := \frac{I_{tr}}{h - y_{b.tr} - t_{slab} - t_{mill} - h_{buildup}} = 59505.2 \cdot in^3$$

Composite section modulus bottom of beam..

$$S_{bot.tr} := \frac{I_{tr}}{y_{b.tr}} = 14226.4 \cdot in^3$$

Eccentricity of strands at midspan for composite section.................................

$$e_{cg.tr} := y_{b.tr} - strand_{cg} \cdot in = 24.3 \cdot in$$

B2. Prestressing Losses [LRFD 5.9.5]

For prestressing members, the total loss, Δf_{pT} , is expressed as:

$$\Delta f_{pT} = \Delta f_{pLT}$$

where... long-term loss shrinkage and creep for concrete, and relaxation of the steel....... Δf_{pLT}

Loss due to elastic shortening is not included in the total loss equation due to the use of transformed section properties.

Initial Stress in Strands

Specified yield strength of the prestressing steel [**LRFD 5.4.4.1**]..............................

$$f_{py} := \begin{vmatrix} (0.85 \cdot f_{pu}) & \text{if } strand_{type} = \text{"StressRelieved"} \\ (0.90 \cdot f_{pu}) & \text{if } strand_{type} = \text{"LowLax"} \end{vmatrix} = 243 \cdot ksi$$

Jacking stress [**LRFD 5.9.3**]...................

$$f_{pj} := \begin{vmatrix} (0.70 \cdot f_{pu}) & \text{if } strand_{type} = \text{"StressRelieved"} \\ (0.75 \cdot f_{pu}) & \text{if } strand_{type} = \text{"LowLax"} \end{vmatrix} = 202.5 \cdot ksi$$

Elastic Shortening

When calculating concrete stresses using transformed section properties, the effects of losses and gains due to elastic deformations are implicitly accounted for and Δf_{pES} should not be included in the prestressing force applied to the transformed section at transfer. However, the elastic shortening loss is needed for calculation of the stress in prestressing and relaxation of prestressing strands. The loss due to elastic shortening in pretensioned members shall be taken as:

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} \cdot f_{cgp}$$ where.........

Modulus of elasticity of concrete at transfer of prestress force......................................

$$E_{ci.beam} = 4016.8 \cdot ksi$$

Modulus elasticity of prestressing steel.....

$$E_p = 28500 \cdot ksi$$

Eccentricity of strands at midspan for non-composite section............................

$$e_{cg.nc.tr} := yb_{nc.tr} - strand_{cg} \cdot in = 11.5 \cdot in$$

Corresponding total prestressing force......

$$F_{ps} := A_{ps} \cdot f_{pj} = 1713.8 \cdot kip$$

Concrete stresses at c.g. of the prestressing force at transfer and the self wt of the beam at maximum moment location..................

$$f_{cgp} := \frac{F_{ps}}{A_{nc.tr}} + \frac{F_{ps} \cdot e_{cg.nc.tr}^{2}}{I_{nc.tr}} - \frac{M_{RelBeam} \cdot e_{cg.nc.tr}}{I_{nc.tr}} = 2.86 \cdot ksi$$

Losses due to elastic shortening...............

$$\Delta f_{pES} := \frac{E_p}{E_{ci.beam}} \cdot f_{cgp} = 20.28 \cdot ksi$$

Prestressing stress at transfer..................

$$f_{pt} := \text{if}\left(f_{pj} - \Delta f_{pES} \geq 0.55 f_{py}, f_{pj} - \Delta f_{pES}, 0.55 f_{py}\right) = 182.22 \cdot ksi$$

Time-Dependent Losses - *Approximate Estimate* LRFD 5.9.5.3

Long-term prestress loss due to creep of concrete, shrinkage of concrete, and relaxation of steel:

$$\Delta f_{pLT} = 10.0 \cdot \frac{f_{pj} \cdot A_{ps}}{A_{nc.tr}} \cdot \gamma_h \cdot \gamma_{st} + 12.0 \cdot \gamma_h \cdot \gamma_{st} + \Delta f_{pR}$$ where...

$$\gamma_h := 1.7 - 0.01 \cdot H = 0.95$$

$$\gamma_{st} := \frac{5}{1 + \frac{f_{ci.beam}}{ksi}} = 0.714$$

$$\Delta f_{pR} := \begin{cases} 2.4ksi & \text{if } strand_{type} = \text{"LowLax"} \\ (10.0ksi) & \text{if } strand_{type} = \text{"StressRelieved"} \end{cases} = 2.4 \cdot ksi$$

$$\Delta f_{pLT} := 10.0 \cdot \frac{f_{pj} \cdot A_{ps}}{A_{nc.tr}} \cdot \gamma_h \cdot \gamma_{st} + 12.0ksi \cdot \gamma_h \cdot \gamma_{st} + \Delta f_{pR} = 24.2 \cdot ksi$$

Total Prestress Loss - *Approximate Estimate*

The total loss, Δf_{pT_0}, is expressed as.....

$$\Delta f_{pT_0} := \Delta f_{pLT} = 24.2 \cdot ksi$$

Percent loss of strand force.....................

$$Loss := \frac{\Delta f_{pT_0}}{f_{pj}} = 12 \cdot \%$$

Time-Dependent Losses - *Refined Estimates* LRFD 5.9.5.4

Long-term prestress loss due to creep of concrete, shrinkage of concrete, and relaxation of steel:

$$\Delta f_{pLT} = \left(\Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1}\right)_{id} + \left(\Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS}\right)_{df}$$

where...

Shrinkage$_{id}$

$$\Delta f_{pSR} = \varepsilon_{bid} \cdot E_p \cdot K_{id}$$

where...

$$\varphi_{b.fi} := 1.9 \cdot k_{vs.g} \cdot k_{hc.g} \cdot k_{f.g} \cdot k_{td.fi} \cdot T_0^{-0.118} = 1.3$$

$$\varphi_{d.fd} := 1.9 \cdot k_{vs.d} \cdot k_{hc.d} \cdot k_{f.d} \cdot k_{td.d.fd} \cdot T_0^{-0.118} = 1.7$$

$$\varphi_{b.di} := 1.9 \cdot k_{vs.g} \cdot k_{hc.g} \cdot k_{f.g} \cdot k_{td.di} \cdot T_0^{-0.118} = 1$$

$$\varphi_{b.fd} := 1.9 \cdot k_{vs.g} \cdot k_{hc.g} \cdot k_{f.g} \cdot k_{td.fd} \cdot T_1^{-0.118} = 0.7$$

Shrinkage strain.................................... $\varepsilon_{bid} := k_{vs.g} \cdot k_{hs} \cdot k_{f.g} \cdot k_{td.di} \cdot 0.00048 = 0.000248$

Transformed section coefficient.........

$$K_{id} := \frac{1}{1 + \frac{E_p}{E_{ci.beam}} \cdot \frac{A_{ps}}{A_{nc.tr}} \cdot \left(1 + \frac{A_{nc.tr} \cdot e_{cg.nc.tr}^{2}}{I_{nc.tr}}\right) \cdot \left(1 + 0.7 \cdot \varphi_{b.fi}\right)} = 0.8$$

Losses due to shrinkage of girder between time of transfer and deck placement......... $\Delta f_{pSR} := \varepsilon_{bid} \cdot E_p \cdot K_{id} = 5.7 \cdot ksi$

Creep$_{id}$

Losses due to creep of girder between time of transfer and deck placement.........

$$\Delta f_{pCR} := \frac{E_p}{E_{ci.beam}} \cdot f_{cgp} \cdot \varphi_{b.di} \cdot K_{id} = 16.1 \cdot ksi$$

$$\Delta f_{pCR} = 16.1 \cdot ksi$$

Relaxation of Prestressing Strands$_{id}$

$$K_L := \begin{cases} 30 & \text{if } strand_{type} = \text{"LowLax"} \\ 7 & \text{otherwise} \end{cases} = 30$$

Per AASHTO LRFD 5.9.5.4.2c Δf_{pR1} may be assumed to equal 1.2 ksi for low-relaxation strands.

Losses due to relaxation of prestressing strands between time of transfer and deck placement.............

$$\Delta f_{pR1} := \begin{cases} \frac{f_{pt}}{K_L} \cdot \left(\frac{f_{pt}}{f_{py}} - 0.55\right) & \text{if } strand_{type} = \text{"StressRelieved"} \\ 1.2ksi & \text{if } strand_{type} = \text{"LowLax"} \end{cases} = 1.2 \cdot ksi$$

Total Time-Dependent Losses -
Between time of transfer and deck placement

$$\Delta f_{pLT.id} := \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR1} = 23 \cdot ksi$$

Shrinkage$_{df}$

$$\Delta f_{pSD} = \varepsilon_{bdf} \cdot E_p \cdot K_{df}$$

Shrinkage Strain.................................. $\varepsilon_{bdf} := k_{vs.g} \cdot k_{hs} \cdot k_{f.g} \cdot k_{td.fd} \cdot 0.00048 = 0.000324$

Transformed section coefficient.............

$$K_{df} := \frac{1}{1 + \frac{E_p}{E_{ci.beam}} \cdot \frac{A_{ps}}{A_{tr}} \cdot \left(1 + \frac{A_{tr} \cdot e_{cg.tr}^{\ 2}}{I_{tr}}\right) \cdot \left(1 + 0.7 \cdot \varphi_{b.fi}\right)} = 0.812$$

Losses due to shrinkage of girder between deck placement and final time................. $\Delta f_{pSD} := \varepsilon_{bdf} \cdot E_p \cdot K_{df} = 7.511 \cdot ksi$

Creep$_{df}$

$$\Delta f_{pCD} = \frac{E_p}{E_{ci.beam}} \cdot f_{cgp} \cdot \left(\varphi_{fi} - \varphi_{di}\right) \cdot K_{df} + \frac{E_p}{E_{c.beam}} \cdot \Delta f_{cd} \cdot \varphi_{fd} \cdot K_{df}$$

Permanent load moments at midspan acting on non-composite section (except beam at transfer).................................. $M_{nc} := M_{Slab} + M_{Forms} = 1081.7 \cdot kip \cdot ft$

Permanent load moments at midspan acting on composite section.................... $M := M_{Trb} + M_{Fws} + M_{Utility} = 141.7 \cdot kip \cdot ft$

Loss in the strands................................ $P_{loss} := A_{ps} \cdot \Delta f_{pLT.id} = 194.7 \cdot kip$

Change in concrete stress at centroid of prestressing strands due to long-term losses between transfer and deck placement......................

$$\Delta f_{cd} := \frac{-P_{loss}}{A_{nc.tr}} - \frac{P_{loss} \cdot e_{cg.nc.tr}^{\ 2}}{I_{nc.tr}} - \frac{M_{nc} \cdot e_{cg.nc.tr}}{I_{nc.tr}} - \frac{M \cdot e_{cg.tr}}{I_{tr}} = -1.65 \cdot ksi$$

Losses due to creep between deck placement and final time.............

$$\Delta f_{pCD} := \frac{E_p}{E_{ci.beam}} \cdot f_{cgp} \cdot \left(\varphi_{b.fi} - \varphi_{b.di}\right) \cdot K_{df} + \frac{E_p}{E_{c.beam}} \cdot \Delta f_{cd} \cdot \varphi_{b.fd} \cdot K_{df} = -0.87 \cdot ksi$$

Relaxation$_{df}$

Losses due to relaxation of prestressing strands in composite section between time of deck placement and final time....... $\Delta f_{pR2} := \Delta f_{pR1} = 1.2 \cdot ksi$

Shrinkage of Decking$_{df}$

$$\Delta f_{pSS} = \frac{E_p}{E_{c.beam}} \cdot \Delta f_{cdf} \cdot K_{df} \cdot \left(1 + 0.7 \cdot \varphi_{b.fd}\right)$$ where...

Shrinkage Strain................................... $\varepsilon_{ddf} := k_{vs.d} \cdot k_{hs} \cdot k_{f.d} \cdot k_{td.fd} \cdot 0.00048 = 0.000413$

Eccentricity of deck with respect to gross composite section................................. $e_d := y'_t - \frac{t_{slab}}{2} = 12 \cdot in$

Change in concrete stress at centroid of prestressing strands due to shrinkage of deck concrete.......................................

$$\Delta f_{cdf} := \frac{\varepsilon_{ddf} \cdot b_{eff.interior} \cdot t_{slab} \cdot E_{c.slab}}{\left(1 + 0.7 \cdot \varphi_{d.fd}\right)} \cdot \left(\frac{1}{A_{tr}} - \frac{e_{cg.tr} \cdot e_d}{I_{tr}}\right) = -0.09 \cdot ksi$$

Losses due to shrinkage of deck composite section.................................

$$\Delta f_{pSS} := \frac{E_p}{E_{c.beam}} \cdot \Delta f_{cdf} \cdot K_{df} \cdot \left(1 + 0.7 \cdot \varphi_{b.fd}\right) = -0.66 \cdot ksi$$

Total Time-Dependent Losses - *Between deck placement and final time*

$$\Delta f_{pLT.df} := \Delta f_{pSD} + \Delta f_{pCD} + \Delta f_{pR2} - \Delta f_{pSS} = 8.5 \cdot ksi$$

Total Time-Dependent Losses $\Delta f_{pLT} := \Delta f_{pLT.id} + \Delta f_{pLT.df} = 31.5 \cdot ksi$

Total Prestress Loss - *Refined Estimates*

The total loss, Δf_{pT_1}, is expressed as..... $\Delta f_{pT_1} := \Delta f_{pLT} = 31.5 \cdot ksi$

Percent loss of strand force..................... $Loss := \frac{\Delta f_{pT_1}}{f_{pj}} = 15.6 \cdot \%$

Per SDG 4.3.1.C.6, use the Approximate Estimate of Time-Dependent Losses for precast, pretensioned, normal weight concrete members designed as simply supported beams (typical condition). For all other members use the Refined Estimates of Time-Dependent Losses.

Conditions := "Approximate Estimate"

(If refined method is preferred type in Refined Estimate" under conditions.)

Total Prestress Loss

Stress Summary Table		
Approximate Estimate		
Prestressing Stress Prior to Transfer	f_{pj}	202.5 ksi
Prestressing Stress at Transfer	f_{pt}	182.22 ksi
Loss Due to Relaxation	Δf_{pR}	2.40 ksi
Total Prestressing Loss	Δf_{pT}	24.24 ksi
Loss		11.97 %

B3. Stress Limits (*Compression* = +, *Tension* = -)

Initial Stresses [SDG 4.3]

Limit of tension in top of beam at release (straight strand only)

Outer 15 percent of design beam........ $f_{top.outer15} = -0.93 \cdot ksi$

Center 70 percent of design beam...... $f_{top.center70} = -0.2 \cdot ksi$

Limit of compressive concrete strength at release... $0.6f_{ci.beam} = 3.6 \cdot ksi$

Total jacking force of strands................. $F_{pj} := f_{pj} \cdot A_{ps} = 1713.8 \cdot kip$

The actual stress in strand after losses at transfer have occured............................ $f_{pe} := f_{pj} = 202.5 \cdot ksi$

Calculate the stress at the top and bottom of beam at release:

Total force of strands............................ $F_{pe} := f_{pe} \cdot A_{ps} = 1713.8 \cdot kip$

Stress at top of beam at center 70%......... $\sigma_{pjTop70} := \frac{M_{RelBeam}}{S_{topnc.tr}} + \left(\frac{F_{pe}}{A_{nc.tr}} - \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{topnc.tr}}\right) = 0.55 \cdot ksi$

Stress at bottom of beam at center 70%... $\sigma_{pjBotBeam} := \frac{-M_{RelBeam}}{S_{botnc.tr}} + \left(\frac{F_{pe}}{A_{nc.tr}} + \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{botnc.tr}}\right) = 3.18 \cdot ksi$

$$Top70Release := \begin{array}{|lll} \text{"OK"} & \text{if} & \sigma_{pjTop70} \le 0 \cdot ksi \wedge \sigma_{pjTop70} \ge f_{top.center70} \\ \text{"OK"} & \text{if} & \sigma_{pjTop70} > 0 \cdot ksi \wedge \sigma_{pjTop70} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{otherwise} & \end{array}$$

where $f_{top.center70} = -0.2 \cdot ksi$ where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ $Top70Release = \text{"OK"}$

$$BotRelease := \begin{array}{|lll} \text{"OK"} & \text{if} & \sigma_{pjBotBeam} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{if} & \sigma_{pjBotBeam} \le 0 \cdot ksi \\ \text{"NG"} & \text{otherwise} & \end{array}$$

$BotRelease = \text{"OK"}$

where $0.6 f_{ci.beam} = 3.6 \cdot ksi$

*(**Note**: Some MathCad equation explanations-*

- *The check for the top beam stresses checks to see if tension is present,* $\sigma_{pjTop70} \le 0 \cdot ksi$*, and then applies the proper allowable. A separate line is used for the compression and tension allowables. The last line,* "NG" otherwise*, is a catch-all statement such that if the actual stress is not within the allowables, it is considered "NG.")*
- *For the bottom beam, the first line,* $\sigma_{pjBotBeam} \le 0.6 f_{ci.beam}$*, checks that the allowable compression is not exceeded. The second line assures that no tension is present, if there is then the variable will be set to "NG." The catch-all statement,* "NG" otherwise*, will be ignored since the first line was satisfied. If the stress were to exceed the allowable, neither of the first two lines will be satisfied therefore the last line would produce the answer of "NG."*

Final Stresses [LRFD Table 5.9.4.2.1-1 & 5.9.4.2.2-1]

(1) Sum of effective prestress and permanent loads

Limit of compression in slab............. $f_{allow1.TopSlab} := 0.45 \cdot f_{c.slab} = 2.03 \cdot ksi$

Limit of compression in top of beam.. $f_{allow1.TopBeam} := 0.45 \cdot f_{c.beam} = 3.83 \cdot ksi$

(2) Sum of effective prestress, permanent loads and transient loads

*(**Note**: The engineer is reminded that this check needs to be made also for stresses during shipping and handling. For purposes of this design example, this calculation is omitted).*

Limit of compression in slab............. $f_{allow2.TopSlab} := 0.60 \cdot f_{c.slab} = 2.7 \cdot ksi$

Limit of compression in top of beam.. $f_{allow2.TopBeam} := 0.60 \cdot f_{c.beam} = 5.1 \cdot ksi$

(3) Tension at bottom of beam only

Limit of tension in bottom of beam.....

$$f_{allow3.BotBeam} := \begin{cases} \left(-0.0948\sqrt{f_{c.beam} \cdot ksi}\right) & \text{if Environment}_{super} = \text{"Extremely"} \\ \left(-0.19\sqrt{f_{c.beam} \cdot ksi}\right) & \text{otherwise} \end{cases} = -0.55 \cdot ksi$$

*(**Note**: For not worse than moderate corrosion conditions.)* $Environment_{super} = \text{"Slightly"}$

B4. Service I and III Limit States

At service, check the stresses of the beam for compression and tension. In addition, the forces in the strands after losses need to be checked.

The actual stress in strand after all losses have occured.. $f_{pe} := f_{pj} - \Delta f_{pT} = 178.3 \cdot ksi$

Allowable stress in strand after all losses have occured.. $f_{pe.Allow} := 0.80 \cdot f_{py} = 194.4 \cdot ksi$

$$LRFD_{5.9.3} := \begin{cases} \text{"OK, stress at service after losses satisfied"} & \text{if } f_{pe} \le f_{pe.Allow} \\ \text{"NG, stress at service after losses not satisfied"} & \text{otherwise} \end{cases}$$

$LRFD_{5.9.3}$ = "OK, stress at service after losses satisfied"

Calculate the stress due to prestress at the top and bottom of beam at midspan:

Total force of strands............................ $F_{pe} := f_{pe} \cdot A_{ps} = 1508.6 \cdot kip$

Stress at top of beam.............................. $\sigma_{peTopBeam} := \frac{F_{pe}}{A_{nc.tr}} - \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{topnc.tr}} = -0.85 \cdot ksi$

Stress at bottom of beam......................... $\sigma_{peBotBeam} := \frac{F_{pe}}{A_{nc.tr}} + \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{botnc.tr}} = 3.85 \cdot ksi$

Service I Limit State

The compressive stresses in the top of the beam will be checked for the following conditions:

(1) Sum of effective prestress and permanent loads

(2) Sum of effective prestress and permanent loads and transient loads

*(**Note**: Transient loads can include loads during shipping and handling. For purposes of this design example, these loads are omitted).*

(1) Sum of effective prestress and permanent loads. The stress due to permanent loads can be calculated as follows:

Stress in top of slab.......................... $\sigma 1_{TopSlab} := \frac{M_{Trb} + M_{Fws} + M_{Utility}}{S_{slab.tr}} = 0.07 \cdot ksi$

Stress in top of beam.......................

$$\sigma 1_{TopBeam} := \frac{M_{Beam} + M_{Slab} + M_{Forms}}{S_{topnc.tr}} + \frac{M_{Trb} + M_{Fws} + M_{Utility}}{S_{top.tr}} + \sigma_{peTopBeam} = 2.61 \cdot ksi$$

Check top slab stresses..................... $TopSlab1 := \text{if}\left(\sigma 1_{TopSlab} \le f_{allow1.TopSlab}, \text{"OK"}, \text{"NG"}\right)$

where $f_{allow1.TopSlab} = 2.03 \cdot ksi$ $TopSlab1 = \text{"OK"}$

Check top beam stresses.................. $TopBeam1 := \text{if}\left(\sigma 1_{TopBeam} \le f_{allow1.TopBeam}, \text{"OK"}, \text{"NG"}\right)$

where $f_{allow1.TopBeam} = 3.83 \cdot ksi$ $TopBeam1 = \text{"OK"}$

(2) Sum of effective prestress, permanent loads and transient loads

Stress in top of slab......................... $\sigma2_{TopSlab} := \sigma1_{TopSlab} + \dfrac{M_{LLI}}{S_{slab.tr}} = 1.15 \cdot ksi$

Stress in top of beam........................ $\sigma2_{TopBeam} := \sigma1_{TopBeam} + \dfrac{M_{LLI}}{S_{top.tr}} = 3.06 \cdot ksi$

Check top slab stresses..................... $TopSlab2 := if\left(\sigma2_{TopSlab} \le f_{allow2.TopSlab}, "OK", "NG"\right)$

where $f_{allow2.TopSlab} = 2.7 \cdot ksi$ $TopSlab2 = "OK"$

Check top beam stresses................... $TopBeam2 := if\left(\sigma2_{TopBeam} \le f_{allow2.TopBeam}, "OK", "NG"\right)$

where $f_{allow2.TopBeam} = 5.1 \cdot ksi$ $TopBeam2 = "OK"$

Service III Limit State total stresses

(3) Tension at bottom of beam only

Stress in bottom of beam..................

$$\sigma3_{BotBeam} := \frac{-M_{Beam} - M_{Slab} - M_{Forms}}{S_{botnc.tr}} + \frac{-M_{Trb} - M_{Fws} - M_{Utility}}{S_{bot.tr}} + \sigma_{peBotBeam} + 0.8 \cdot \frac{-M_{LLI}}{S_{bot.tr}} = -0.5 \cdot ksi$$

Check bottom beam stresses............ $BotBeam3 := if\left(\sigma3_{BotBeam} \ge f_{allow3.BotBeam}, "OK", "NG"\right)$

where $f_{allow3.BotBeam} = -0.55 \cdot ksi$ $BotBeam3 = "OK"$

B5. Strength I Limit State moment capacity [LRFD 5.7.3]

Strength I Limit State design moment.......

$M_r = 6479.1\ \text{ft}\cdot\text{kip}$

Factored resistance

$M_r = \phi\cdot M_n$

Nominal flexural resistance

$$M_n = A_{ps}\cdot f_{ps}\cdot\left(d_p - \frac{a}{2}\right) + A_s\cdot f_y\cdot\left(d_s - \frac{a}{2}\right) - A'_s\cdot f'_s\cdot\left(d'_s - \frac{a}{2}\right) + 0.85\cdot f'_c\cdot\left(b - b_w\right)\cdot h_f\cdot\left(\frac{a}{2} - \frac{h_f}{2}\right)$$

For a rectangular, section without compression reinforcement,

$$M_n = A_{ps}\cdot f_{ps}\cdot\left(d_p - \frac{a}{2}\right) + A_s\cdot f_y\cdot\left(d_s - \frac{a}{2}\right)$$

where $a = \beta_1\cdot c$

and

$$c = \frac{A_{ps}\cdot f_{pu} + A_s\cdot f_s}{0.85\cdot f'_c\cdot\beta_1\cdot b + k\cdot A_{ps}\cdot\frac{f_{pu}}{d_p}}$$

In order to determine the average stress in the prestressing steel to be used for moment capacity, a factor "k" needs to be computed.

Value for "k"... $k := 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) = 0.28$

Stress block factor................................ $\beta_1 := \min\left[\max\left[0.85 - 0.05\cdot\left(\frac{f_{c.beam} - 4000\cdot\text{psi}}{1000\cdot\text{psi}}\right), 0.65\right], 0.85\right] = 0.65$

Distance from the compression fiber to cg of prestress.. $d_p := h - strand_{cg}\cdot\text{in} = 40.6\cdot\text{in}$

Area of reinforcing mild steel.................. $A_s := 0\cdot\text{in}^2$

*(**Note**: For strength calculations, deck reinforcement is conservatively ignored.)*

Distance from compression fiber to reinforcing mild steel.............................. $d_s := 0\cdot\text{in}$

Assume $f_s = f_y$ **[LRFD 5.7.2.1]**............... $f_s := f_y = 60 \cdot ksi$

Distance between the neutral axis and compressive face..................................

$$c := \frac{A_{ps} \cdot f_{pu} + A_s \cdot f_s}{0.85 \cdot f_{c.beam} \cdot \beta_1 \cdot b_{tr.exterior} + k \cdot A_{ps} \cdot \frac{f_{pu}}{d_p}} = 4.64 \cdot in$$

Depth of equivalent stress block............... $a := \beta_1 \cdot c = 3.02 \cdot in$

Average stress in prestressing steel...........

$$f_{ps} := f_{pu} \cdot \left(1 - k \cdot \frac{c}{d_p}\right) = 261.4 \cdot ksi$$

Resistance factor for tension and flexure of prestressed members **[LRFD 5.5.4.2]**......

$\phi' = 1.00$

Moment capacity provided......................

$$M_{r.prov} := \phi' \cdot \left[A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2}\right) + A_s \cdot f_s \cdot \left(d_s - \frac{a}{2}\right)\right] = 7208.2 \cdot kip \cdot ft$$

Check moment capacity provided exceeds required..

$$Moment_{Capacity} := \left(\begin{array}{ll} \text{"OK"} & \text{if } M_{r.prov} \geq M_r \\ \text{"NG"} & \text{otherwise} \end{array} \right)$$

where

$M_r = 6479.1 \cdot ft \cdot kip$

$Moment_{Capacity} = \text{"OK"}$

Check assumption that $f_s = f_y$................

$$Check\_f_s := \text{if}\left(A_s = 0, \text{"OK"}, \text{if}\left(\frac{c}{d_s} < 0.6, \text{"OK"}, \text{"Not OK"}\right)\right)$$

$Check\_f_s = \text{"OK"}$

B6. Limits for Reinforcement [LRFD 5.7.3.3]

Minimum Reinforcement

The minimum reinforcement requirements ensure the factored moment capacity provided is at least equal to the lesser of the cracking moment and 1.33 times the factored moment required by the applicable strength load combinations.

Modulus of Rupture.............................. $f_r := -0.24 \cdot \sqrt{f_{c.beam} \cdot ksi} = -0.7 \cdot ksi$ [SDG 1.4.1.B]

Total unfactored dead load moment on noncomposite section............................ $M_{dnc} := M_{Beam} + M_{nc} = 1890.8 \cdot kip \cdot ft$

Flexural cracking variability factor (1.2 for precast segmental structures, 1.6 otherwise)...................................... $\gamma_1 := 1.6$

Prestress variability (1.1 for bonded tendons, 1.0 for unbonded tendons)..................... $\gamma_2 := 1.1$

Ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement (0.67 for A615 Grade 60 reinforcement, 0.75 for A706 Grade 60 reinforcement, 1.00 for prestressed concrete structures).. $\gamma_3 := 1.00$

Cracking moment..................................

$$M_{cr} := \gamma_3 \cdot \left[\left(\gamma_1 \cdot f_r + \gamma_2 \cdot \sigma_{peBotBeam} \right) \cdot S_{bot.tr} - M_{d.nc} \cdot \left(\frac{S_{bot.tr}}{S_{botnc.tr}} - 1 \right) \right] = 2299 \cdot kip \cdot ft$$

Required flexural resistance............ $M_{r.reqd} := \min\left(M_{cr}, 133 \cdot \% \cdot M_r\right) = 2299 \cdot kip \cdot ft$

Check that the capacity provided, $M_{r.prov} = 7208.2 \cdot ft \cdot kip$, exceeds minimum requirements, $M_{r.reqd} = 2299 \cdot ft \cdot kip$.

$$LRFD_{5.7.3.3.2} := \begin{cases} \text{"OK, minimum reinforcement for positive moment is satisfied"} & \text{if } M_{r.prov} \ge M_{r.reqd} \\ \text{"NG, reinforcement for positive moment is less than minimum"} & \text{otherwise} \end{cases}$$

$LRFD_{5.7.3.3.2} =$ "OK, minimum reinforcement for positive moment is satisfied"

C. Interior Beam Debonding Requirements

C1. Strand Pattern definition at Support

Define the number of strands and eccentricity of strands from bottom of beam at Support = 0

SUPPORT Strand Pattern Data						
Rows of strand from bottom of beam	Input (inches)	Number of strands per row	MIDSPAN	Number of strands per row	SUPPORT	COMMENTS
y9 =	19	n9 =	0	n9 =	0	
y8 =	17	n8 =	0	n8 =	0	
y7 =	15	n7 =	0	n7 =	0	
y6 =	13	n6 =	0	n6 =	0	
y5 =	11	n5 =	0	n5 =	0	
y4 =	9	n4 =	0	n4 =	0	
y3 =	7	n3 =	5	n3 =	5	
y2 =	5	n2 =	17	n2 =	15	
y1 =	3	n1 =	17	n1 =	11	
Strand c.g. =	**4.61**		**39**	**Total strands =**	**31**	

Area of prestressing steel........................ $A_{ps.Support} := \left(strands_{total} \cdot StrandArea\right) = 6.73 \cdot in^2$

Non-composite area transformed............. $A_{nc.tr} := A_{nc} + \left(n_p - 1\right) \cdot A_{ps.Support} = 840.4 \cdot in^2$

Non-composite neutral axis transformed... $yb_{nc.tr} := \dfrac{yb_{nc} \cdot A_{nc} + strand_{cg} \cdot in \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Support}\right]}{A_{nc.tr}} = 16 \cdot in$

Non-composite inertia transformed..... $I_{nc.tr} := I_{nc} + \left(yb_{nc.tr} - strand_{cg} \cdot in\right)^2 \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Support}\right] = 131886 \cdot in^4$

Non-composite section modulus top......... $S_{topnc.tr} := \dfrac{I_{nc.tr}}{h_{nc} - yb_{nc.tr}} = 6600.4 \cdot in^3$

Non-composite section modulus bottom.... $S_{botnc.tr} := \dfrac{I_{nc.tr}}{yb_{nc.tr}} = 8233.5 \cdot in^3$

C2. Stresses at support at release

Total jacking force of strands.................. $F_{pj} := f_{pj} \cdot A_{ps.Support} = 1362.2 \cdot kip$

The actual stress in strand after losses at transfer have occured............................. $f_{pe} := f_{pj} = 202.5 \cdot ksi$

Calculate the stress due to prestress at the top and bottom of beam at release:

Total force of strands............................. $F_{pe.s} := f_{pe} \cdot A_{ps.Support} = 1362.2 \cdot kip$

Eccentricity of strands at support............ $e_{cg.nc.tr.s} := yb_{nc.tr} - strand_{cg} \cdot in = 11.4 \cdot in$

Stress at top of beam at support............... $\sigma_{pjTopEnd} := \left(\frac{F_{pe.s}}{A_{nc.tr}} - \frac{F_{pe.s} \cdot e_{cg.nc.tr.s}}{S_{topnc.tr}} \right) = -0.73 \cdot ksi$

Stress at bottom of beam at support... $\sigma_{pjBotEnd} := \left(\frac{F_{pe.s}}{A_{nc.tr}} + \frac{F_{pe.s} \cdot e_{cg.nc.tr.s}}{S_{botnc.tr}} \right) = 3.51 \cdot ksi$

$$TopRelease := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjTopEnd} \le 0 \cdot ksi \wedge \sigma_{pjTopEnd} \ge f_{top.outer15} \\ \text{"OK"} & \text{if } \sigma_{pjTopEnd} > 0 \cdot ksi \wedge \sigma_{pjTopEnd} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $f_{top.outer15} = -0.93 \cdot ksi$ where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ $TopRelease = \text{"OK"}$

*(**Note**: See **Sect D3** - By inspection, if the factor to account for the strand force varying up to the transfer length of the strands is applied, the stresses at the top will be within the allowable limit.)*

$$BotRelease := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjBotEnd} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{if } \sigma_{pjBotEnd} \le f_{top.center70} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ where $f_{top.center70} = -0.2 \cdot ksi$ $BotRelease = \text{"OK"}$

C3. Strand Pattern definition at Debond1

Define the number of strands and eccentricity of strands from bottom of beam at $Debond1 = 10\,ft$

DEBOND1 Strand Pattern Data

Rows of strand from bottom of beam	Input (inches)	Number of strands per row	MIDSPAN	SUPPORT	Number of strands per row	DEBOND1	COMMENTS
y9 =	19	n9 =	0	0	n9 =	0	
y8 =	17	n8 =	0	0	n8 =	0	
y7 =	15	n7 =	0	0	n7 =	0	
y6 =	13	n6 =	0	0	n6 =	0	
y5 =	11	n5 =	0	0	n5 =	0	
y4 =	9	n4 =	0	0	n4 =	0	
y3 =	7	n3 =	5	5	n3 =	5	
y2 =	5	n2 =	17	15	n2 =	17	
y1 =	3	n1 =	17	11	n1 =	13	
Strand c.g. =	**4.54**		**39**	**31**	**Total strands =**	**35**	

Area of prestressing steel........................ $A_{ps.Debond1} := \left(strands_{total} \cdot StrandArea\right) = 7.59 \cdot in^2$

Non-composite area transformed............. $A_{nc.tr} := A_{nc} + \left(n_p - 1\right) \cdot A_{ps.Debond1} = 844.7 \cdot in^2$

Non-composite neutral axis transformed... $yb_{nc.tr} := \dfrac{yb_{nc} \cdot A_{nc} + strand_{cg} \cdot in \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Debond1}\right]}{A_{nc.tr}} = 16 \cdot in$

Non-composite inertia transformed...........

$$I_{nc.tr} := I_{nc} + \left(yb_{nc.tr} - strand_{cg} \cdot in\right)^2 \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Debond1}\right] = 132454 \cdot in^4$$

Non-composite section modulus top......... $S_{topnc.tr} := \dfrac{I_{nc.tr}}{h_{nc} - yb_{nc.tr}} = 6608.5 \cdot in^3$

Non-composite section modulus bottom.... $S_{botnc.tr} := \dfrac{I_{nc.tr}}{yb_{nc.tr}} = 8300.7 \cdot in^3$

C4. Stresses at Debond1 at Release

Total jacking force of strands.................. $F_{pj} := f_{pj} \cdot A_{ps.Debond1} = 1538 \cdot kip$

The actual stress in strand after losses at transfer have occured............................. $f_{pe} := f_{pj} = 202.5 \cdot ksi$

Calculate the stress due to prestress at the top and bottom of beam at release:

Total force of strands............................. $F_{pe} := f_{pe} \cdot A_{ps.Debond1} = 1538 \cdot kip$

Eccentricity of strands at Debond1.......... $e_{cg.nc.tr} := yb_{nc.tr} - strand_{cg} \cdot in = 11.41 \cdot in$

Stress at top of beam at Debond1............ $\sigma_{pjTop15} := \frac{M_{RelBeamD1}}{S_{topnc.tr}} + \left(\frac{F_{pe}}{A_{nc.tr}} - \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{topnc.tr}} \right) = -0.23 \cdot ksi$

Stress at bottom of beam at Debond1....... $\sigma_{pjBotBeam} := \frac{-M_{RelBeamD1}}{S_{botnc.tr}} + \left(\frac{F_{pe}}{A_{nc.tr}} + \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{botnc.tr}} \right) = 3.46 \cdot ksi$

Top stress limit............................. $f_{top.limit} := \text{if}\left(Debond1 \le 0.15 \cdot L_{beam}, f_{top.outer15}, f_{top.center70}\right) = -0.93 \cdot ksi$

$$TopRelease := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjTop15} \le 0 \cdot ksi \wedge \sigma_{pjTop15} \ge f_{top.limit} \\ \text{"OK"} & \text{if } \sigma_{pjTop15} > 0 \cdot ksi \wedge \sigma_{pjTop15} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $f_{top.limit} = -0.93 \cdot ksi$ where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ $TopRelease = \text{"OK"}$

$$BotRelease := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjBotBeam} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{if } \sigma_{pjBotBeam} \le f_{top.center70} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $f_{top.center70} = -0.2 \cdot ksi$ where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ $BotRelease = \text{"OK"}$

C5. Strand Pattern definition at Debond2

Define the number of strands and eccentricity of strands from bottom of beam at $\text{Debond2} = 20\,\text{ft}$

DEBOND2 Strand Pattern Data

Rows of strand from bottom of beam	Input (inches)	Number of strands per	MIDSPAN	SUPPORT	DEBOND1	Number of strands per row	DEBOND2	COMMENTS
y9 =	19	n9 =	0	0	0	n9 =	0	
y8 =	17	n8 =	0	0	0	n8 =	0	
y7 =	15	n7 =	0	0	0	n7 =	0	
y6 =	13	n6 =	0	0	0	n6 =	0	
y5 =	11	n5 =	0	0	0	n5 =	0	
y4 =	9	n4 =	0	0	0	n4 =	0	
y3 =	7	n3 =	5	5	5	n3 =	5	
y2 =	5	n2 =	17	15	17	n2 =	17	
y1 =	3	n1 =	17	11	13	n1 =	17	
Strand c.g. =	**4.38**		**39**	**31**	**35**	**Total strands =**	**39**	**All strands are active beyond this point**

Area of prestressing steel........................ $A_{ps.Debond2} := \left(\text{strands}_{total} \cdot \text{StrandArea}\right) = 8.46 \cdot \text{in}^2$

Non-composite area transformed............ $A_{nc.tr} := A_{nc} + \left(n_p - 1\right) \cdot A_{ps.Debond2} = 849 \cdot \text{in}^2$

Non-composite neutral axis transformed... $yb_{nc.tr} := \dfrac{yb_{nc} \cdot A_{nc} + \text{strand}_{cg} \cdot \text{in} \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Debond2}\right]}{A_{nc.tr}} = 15.9 \cdot \text{in}$

Non-composite inertia transformed.... $I_{nc.tr} := I_{nc} + \left(yb_{nc.tr} - \text{strand}_{cg} \cdot \text{in}\right)^2 \cdot \left[\left(n_p - 1\right) \cdot A_{ps.Debond2}\right] = 133104 \cdot \text{in}^4$

Non-composite section modulus top......... $S_{topnc.tr} := \dfrac{I_{nc.tr}}{h_{nc} - yb_{nc.tr}} = 6619.2 \cdot \text{in}^3$

Non-composite section modulus bottom.... $S_{botnc.tr} := \dfrac{I_{nc.tr}}{yb_{nc.tr}} = 8375.9 \cdot \text{in}^3$

C6. Stresses at Debond2 at Release

Total jacking force of strands.................. $F_{pj} := f_{pj} \cdot A_{ps.Debond2} = 1713.8 \cdot kip$

The actual stress in strand after losses at transfer have occured............................. $f_{pe} := f_{pj} = 202.5 \cdot ksi$

Calculate the stress due to prestress at the top and bottom of beam at release:

Total force of strands............................. $F_{pe} := f_{pe} \cdot A_{ps.Debond2} = 1713.8 \cdot kip$

Eccentricity of strands at Debond2.......... $e_{cg.nc.tr} := yb_{nc.tr} - strand_{cg} \cdot in = 11.51 \cdot in$

Stress at top of beam at Debond2............ $\sigma_{pjTop15} := \dfrac{M_{RelBeamD2}}{S_{topnc.tr}} + \left(\dfrac{F_{pe}}{A_{nc.tr}} - \dfrac{F_{pe} \cdot e_{cg.nc.tr}}{S_{topnc.tr}}\right) = 0.09 \cdot ksi$

Stress at bottom of beam at Debond2....... $\sigma_{pjBotBeam} := \dfrac{-M_{RelBeamD2}}{S_{botnc.tr}} + \left(\dfrac{F_{pe}}{A_{nc.tr}} + \dfrac{F_{pe} \cdot e_{cg.nc.tr}}{S_{botnc.tr}}\right) = 3.54 \cdot ksi$

Top stress limit................................ $f_{top.limit} := \text{if}\left(Debond2 \le 0.15 \cdot L_{beam}, f_{top.outer15}, f_{top.center70}\right) = -0.2 \cdot ksi$

$$TopRelease := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjTop15} \le 0 \cdot ksi \wedge \sigma_{pjTop15} \ge f_{top.limit} \\ \text{"OK"} & \text{if } \sigma_{pjTop15} > 0 \cdot ksi \wedge \sigma_{pjTop15} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $f_{top.limit} = -0.2 \cdot ksi$ where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ $TopRelease = \text{"OK"}$

$$BotRelease := \begin{cases} \text{"OK"} & \text{if } \sigma_{pjBotBeam} \le 0.6 f_{ci.beam} \\ \text{"NG"} & \text{if } \sigma_{pjBotBeam} \le f_{top.center70} \\ \text{"NG"} & \text{otherwise} \end{cases}$$

where $f_{top.center70} = -0.2 \cdot ksi$ where $0.6 f_{ci.beam} = 3.6 \cdot ksi$ $BotRelease = \text{"OK"}$

D. Shear Design

D1. Determine Nominal Shear Resistance [LRFD 5.8.3.3]

The nominal shear resistance, Vn, shall be determined as the lesser of:

$$V_n = V_c + V_s + V_p$$

$$V_n = 0.25 \cdot f'_c \cdot b_v \cdot d_v + V_p$$

The shear resistance of a concrete member may be separated into a component, V_c, that relies on tensile stresses in the concrete, a component, V_s, that relies on tensile stresses in the transverse reinforcement, and a component, V_p, that is the vertical component of the prestressing force.

Nominal shear resistance of concrete section.. $V_c = 0.0316 \cdot \beta \cdot \sqrt{f_c} \cdot b_v \cdot d_v$

Nominal shear resistance of shear reinforcement section............................. $V_s = \dfrac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{s}$

Nominal shear resistance from prestressing for straight strands (non-draped)............. $V_p := 0 \cdot kip$

Effective shear depth............................. $d_v := \max\left(d_p - \dfrac{a}{2}, 0.9 \cdot d_p, 0.72 \cdot h\right)$

$d_v = 39.1 \cdot in$ or $d_v = 3.3\,ft$

*(**Note**: This location is close to the same location as previously estimated for* $ShearChk = 3.2\,ft$ *.)*

D2.1 β and θ Parameters Method 1 [LRFD Appendix B-5]

Tables are given in LRFD to determine β from the longitudinal strain and $\dfrac{v}{f'_c}$ parameter, so these values need to be calculated.

Longitudinal strain for sections with prestressing and transverse reinforcement.

$$\varepsilon_x = \frac{\dfrac{|M_u|}{d_v} + 0.5 \cdot |V_u - V_p| \cdot \cot(\theta) - A_{ps} \cdot f_{po}}{2 \cdot (E_s \cdot A_s + E_p \cdot A_{ps})}$$

Effective width............................... $b_v := b_w$ where $b_v = 7 \cdot in$

Effective shear depth....................... $d_v = 3.3 \cdot ft$

Factor indicating ability of diagonally cracked concrete to transmit tension.. β

Angle of inclination for diagonal compressive stresses........................ θ

*(**Note**: Values of* $\beta = 2$ *and* $\theta = 45 \cdot deg$ *cannot be assumed since beam is prestressed.)*

LRFD Table B5.2-1 presents values of θ and β for sections with transverse reinforcement. LRFD Appendix B5 states that data given by the table may be used over a range of values. Linear interpolation may be used, but is not recommended for hand calculations.

$\frac{v_u}{f'_c}$	$\varepsilon_x \times 1{,}000$								
	≤-0.20	≤-0.10	≤-0.05	≤0	≤0.125	≤0.25	≤0.50	≤0.75	≤1.00
≤0.075	22.3 6.32	20.4 4.75	21.0 4.10	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23
≤0.100	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18
≤0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13
≤0.150	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08
≤0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96
≤0.200	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79
≤0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64
≤0.250	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50

The initial value of ε_x should not be taken greater than 0.001.

The longitudinal strain and $\frac{v}{f'_c}$ parameter are calculated for the appropriate critical sections.

The shear stress on the concrete shall be determined as [LRFD equation 5.8.2.9-1]:

$$v = \frac{V_u - \phi \cdot V_p}{\phi \cdot b_v \cdot d_v}$$

Factored shear force at the critical section

$V_u = 300.6 \cdot kip$

Shear stress on the section...................... $v := \frac{V_u - \phi_v \cdot V_p}{\phi_v \cdot b_v \cdot d_v} = 1.2 \cdot ksi$

Parameter for locked in difference in strain between prestressing tendon and concrete. $f_{po} := 0.7 \cdot f_{pu} = 189 \cdot ksi$

The prestressing strand force becomes effective with the transfer length.............

$L_{transfer} := 60 \cdot strand_{dia} = 36 \cdot in$

Since the transfer length, $L_{transfer} = 3\,ft$, is less than the shear check location, $ShearChk = 3.2\,ft$, from the end of the beam, the full force of the strands are effective.

Factored moment on section...................

$M_u := \max(M_{shr}, V_u \cdot d_v) = 979.5 \cdot kip \cdot ft$

For the longitudinal strain calculations, an initial assumption for θ must be made.......

$\theta := 23.3 \cdot deg$

Area of concrete on the tension side of the member...

$$A_c := A_{nc} - b_{tf} \cdot (h_{tf} + 1.5in) - b_w \cdot (h_{nc} - 0.5 \cdot h - h_{tf}) - A_{ps.Support} - A_s = 490.3 \cdot in^2$$

Longitudinal strain............................

$$\varepsilon_{x1} := \min\left[\frac{\dfrac{|M_u|}{d_v} + 0.5 \cdot |V_u| \cdot \cot(\theta) - A_{ps.Support} \cdot f_{po}}{2 \cdot (E_s \cdot A_s + E_p \cdot A_{ps.Support})} \cdot 1000, 1\right] = -1.622$$

$$\varepsilon_x := \text{if}\left[\varepsilon_{x1} < 0, \max\left[-0.2, \frac{\dfrac{|M_u|}{d_v} + 0.5 \cdot |V_u| \cdot \cot(\theta) - A_{ps.Support} \cdot f_{po}}{2 \cdot (E_{c.beam} \cdot A_c + E_s \cdot A_s + E_p \cdot A_{ps.Support})} \cdot 1000\right], \varepsilon_{x1}\right] = -0.123$$

$\dfrac{v}{f'_c}$ parameter.....................................

$\dfrac{v}{f_{c.beam}} = 0.144$

Based on **LRFD Table B5.2-1**, the values of θ and β can be approximately taken as:

Angle of inclination of compression stresses

$\theta = 23.3 \cdot deg$

Factor relating to longitudinal strain on the shear capacity of concrete.....................

$\beta := 2.79$

Nominal shear resistance of concrete section...

$V_c := 0.0316 \cdot \beta \cdot \sqrt{f_{c.beam} \cdot ksi} \cdot b_v \cdot d_v = 70.4 \cdot kip$

D2.2 β and θ Parameters Method 2 [LRFD 5.8.3.4.2]

The strain in nonprestressed long tension reinforcement.........................

$$\varepsilon_{s1} := \min\left(\frac{\frac{|M_u|}{d_v} + |V_u - V_p| - A_{ps.Support} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps.Support}}, 0.006\right) = -0.003496$$

$$\varepsilon_s := \text{if}\left(\varepsilon_{s1} < 0, \max\left(-.0004, \frac{\frac{|M_u|}{d_v} + |V_u - V_p| - A_{ps.Support} \cdot f_{po}}{E_s \cdot A_s + E_p \cdot A_{ps.Support} + E_{c.beam} \cdot A_c}\right), \varepsilon_{s1}\right) = -0.000264$$

Angle of inclination of compression stresses...

$$\beta_1 := \frac{4.8}{1 + 750 \cdot |\varepsilon_s|} = 4$$

Factor relating to longitudinal strain on the shear capacity of concrete.....................

$$\theta_1 := (29 + 3500 \cdot \varepsilon_s) = 28.1$$

Nominal shear resistance of concrete section...

$$V_{c1} := 0.0316 \cdot \beta_1 \cdot \sqrt{f_{c.beam} \cdot ksi} \cdot b_v \cdot d_v = 101 \cdot kip$$

D2.3 β and θ Parameters Method 3 [LRFD 5.8.3.4.3]

Nominal shear resistance provided by concrete when inclined cracking results from combined shear and moment...........

$$V_{ci} = 0.02 \cdot \sqrt{f'_c} \cdot b_v \cdot d_v + V_d + \frac{V_i \cdot M_{cr}}{M_{max}} \geq 0.06 \cdot \sqrt{f'_c} \cdot b_v \cdot d_v$$

Nominal shear resistance provided by concrete when inclined cracking results from excessive principal tensions in web..

$$V_{cw} = \left(0.06 \cdot \sqrt{f'_c} + 0.30 \cdot f_{pc}\right) \cdot b_v \cdot d_v + V_p$$

Radius of gyration................................

$$r := \sqrt{\frac{I_{nc}}{A_{nc}}} = 12.6 \cdot in$$

Modulus of rupture for LRFD Section 5.8.3.4.3..

$$f_{r.5.8.3.4.3} := -0.2 \cdot \sqrt{f_{c.beam} \cdot ksi} = -0.58 \cdot ksi$$

Shear force due to unfactored dead load (DC and DW).......................................

$$V_d := V_{DC.BeamExt}(ShearChk) + V_{DW.BeamExt}(ShearChk) = 85.9 \cdot k$$

Moment due to unfactored dead load... $M_d := M_{DC.BeamExt}(ShearChk) + M_{DW.BeamExt}(ShearChk) = 285.7 \cdot kip \cdot ft$

Compressive stress in concrete after allowance for all prestress losses at centroid of cross section..................

$$f_{pc} := \frac{-F_{pe.s}}{A_{nc}} \cdot \left[1 - \frac{e_{cg.nc.tr.s} \cdot (y'_b - yb_{nc})}{r^2}\right] + \frac{M_d \cdot (y'_b - yb_{nc})}{I_{nc}} = 0.172 \cdot ksi$$

Compressive stress due to effective prestress forces only at the extreme fiber where tensile stress is caused by externally applied loads...........................

$$f_{cpe} := \frac{-F_{pe.s}}{A_{nc}} \cdot \left(1 + \frac{e_{cg.nc.tr.s} \cdot yb_{nc}}{r^2}\right) = -3.7 \cdot ksi$$

Moment causing flexural cracking at section due to externally applied loads.......

$$M_{cre} := \left| S_b \cdot \left(f_{r.5.8.3.4.3} + f_{cpe} + \frac{M_d}{S_{bnc}}\right)\right| = 4164.2 \cdot kip \cdot ft$$

Maximum factored moment at section due to externally applied loads..................

$$M_{max} := 1.75 \cdot M_{LLI.Exterior}(ShearChk) = 586.7 \cdot kip \cdot ft$$

Factored shear force at section due to externally applied loads...........................

$$V_i := 1.75 \cdot V_{LLI.Exterior}(ShearChk) = 193.2 \cdot kip$$

Nominal shear resistance provided by concrete when inclined cracking results from combined shear and moment...........

$$V_{ci} := \max\left(0.02 \cdot \sqrt{f_{c.beam} \cdot ksi} \cdot b_v \cdot d_v + V_d + \frac{V_i \cdot M_{cre}}{M_{max}}, 0.06 \cdot \sqrt{f_{c.beam} \cdot ksi} \cdot b_v \cdot d_v\right) = 1473.1 \cdot kip$$

Nominal shear resistance provided by concrete when inclined cracking result from excessive principal tensions in web..

$$V_{cw} := \left[\left(0.06 \cdot \sqrt{\frac{f_{c.beam}}{ksi}} \cdot ksi - 0.30 \cdot f_{pc}\right) \cdot b_v \cdot d_v + V_p\right] = 33.7 \cdot kip$$

Nominal shear resistance of concrete section...

$V_{c.2} := \min(V_{ci}, V_{cw}) = 33.7 \cdot kip$

$$\cot\theta := \left| \begin{array}{l} \min\left[1 + 3 \cdot \left(\frac{|f_{pc}|}{\sqrt{f_{c.beam} \cdot ksi}}\right), 1.8\right] \text{ if } V_{ci} > V_{cw} \\ 1.0 \text{ otherwise} \end{array} \right. = 1.18$$

$\theta_2 := \operatorname{atan}\left(\frac{1}{\cot\theta}\right) = 40.3 \cdot deg$

$\beta_2 :=$ "NA"

	θ	β	V_c
Method 1	23.30 deg	2.79	70.36 kip
Method 2	28.07 deg	4.01	101.03 kip
Method 3	40.34 deg	NA	33.73 kip

Stirrups

Size of stirrup bar ("4" "5" "6")...

bar := "5"

Area of shear reinforcement...................

$A_v = 0.620 \cdot in^2$

Diameter of shear reinforcement.............

$dia = 0.625 \cdot in$

Nominal shear strength provided by shear reinforcement

$$V_n = V_c + V_p + V_s$$

where...

$V_n := \min\left(\frac{V_u}{\phi_v}, 0.25 \cdot f_{c.beam} \cdot b_v \cdot d_v + V_p\right) = 334 \cdot kip$

and...

$V_s := V_n - V_c - V_p = 263.6 \cdot kip$

Spacing of stirrups

Minimum transverse reinforcement.........

$s_{min} := \frac{A_v \cdot f_y}{0.0316 \cdot b_v \cdot \sqrt{f_{c.beam} \cdot ksi}} = 57.7 \cdot in$

Transverse reinforcement required..........

$s_{req} := \text{if}\left(V_s \le 0, s_{min}, \frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{V_s}\right) = 12.8 \cdot in$

Minimum transverse reinforcement required..

$s := \min(s_{min}, s_{req}) = 12.8 \cdot in$

Maximum transverse reinforcement spacing

$$s_{max} := \text{if}\left[\frac{V_u - \phi_v \cdot V_p}{\phi_v \cdot (b_v \cdot d_v)} < 0.125 \cdot f_{c.beam}, \min(0.8 \cdot d_v, 24 \cdot in), \min(0.4 \cdot d_v, 12 \cdot in)\right] = 12 \cdot in$$

Spacing of transverse reinforcement cannot exceed the following spacing........ $spacing := \text{if}(s_{max} > s, s, s_{max}) = 12 \cdot in$

D3. Longitudinal Reinforcement [LRFD 5.8.3.5]

For sections not subjected to torsion, longitudinal reinforcement shall be proportioned so that at each section the tensile capacity of the reinforcement on the flexural tension side of the member, taking into account any lack of full development of that reinforcement, shall be proportioned to satisfy:

General equation for force in longitudinal reinforcement

$$T = \frac{|M_u|}{d_v \cdot \phi_b} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5 \cdot V_s\right) \cdot \cot(\theta)$$

where.. $V_s := \min\left(\frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{spacing}, \frac{V_u}{\phi_v}\right) = 281.5 \cdot kip$

and.. $T := \frac{|M_u|}{d_v \cdot \phi'} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5 \cdot V_s\right) \cdot \cot(\theta) = 749.2 \cdot kip$

At the shear check location

Longitudinal reinforcement, previously computed for positive moment design.......

$A_{ps.Support} = 6.7 \cdot in^2$

Equivalent force provided by this steel...... $T_{psShearChk} := A_{ps.Support} \cdot f_{pe} = 1362.2 \cdot kip$

$LRFD_{5.8.3.5} :=$ | "Ok, positive moment longitudinal reinforcement is adequate" if $T_{psShearChk} \geq T$
| "NG, positive moment longitudinal reinforcement shall be provided" otherwise

$LRFD_{5.8.3.5} =$ "Ok, positive moment longitudinal reinforcement is adequate"

At the support location

General equation for force in longitudinal reinforcement

$$T = \frac{M_u}{d_v \cdot \phi_b} + \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right) \cdot \cot(\theta) \qquad \text{where } M_u = 0 \cdot ft \cdot kip$$

where.. $V_s := \min\left(\frac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{spacing}, \frac{V_u}{\phi_v}\right) = 281.5 \cdot kip$

and... $T := \left(\frac{V_u}{\phi_v} - 0.5 \cdot V_s - V_p\right) \cdot \cot(\theta) = 448.6 \cdot kip$

In determining the tensile force that the reinforcement is expected to resist at the inside edge of the bearing area, the values calculated at $d_v = 3.3\,ft$ from the face of the support **may** be used. Note that the force is greater due to the contribution of the moment at d_v. For this example, the actual values at the face of the support will be used.

Longitudinal reinforcement, previously computed for positive moment design.......

$A_{ps.Support} = 6.7 \cdot in^2$

The prestressing strand force is not all effective at the support area due to the transfer length required to go from zero force to maximum force. A factor will be applied that takes this into account.

Transfer length...................................... $L_{transfer} = 36 \cdot in$

Distance from center line of bearing to end of beam... $J = 8 \cdot in$

Estimated length of bearing pad.............. $L_{pad} := 8 \cdot in$

Determine the force effective at the inside edge of the bearing area.

Factor to account for effective force........ $Factor := \dfrac{J + \dfrac{L_{pad}}{2}}{L_{transfer}} = 0.33$

Equivalent force provided by this steel...... $T_{psSupport} := A_{ps.Support} \cdot f_{pe} \cdot Factor = 454.1 \cdot kip$

$LRFD_{5.8.3.5} :=$ | "Ok, positive moment longitudinal reinforcement is adequate" if $T_{psSupport} \geq T$
| "NG, positive moment longitudinal reinforcement shall be provided" otherwise

$LRFD_{5.8.3.5}$ = "Ok, positive moment longitudinal reinforcement is adequate"

D4. Interface Shear Reinforcement [LRFD 5.8.4]

Assumed a roughened surface per LRFD 5.8.4.3: $c := 0.28 \cdot ksi$ $\mu := 1.00$ $K_1 := 0.3$ $K_2 := 1.8ksi$

Distance between the centroid of the steel in the tension side of the beam to the center of the compression blocks in the deck......

$$d_{interface.avg} := d_p - \frac{t_{slab}}{2} = 36.6 \cdot in$$

$$V_{u.interface} := V_u = 300.6 \cdot kip$$

Interface shear force per unit length per LRFD C5.8.4.2-7..................................

$$V_{uh} := \frac{V_{u.interface}}{d_{interface.avg}} = 8.21 \cdot \frac{kip}{in}$$

$$V_{nh.reqd} := \frac{V_{uh}}{\phi_v} = 9.1 \cdot \frac{kip}{in}$$

$$A_{cv} := b_{tf} \cdot \left(\frac{in}{in}\right) = 48 \cdot \frac{in^2}{in}$$

$$V_{nh.max} := \min\left(K_2 \cdot A_{cv}, K_1 \cdot f_{c.slab} \cdot A_{cv}\right) ft = 777.6 \cdot kip$$

$$CheckV_{nh.reqd} := \text{if}\left(V_{nh.reqd} \cdot ft < V_{nh.max}, \text{"OK"}, \text{"Check V.nh.reqd"}\right) = \text{"OK"}$$

The minimum reinforcement requirement may be waived if $\frac{V_{uh}}{A_{cv}} < 0.21\ ksi$ assuming requirements of **LRFD 5.8.4.4**.are satisfied.

$$MinInterfaceReinfReqd := \text{if}\left(\frac{V_{uh}}{A_{cv}} < 0.21 \cdot ksi, \text{"No"}, \text{"Yes"}\right) = \text{"No"}$$

Minimum interface steel, if required

$$A_{vf.min} := \text{if}\left(MinInterfaceReinfReqd = \text{"Yes"}, \min\left(0.05 \cdot A_{cv} \cdot \frac{ksi}{f_y}, \frac{1.33 \cdot V_{nh.reqd} - c \cdot A_{cv}}{\mu \cdot f_y}\right), 0 \cdot \frac{in^2}{ft}\right) = 0 \cdot \frac{in^2}{ft}$$

Design interface steel per LRFD 5.8.4.1-3, if required

$$A_{vf.des} := \max\left(\frac{V_{nh.reqd} - c\cdot A_{cv}}{\mu\cdot f_y}, 0\cdot\frac{in^2}{ft}\right) = 0\cdot\frac{in^2}{ft}$$

$$A_{vf.reqd} := \max(A_{vf.min}, A_{vf.des}) = 0\cdot\frac{in^2}{ft}$$

Actual stirrup spacing per Design Standard

$$spacing := 3in$$

Area of reinforcement passing through the interface between the deck and the girder. Minimum interface steel, if required:

$$A_{v.prov.interface} := \frac{A_v}{spacing}$$

$$TotalInterfaceSteelProvided := A_{v.prov.interface} = 2.5\cdot\frac{in^2}{ft}$$

$$TotalInterfaceSteelRequired := A_{vf.reqd} = 0\cdot\frac{in^2}{ft}$$

$$CheckInterfaceSteel := if\left(\frac{TotalInterfaceSteelProvided}{TotalInterfaceSteelRequired + 0.001\cdot\frac{in^2}{ft}} \geq 1, \text{"OK"}, \text{"Add Interface Steel"}\right) = \text{"OK"}$$

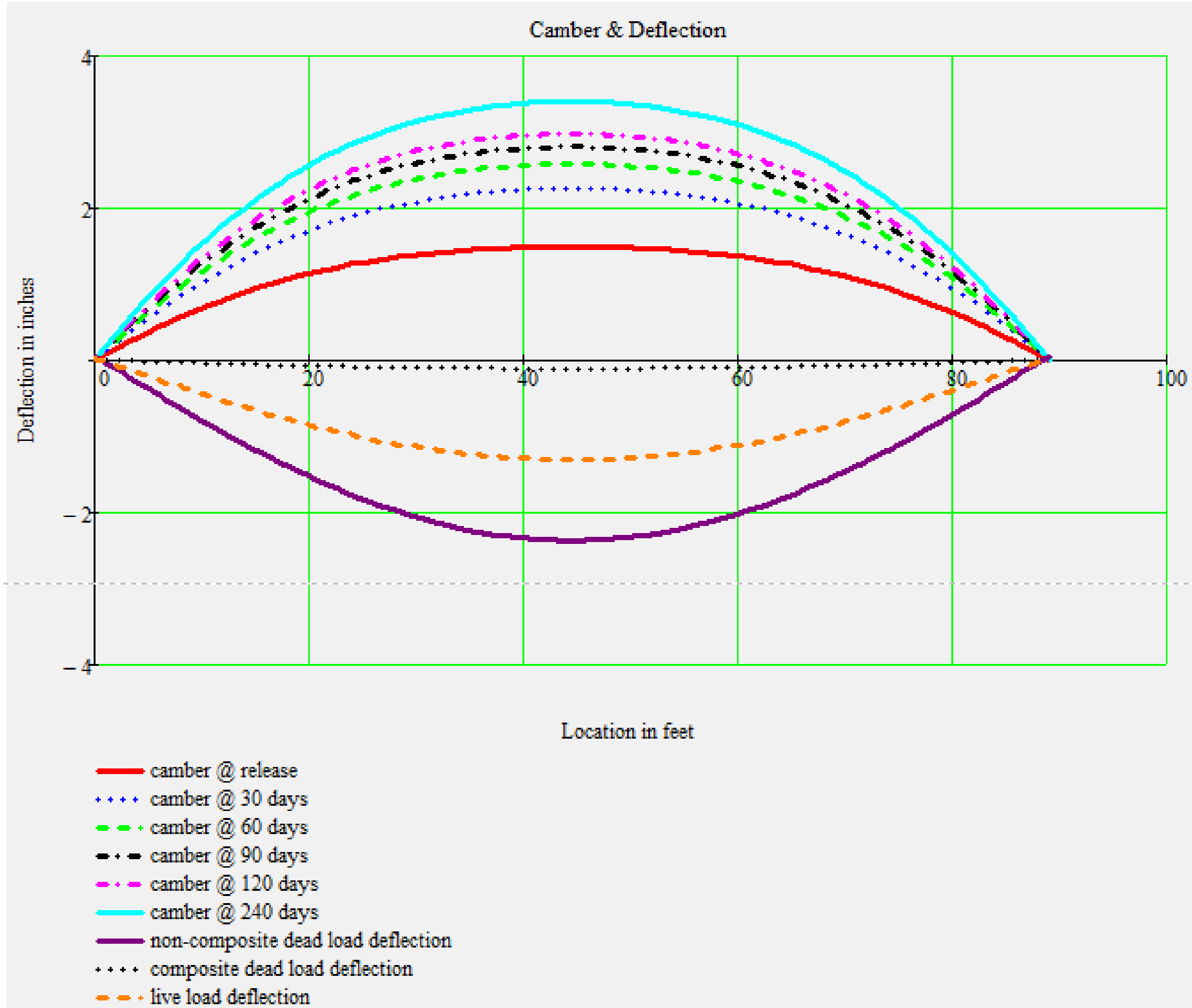
$$CR_{InterfaceSteel} := \frac{TotalInterfaceSteelProvided}{TotalInterfaceSteelRequired + 0.001\cdot\frac{in^2}{ft}} = 2480$$

Note:
Typically shear steel is extended up into the deck slab.
These calculations are based on shear steel functioning as interface reinforcing.
The interface_factor can be used to adjust this assumption.

E. Deflection Check

The FDOT Prestressed beam program version 3.21 was utilized to quickly determine if the FIB36 used in the interior and exterior beam design will satisfy the deflection criteria per **SDG 1.2**. Shown below is the calculated deflection of the exterior beam, which has a higher deflection than the interior beam.

Camber and Shrinkage and Dead Load Deflections



SlopeData =

"Stage"	"Change in L @ Top (in)"	"Change in L @ Bot. (in)"	"Slope at End (deg)"	"midspan defl (in)"
"Release"	−0.0411	−0.7397	0.3753	1.4947
"30 Days"	−0.1707	−1.1287	0.5584	2.2613
"60 Days"	−0.2426	−1.3082	0.6345	2.5797
"90 Days"	−0.2847	−1.4265	0.6883	2.8049
"120 Days"	−0.3257	−1.5257	0.7293	2.9767
"240 Days"	−0.4042	−1.747	0.8301	3.3988
"non-comp DL"	−0.2875	0.2294	−0.4116	−2.3631
"comp DL"	−0.0056	0.0182	−0.0189	−0.1088
"LL"	−0.0678	0.2196	−0.2288	$\boxed{-1.3033}$

Live load deflection.............. $\Delta_{LL} := 1.3033in$

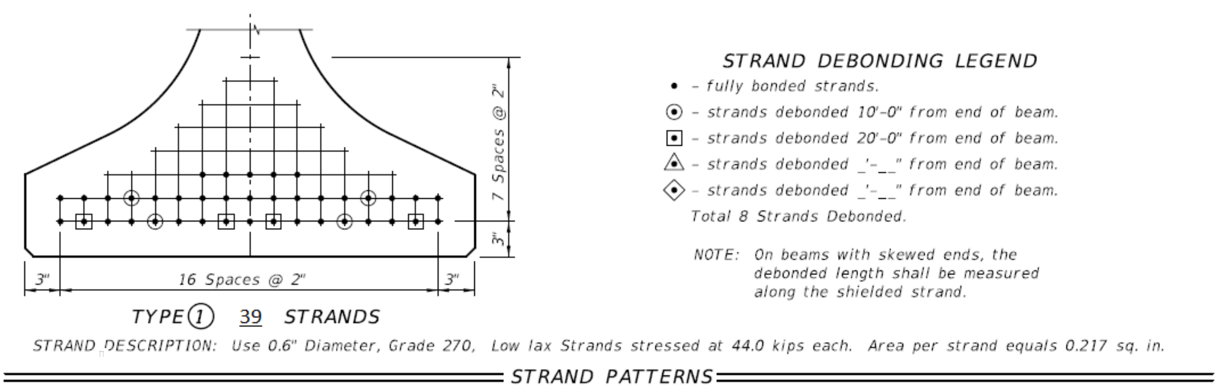
Deflection limit.................... $\Delta_{Allow} := \frac{L_{span}}{800} = 1.35 \cdot in$

$Check\_\Delta := if\left(\Delta_{Allow} > \Delta_{LL}, "OK", "Not\ OK"\right)$ $Check\_\Delta = "OK"$

Several important design checks were not performed in this design example (to reduce the length of calculations). However, the engineer should assure that the following has been done at a minimum:

- Design for anchorage steel
- Design for camber
- Design check for beam transportation loads
- Design for fatigue checks when applicable

F. Summary



Traditional Deck Design

References

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\205PSBeam2.xmcd(R)

Description

This section provides the criteria for the traditional deck design.

Page Contents

LRFD Criteria

Live Loads - Application of Design Vehicular Live Loads - Deck Overhang Load [LRFD 3.6.1.3.4]

This section is not applicable for Florida designs, since the barriers are not designed as structurally continuous and composite with the deck slab.

Static Analysis - Approximate Methods of Analysis - Decks [LRFD 4.6.2.1]

Deck Slab Design Table [LRFD Appendix A4]

Table A4-1 in Appendix 4 may be used to determine the design live load moments.

STRENGTH I - Basic load combination relating to the normal vehicular use of the bridge without wind.

$WA, FR = 0$ — For superstructure design, water load / stream pressure and friction forces are not applicable.

$TU, CR, SH, FR = 0$ — Uniform temperature, creep, shrinkage are generally ignored.

$$Strength1 = 1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL$$

SERVICE I - Load combination relating to the normal operational use of the bridge with a 55 MPH wind and all loads taken at their nominal values.

$BR, WS, WL = 0$ — For superstructure design, braking forces, wind on structure and wind on live load are not applicable.

$$Service1 = 1.0 \cdot DC + 1.0 \cdot DW + 1.0 \cdot LL$$

FATIGUE - Fatigue load combination relating to repetitive gravitational vehicular live load under a single design truck.

"Not applicable for deck slabs on multi-beam bridges"

FDOT Criteria

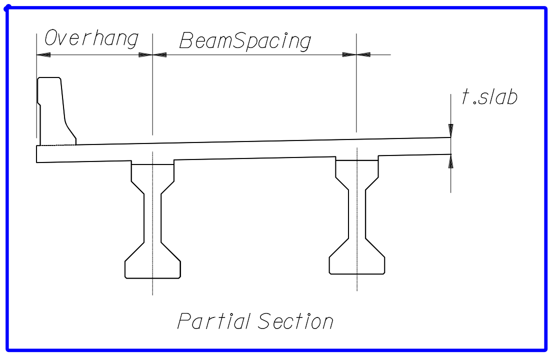
Skewed Decks [SDG 4.2.10]

Transverse steel.................................... "Perpendicular to CL of span "

Top reinforcement for deck slab along skew.. "Use 3 #5 bars @ 6 inch spacing"

Top and bottom longitudinal reinforcement for deck slab at skew.............................. "Double required amount of reinforcing"

A. Input Variables



Bridge design span length........................ $L_{span} = 90\,ft$

Number of beams.................................. $N_{beams} = 9$

Beam Spacing....................................... $BeamSpacing = 10\,ft$

$S := BeamSpacing$

Average Buildup.................................... $h_{buildup} = 1 \cdot in$

Beam top flange width............................ $b_{tf} = 48 \cdot in$

Thickness of deck slab........................... $t_{slab} = 8 \cdot in$

Milling surface thickness......................... $t_{mill} = 0.5 \cdot in$

Deck overhang...................................... $Overhang = 4.542\,ft$

Dynamic Load Allowance........................ $IM = 1.33$

Bridge skew... $Skew = -20 \cdot deg$

B. Approximate Methods of Analysis - Decks [LRFD 4.6.2]

B1. Width of Equivalent Interior Strips [LRFD 4.6.2.1.3]

The deck is designed using equivalent strips of deck width. The equivalent strips account for the longitudinal distribution of LRFD wheel loads and are not subject to width limitations. The width in the transverse direction is calculated for both positive and negative moments. The overhangs will not be addressed in this section.

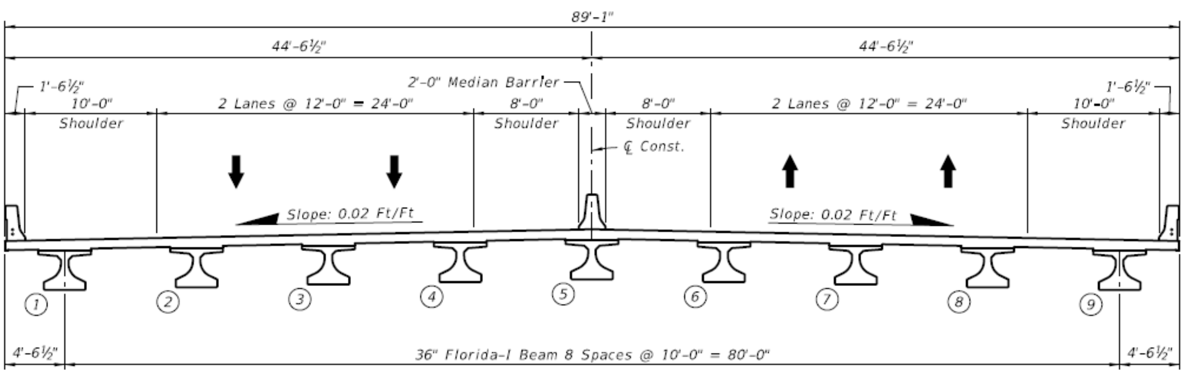
Width of equivalent strip for positive moment..

$$E_{pos} := \left(26 + 6.6 \cdot \frac{S}{ft}\right) \cdot in = 92 \cdot in$$

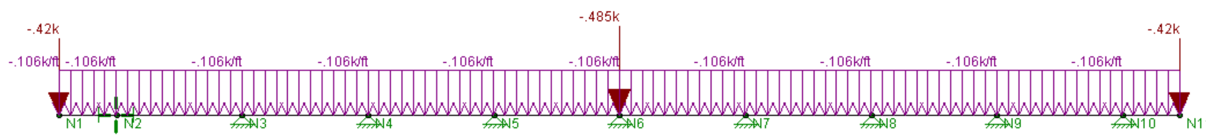
Width of equivalent strip for negative moment..

$$E_{neg} := \left(48 + 3.0 \cdot \frac{S}{ft}\right) \cdot in = 78 \cdot in$$

The equivalent strips can be modeled as continuous beams on rigid supports, since typical Florida-I beam bridges do not have any transverse beams.



TYPICAL SECTION



B2. Live Loads for Equivalent Strips

All HL-93 wheel loads shall be applied to the equivalent strip of deck width, since the spacing of supporting components in the secondary direction (longitudinal to beams) exceeds 1.5 times the spacing in the primary direction (transverse to beams). **[LRFD 4.6.2.1.5]**

HL-93 wheel load.................................. $P := 16kip$

HL-93 wheel load for negative moment..... $P_{neg} := \frac{P}{E_{neg}} \cdot (IM) = 3.27 \cdot klf$

HL-93 wheel load for positive moment..... $P_{pos} := \frac{P}{E_{pos}} \cdot (IM) = 2.78 \cdot klf$

Location of Negative Live Load Design Moment

The negative live load design moment is taken at a distance from the supports....... $Loc_{negative} := \min\left(\frac{1}{3} \cdot b_{tf}, 15 \cdot in\right) = 15 \cdot in$

HL-93 Live Load Design Moments

Instead of performing a continuous beam analysis, Table A4-1 in Appendix A4 may be used to determine the live load design moments. The following assumptions and limitations should be considered when using these moments:

- The moments are calculated by applying the equivalent strip method to concrete slabs supported on parallel beams.
- Multiple presence factors and dynamic load allowance are included.
- The values are calculated according to the location of the design section for negative moments in the deck (LRFD 4.6.2.1.6). For distances between the listed values, interpolation may be used.
- The moments are applicable for decks supported by at least three beams with a width between the centerlines of the exterior beams of not less than 14.0 ft.
- The values represent the upper bound for moments in the interior regions of the slab.
- A minimum and maximum total overhang width from the center of the exterior girder are evaluated. The minimum is 21 in. and the maximum is the smaller of $(0.625 \cdot BeamSpacing)$ and 6 ft.
- A railing barrier width of 21.0 in. is used to determine the clear overhang width. Florida utilizes a railing width of 18.5 in. The difference in moments from the different railing width is expected to be within acceptable limits for practical design.
- The moments do not apply to deck overhangs, which may be detailed in accordance with the provisions of SDG 4.2.4.B without further analysis (see section Deck Overhang Design of these calculations).

S		Positive Moment	NEGATIVE MOMENT						
			Distance from CL of Girder to Design Section for Negative Moment						
			0.0 in.	3 in.	6 in.	9 in.	12 in.	18 in.	24 in.
4'	-0"	4.68	2.68	2.07	1.74	1.60	1.50	1.34	1.25
4'	-3"	4.66	2.73	2.25	1.95	1.74	1.57	1.33	1.20
4'	-6"	4.63	3.00	2.58	2.19	1.90	1.65	1.32	1.18
4'	-9"	4.64	3.38	2.90	2.43	2.07	1.74	1.29	1.20
5'	-0"	4.65	3.74	3.20	2.66	2.24	1.83	1.26	1.12
5'	-3"	4.67	4.06	3.47	2.89	2.41	1.95	1.28	0.98
5'	-6"	4.71	4.36	3.73	3.11	2.58	2.07	1.30	0.99
5'	-9"	4.77	4.63	3.97	3.31	2.73	2.19	1.32	1.02
6'	-0"	4.83	4.88	4.19	3.50	2.88	2.31	1.39	1.07
6'	-3"	4.91	5.10	4.39	3.68	3.02	2.42	1.45	1.13
6'	-6"	5.00	5.31	4.57	3.84	3.15	2.53	1.50	1.20
6'	-9"	5.10	5.50	4.74	3.99	3.27	2.64	1.58	1.28
7'	-0"	5.21	5.98	5.17	4.36	3.56	2.84	1.63	1.37
7'	-3"	5.32	6.13	5.31	4.49	3.68	2.96	1.65	1.51
7'	-6"	5.44	6.26	5.43	4.61	3.78	3.15	1.88	1.72
7'	-9"	5.56	6.38	5.54	4.71	3.88	3.30	2.21	1.94
8'	-0"	5.69	6.48	5.65	4.81	3.98	3.43	2.49	2.16
8'	-3"	5.83	6.58	5.74	4.90	4.06	3.53	2.74	2.37
8'	-6"	5.99	6.66	5.82	4.98	4.14	3.61	2.96	2.58
8'	-9"	6.14	6.74	5.90	5.06	4.22	3.67	3.15	2.79
9'	-0"	6.29	6.81	5.97	5.13	4.28	3.71	3.31	3.00
9'	-3"	6.44	6.87	6.03	5.19	4.40	3.82	3.47	3.20
9'	-6"	6.59	7.15	6.31	5.46	4.66	4.04	3.68	3.39
9'	-9"	6.74	7.51	6.65	5.80	4.94	4.21	3.89	3.58
10'	-0"	6.89	7.85	6.99	6.13	5.26	4.41	4.09	3.77
10'	-3"	7.03	8.19	7.32	6.45	5.58	4.71	4.29	3.96
10'	-6"	7.17	8.52	7.64	6.77	5.89	5.02	4.48	4.15
10'	-9"	7.32	8.83	7.95	7.08	6.20	5.32	4.68	4.34
11'	-0"	7.46	9.14	8.26	7.38	6.50	5.62	4.86	4.52
11'	-3"	7.60	9.44	8.55	7.67	6.79	5.91	5.04	4.70
11'	-6"	7.74	9.72	8.84	7.96	7.07	6.19	5.22	4.87
11'	-9"	7.88	10.01	9.12	8.24	7.36	6.47	5.40	5.05
12'	-0"	8.01	10.28	9.40	8.51	7.63	6.74	5.56	5.21
12'	-3"	8.15	10.55	9.67	8.78	7.90	7.02	5.75	5.38
12'	-6"	8.28	10.81	9.93	9.04	8.16	7.28	5.97	5.54
12'	-9"	8.41	11.06	10.18	9.30	8.42	7.54	6.18	5.70
13'	-0"	8.54	11.31	10.43	9.55	8.67	7.79	6.38	5.86
13'	-3"	8.66	11.55	10.67	9.80	8.92	8.04	6.59	6.01
13'	-6"	8.78	11.79	10.91	10.03	9.16	8.28	6.79	6.16
13'	-9"	8.90	12.02	11.14	10.27	9.40	8.52	6.99	6.30
14'	-0"	9.02	12.24	11.37	10.50	9.63	8.76	7.18	6.45
14'	-3"	9.14	12.46	11.59	10.72	9.85	8.99	7.38	6.58
14'	-6"	9.25	12.67	11.81	10.94	10.08	9.21	7.57	6.72
14'	-9"	9.36	12.88	12.02	11.16	10.30	9.44	7.76	6.86

For this example.................................... $BeamSpacing = 10\,ft$

$Loc_{negative} = 15 \cdot in$

Positive Live Load Design Moment........... $M_{LL.pos} := 6.89 \cdot ft \cdot kip$

Negative Live Load Design Moment.....

$$M_{LL.neg} := (15in - 12 \cdot in) \cdot \left[\frac{(4.09 \cdot ft \cdot kip - 4.41 \cdot ft \cdot kip)}{(18 \cdot in - 12 \cdot in)} \right] + 4.41 \cdot (ft \cdot kip) = 4.25 \cdot kip \cdot$$

*(**Note**: Interpolated value)*

B3. Dead Load Design Moments

Design width of deck slab....................... $b_{slab} := 1ft$

"DC" loads include the dead load of structural components and non-structural attachments

Self-weight of deck slab.................... $w_{slab} := \left[\left(t_{slab} + t_{mill}\right)\cdot b_{slab}\right]\cdot\gamma_{conc} = 0.106\cdot klf$

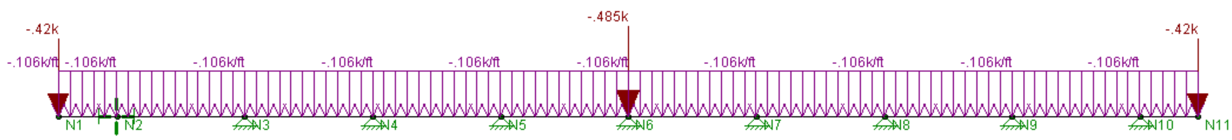
Weight of traffic barriers.................. $P_{barrier} := w_{barrier.ea}\cdot b_{slab} = 0.42\cdot kip$

Weight of median barrier.................. $P_{median.barrier} := w_{median.bar}\cdot b_{slab} = 0.485\cdot kip$

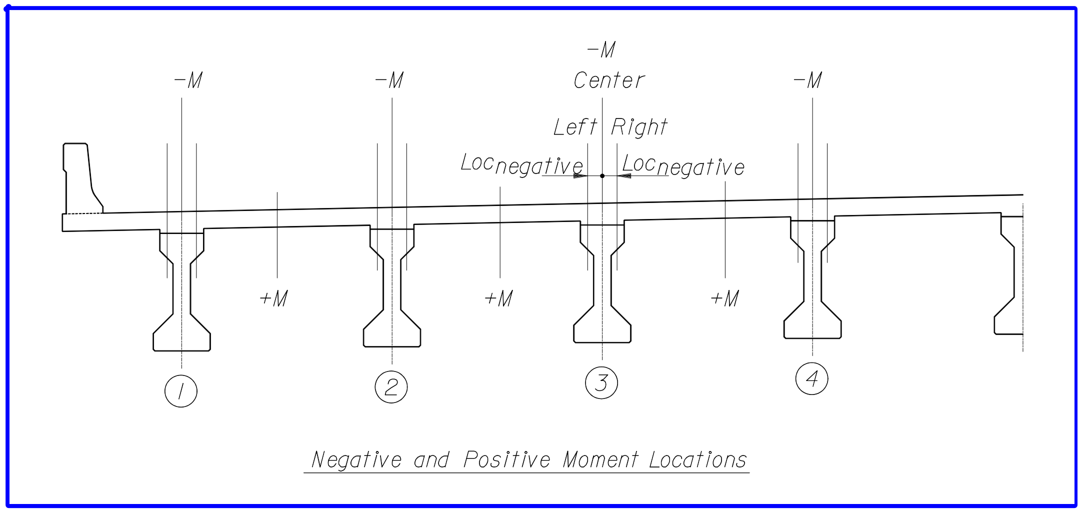
"DW" loads include the dead load of a future wearing surface and utilities

Weight of Future Wearing Surface..... $w_{fws} := \rho_{fws}\cdot b_{slab} = 0\cdot klf$

Analysis Model for Dead Loads



Any plane frame program can be utilized to develop the moments induced by the dead loads. For this example, RISA was used to determine the dead load design moments for both the DC and DW loads.



Beam / Span	Design Moments for DC Loads Positive Moment (k-ft)	Negative Moment (k-ft) Center	Left	Right
1	0.00	-3.00	-2.04	-2.09
2	0.67	-0.32	-0.07	0.17
3	0.39	-1.04	-0.37	-0.43
4	0.45	-0.84	-0.29	-0.27

The governing negative design moment for DC loads occurs at beam 1. However, this moment is due to the overhang, which typically has more negative moment steel requirements than the interior regions of the deck. Since the overhang is designed separately, the overhang moments are not considered here. For the interior regions, the positive moment in Span 2 and the negative moment to the right of beam 3 govern.

Positive moment.............................. $M_{DC.pos} := 0.67 \cdot kip \cdot ft$

Negative moment............................ $M_{DC.neg} := 0.43 \cdot kip \cdot ft$

Beam / Span	Design Moments for DW Loads Positive Moment (k-ft)	Negative Moment (k-ft) Center	Left	Right
1	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00
4	0.00	0.00	0.00	0.00

Positive moment.............................. $M_{DW.pos} := 0.0 \cdot kip \cdot ft$

Negative moment............................. $M_{DW.neg} := 0.0 \cdot kip \cdot ft$

B4. Limit State Moments

The service and strength limit states are used to design the section

Service I Limit State

Positive Service I Moment...................... $M_{serviceI.pos} := M_{DC.pos} + M_{DW.pos} + M_{LL.pos} = 7.56 \cdot kip \cdot ft$

Negative Service I Moment..................... $M_{serviceI.neg} := M_{DC.neg} + M_{DW.neg} + M_{LL.neg} = 4.68 \cdot kip \cdot ft$

Strength I Limit State

Positive Strength I Moment......... $M_{strengthI.pos} := 1.25M_{DC.pos} + 1.50 \cdot M_{DW.pos} + 1.75 \cdot M_{LL.pos} = 12.89 \cdot kip \cdot ft$

Negative Strength I Moment.......... $M_{strengthI.neg} := 1.25M_{DC.neg} + 1.50 \cdot M_{DW.neg} + 1.75 \cdot M_{LL.neg} = 7.98 \cdot kip \cdot ft$

C. Moment Design

A few recommendations on bar size and spacing are available to minimize problems during construction.

- The same size and spacing of reinforcing should be utilized for both the negative and positive moment regions.
- If this arrangement is not possible, the top and bottom reinforcement should be spaced as a multiple of each other. This pattern places the top and bottom bars in the same grid pattern, and any additional steel is placed between these bars.

The design procedure consists of calculating the reinforcement required to satisfy the design moment, then checking this reinforcement against criteria for crack control, minimum reinforcement, maximum reinforcement, shrinkage and temperature reinforcement, and distribution of reinforcement. The procedure is the same for both positive and negative moment regions.

C1. Positive Moment Region Design - Flexural Resistance [LRFD 5.7.3.2]

Factored resistance

$$M_r = \phi \cdot M_n$$

Nominal flexural resistance

$$M_n = A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2}\right) + A_s \cdot f_y \cdot \left(d_s - \frac{a}{2}\right) - A'_s \cdot f'_y \cdot \left(d'_s - \frac{a}{2}\right) + 0.85 \cdot f'_c \cdot (b - b_w) \cdot h_f \cdot \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

Simplifying the nominal flexural resistance

$$M_n = A_s \cdot f_y \cdot \left(d_s - \frac{a}{2}\right) \qquad \text{where} \qquad a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b}$$

Using variables defined in this example..... $M_r = \phi \cdot A_{s.pos} \cdot f_y \cdot \left[d_s - \frac{1}{2} \cdot \left(\frac{A_{s.pos} \cdot f_y}{0.85 \cdot f_{c.slab} \cdot b}\right)\right]$

where $M_r := M_{strengthI.pos}$

$f_{c.slab} = 4.5 \cdot ksi$

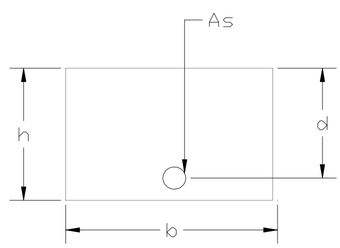
$f_y = 60 \cdot ksi$

$\phi = 0.9$

$t_{slab} = 8 \cdot in$

$b := b_{slab}$

$t_{slab} = h$



$b = 12 \cdot in$

Initial assumption for area of steel required

Size of bar..................................... bar := "5"

Proposed bar spacing...................... $spacing_{pos} := 6 \cdot in$

Bar area.. $A_{bar} = 0.310 \cdot in^2$

Bar diameter.................................. $dia = 0.625 \cdot in$

Area of steel provided per foot of slab...... $A_{s.pos} := \dfrac{A_{bar} \cdot 1ft}{spacing_{pos}} = 0.62 \cdot in^2$

Distance from extreme compressive fiber to centroid of reinforcing steel.................... $d_{s.pos} := t_{slab} + t_{mill} - cover_{deck} - \dfrac{dia}{2} = 5.69 \cdot in$

Solve the quadratic equation for the area of steel required....................................... Given $M_r = \phi \cdot A_{s.pos} \cdot f_y \cdot \left[d_{s.pos} - \dfrac{1}{2} \cdot \left(\dfrac{A_{s.pos} \cdot f_y}{0.85 \cdot f_{c.slab} \cdot b} \right) \right]$

Reinforcing steel required....................... $A_{s.reqd.pos} := Find(A_{s.pos}) = 0.54 \cdot in^2$

The area of steel provided, $A_{s.pos} = 0.62 \cdot in^2$, should be greater than the area of steel required, $A_{s.reqd.pos} = 0.54 \cdot in^2$. If not, decrease the spacing of the reinforcement. Once $A_{s.pos}$ is greater than $A_{s.reqd}$, the proposed reinforcing is adequate for the design moments.

C2. Negative Moment Region Design - Flexural Resistance [LRFD 5.7.3.2]

Variables: $M_r := M_{strengthI.neg} = 7.98 \cdot kip \cdot ft$

$f_{c.slab} = 4.5 \cdot ksi$

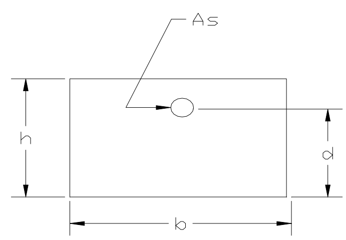
$f_y = 60 \cdot ksi$

$\phi = 0.9$

$t_{slab} = 8 \cdot in$

$b := b_{slab}$

$t_{slab} = h$



$b = 12 \cdot in$

Initial assumption for area of steel required

Size of bar...................................... bar = "5"

Proposed bar spacing...................... $spacing_{neg} := 6in$

Bar area.. $A_{bar} = 0.310 \cdot in^2$

Bar diameter.................................. $dia = 0.625 \cdot in$

Area of steel provided per foot of slab...... $A_{s.neg} := \dfrac{A_{bar} \cdot 1ft}{spacing_{neg}} = 0.62 \cdot in^2$

Distance from extreme compressive fiber to centroid of reinforcing steel................ $d_{s.neg} := t_{slab} + t_{mill} - cover_{deck} - \dfrac{dia}{2} = 5.69 \cdot in$

Solve the quadratic equation for the area of steel required....................................... Given $M_r = \phi \cdot A_{s.neg} \cdot f_y \cdot \left[d_{s.neg} - \dfrac{1}{2} \cdot \left(\dfrac{A_{s.neg} \cdot f_y}{0.85 \cdot f_{c.slab} \cdot b} \right) \right]$

Reinforcing steel required....................... $A_{s.reqd.neg} := Find(A_{s.neg}) = 0.324 \cdot in^2$

The area of steel provided, $A_{s.neg} = 0.62 \cdot in^2$, should be greater than the area of steel required, $A_{s.reqd.neg} = 0.32 \cdot in^2$. If not, decrease the spacing of the reinforcement. Once $A_{s.neg}$ is greater than $A_{s.reqd}$, the proposed reinforcing is adequate for the design moments.

C3. Crack Control by Distribution Reinforcement [LRFD 5.7.3.4]

Concrete is subjected to cracking. Limiting the width of expected cracks under service conditions increases the longevity of the structure. Potential cracks can be minimized through proper placement of the reinforcement. The check for crack control requires that the actual stress in the reinforcement should not exceed the service limit state stress (LRFD 5.7.3.4). The stress equations emphasize bar spacing rather than crack widths.

The maximum spacing of the mild steel reinforcement for control of cracking at the service limit state shall satisfy......... $s \le \dfrac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$

where $\beta_s = 1 + \dfrac{d_c}{0.7(h - d_c)}$

Exposure factor for Class 1 exposure condition....................................... $\gamma_e := 1.00$ **[SDG 3.10]**

Overall thickness or depth of the component............................. $t_{slab} = 8 \cdot in$

Positive Moment

Distance from extreme tension fiber to center of closest bar.............................. $d_c := cover_{deck} - t_{mill} + \dfrac{dia}{2} = 2.31 \cdot in$

$$\beta_s := 1 + \frac{d_c}{0.7\left(t_{slab} - d_c\right)} = 1.581$$

The neutral axis of the section must be determined to determine the actual stress in the reinforcement. This process is iterative, so an initial assumption of the neutral axis must be made.

Guess $x := 1.8 \cdot in$

Given $\frac{1}{2} \cdot b \cdot x^2 = \frac{E_s}{E_{c.slab}} \cdot A_{s.pos} \cdot \left(d_{s.pos} - x\right)$

$$x_{na.pos} := \text{Find}(x) = 1.824 \cdot in$$

Tensile force in the reinforcing steel due to service limit state moment......................

$$T_s := \frac{M_{serviceI.pos}}{d_{s.pos} - \frac{x_{na.pos}}{3}} = 17.86 \cdot kip$$

Actual stress in the reinforcing steel due to service limit state moment......................

$$f_{s.actual} := \frac{T_s}{A_{s.pos}} = 28.81 \cdot ksi$$

Required reinforcement spacing..........

$$s_{required} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{s.actual}} - 2 \cdot d_c = 10.75 \cdot in$$

Provided reinforcement spacing..........

$$spacing_{pos} = 6 \cdot in$$

The required spacing of mild steel reinforcement in the layer closest to the tension face shall not be less than the reinforcement spacing provided due to the service limit state moment.

$$LRFD_{5.7.3.4} := \begin{cases} \text{"OK, crack control for +M is satisfied"} & \text{if } s_{required} \geq spacing_{pos} \\ \text{"NG, crack control for +M not satisfied, provide more reinforcement"} & \text{otherwise} \end{cases}$$

$$LRFD_{5.7.3.4} = \text{"OK, crack control for +M is satisfied"}$$

Negative Moment

Distance from extreme tension fiber to center of closest bar ....

$$d_c := cover_{deck} - t_{mill} + \frac{dia}{2} = 2.31 \cdot in$$

$$\beta_s := 1 + \frac{d_c}{0.7(t_{slab} - d_c)} = 1.58$$

The neutral axis of the section must be determined to determine the actual stress in the reinforcement. This process is iterative, so an initial assumption of the neutral axis must be made.

Guess $x := 1.8 \cdot in$

Given $\frac{1}{2} \cdot b \cdot x^2 = \frac{E_s}{E_{c.slab}} \cdot A_{s.neg} \cdot (d_{s.neg} - x)$

$$x_{na.neg} := Find(x) = 1.824 \cdot in$$

Tensile force in the reinforcing steel due to service limit state moment........................ $T_s := \frac{M_{serviceI.neg}}{d_{s.neg} - \frac{x_{na.neg}}{3}} = 11.06 \cdot kip$

Actual stress in the reinforcing steel due to service limit state moment..................... $f_{s.actual} := \frac{T_s}{A_{s.neg}} = 17.83 \cdot ksi$

Required reinforcement spacing........... $s_{required} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{s.actual}} - 2 \cdot d_c = 20.21 \cdot in$

Provided reinforcement spacing........... $spacing_{neg} = 6 \cdot in$

The required spacing of mild steel reinforcement in the layer closest to the tension face shall not be less than the reinforcement spacing provided due to the service limit state moment.

$LRFD_{5.7.3.4} :=$ | "OK, crack control for -M is satisfied" if $s_{required} \geq spacing_{neg}$
| "NG, crack control for -M not satisfied, provide more reinforcement" otherwise

$LRFD_{5.7.3.4} =$ "OK, crack control for -M is satisfied"

C4. Limits for Reinforcement [LRFD 5.7.3.3]

Minimum Reinforcement [5.7.3.3.2]

Area of steel provided............................. $A_s := max(A_{s.pos}, A_{s.neg}) = 0.62 \cdot in^2$

Area of steel required for bending............ $A_{s.reqd} := \min\left(A_{s.reqd.pos}, A_{s.reqd.neg}\right) = 0.32 \cdot in^2$

The minimum reinforcement requirements ensure the moment capacity provided is at least 1.2 times greater than the cracking moment.

Modulus of Rupture.............................. $f_r := 0.24 \cdot \sqrt{f_{c.slab} \cdot ksi} = 0.509 \cdot ksi$ [SDG 1.4.1.B]

Distance from the extreme tensile fiber to the neutral axis of the composite section... $y := \frac{t_{slab}}{2} = 4 \cdot in$

Moment of inertia for the section............ $I_{slab} := \frac{1}{12} \cdot b \cdot t_{slab}^{\ 3} = 512 \cdot in^4$

Section modulus.................................... $S := \frac{I_{slab}}{y} = 128 \cdot in^3$

Flexural cracking variability factor........... $\gamma_1 := 1.6$

Ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement....................................... $\gamma_3 := 0.67$

Cracking moment.................................. $M_{cr} := \gamma_3 \cdot \left[\left(\gamma_1 \cdot f_r\right) \cdot S\right] = 5.82 \cdot kip \cdot ft$

Minimum distance to reinforcing steel...... $d_s := \min\left(d_{s.pos}, d_{s.neg}\right) = 5.69 \cdot in$

Minimum reinforcement required............

$$A_{min} := \frac{\dfrac{M_{cr}}{\phi}}{f_y \cdot \left[d_s - \dfrac{1}{2}\left(\dfrac{A_s \cdot f_y}{0.85 \cdot f_{c.slab} \cdot b}\right)\right]} = 0.24 \cdot in^2$$

Required area of steel for minimum reinforcement should not be less than $A_{s.reqd} \cdot 133\%$ or A_{min} .... $A_{s.req} := \min\left(A_{s.reqd} \cdot 133 \cdot \%, A_{min}\right) = 0.24 \cdot in^2$

Maximum bar spacing for mimimum reinforcement.......................................

$$spacing_{max} := \frac{b}{\left(\frac{A_{s.req}}{A_{bar}}\right)} = 15.19 \cdot in$$

Greater bar spacing from positive and negative moment section.........................

$$spacing := \max(spacing_{pos}, spacing_{neg}) = 6 \cdot in$$

The bar spacing should be less than the maximum bar spacing for minimum reinforcement

$$LRFD_{5.7.3.3.2} := \begin{cases} \text{"OK, minimum reinforcement requirements are satisfied"} & \text{if } spacing \le spacing_{max} \\ \text{"NG, section is under-reinforced, so redesign!"} & \text{otherwise} \end{cases}$$

$$LRFD_{5.7.3.3.2} = \text{"OK, minimum reinforcement requirements are satisfied"}$$

C5. Shrinkage and Temperature Reinforcement [LRFD 5.10.8]

Shrinkage and temperature reinforcement provided

Size of bar ("4" "5" "6")........ $bar_{st} := \text{"5"}$

Bar spacing..................................... $bar_{spa.st} := 8 \cdot in$

Bar area... $A_{bar.st} = 0.31 \cdot in^2$

Bar diameter.................................... $dia = 0.625 \cdot in$

Gross area of section............................. $A_g := b_{slab} \cdot t_{slab} = 96 \cdot in^2$

Minimum area of shrinkage and temperature reinforcement.......................................

$$A_{ST} := \min\left[\begin{bmatrix} 0.60 \frac{in^2}{ft} \\ \max\left[\begin{bmatrix} \left(0.11 \frac{in^2}{ft}\right) \\ \left[\frac{1.30 \cdot \frac{kip}{in \cdot ft} \cdot A_g}{2 \cdot (b_{slab} + t_{slab}) f_y}\right] \end{bmatrix}\right] \end{bmatrix}\right] = 0.11 \cdot \frac{in^2}{ft}$$

Maximum spacing for shrinkage and temperature reinforcement......................

$$spacing_{ST} := \min\left(\frac{A_{bar.st}}{A_{ST}}, 3 \cdot t_{slab}, 18 \cdot in\right) = 18 \cdot in$$

The bar spacing should be less than the maximum spacing for shrinkage and temperature reinforcement

$$LRFD_{5.7.10.8} := \begin{cases} \text{"OK, minimum shrinkage and temperature requirements"} & \text{if } bar_{spa.st} \le spacing_{ST} \\ \text{"NG, minimum shrinkage and temperature requirements"} & \text{otherwise} \end{cases}$$

$$LRFD_{5.7.10.8} = \text{"OK, minimum shrinkage and temperature requirements"}$$

C6. Distribution of Reinforcement [LRFD 9.7.3.2]

The primary reinforcement is placed perpendicular to traffic, since the effective strip is perpendicular to traffic. Reinforcement shall also be placed in the secondary direction (parallel to traffic) for load distribution purposes. This reinforcement is placed in the bottom of the deck slab as a percentage of the primary reinforcement.

Distribution reinforcement provided

Size of bar ("4" "5" "6")........ $bar_{dist} := \text{"5"}$

Bar spacing..................................... $bar_{spa.dist} := bar_{spa.st} = 8 \cdot in$

Bar area... $A_{bar.dist} = 0.31 \cdot in^2$

Bar diameter................................... $dia = 0.625 \cdot in$

The effective span length (LRFD 9.7.2.3) is the distance between the flange tips plus the flange overhang............................... $Slab_{eff.Length} := (BeamSpacing - b_w) = 9.417 \cdot ft$

The area for secondary reinforcement should not exceed 67% of the area for primary reinforcement...........................

$$\%A_{steel} := \min\left(\frac{220}{\sqrt{\frac{Slab_{eff.Length}}{ft}}}\%, 67\%\right) = 67 \cdot \%$$

Required area for secondary reinforcement....................................... $A_{s.DistR} := A_{s.pos} \cdot \%A_{steel} = 0.42 \cdot in^2$

Maximum spacing for secondary reinforcement.......................................

$$MaxSpacing_{DistR} := \frac{b}{\left(\frac{A_{s.DistR}}{A_{bar.dist}}\right)} = 8.96 \cdot in$$

The bar spacing should not exceed the maximum spacing for secondary reinforcement

$$LRFD_{9.7.3.2} := \begin{array}{|ll} \text{"OK, distribution reinforcement requirements"} & \text{if } bar_{spa.dist} \le MaxSpacing_{DistR} \\ \text{"NG, distribution reinforcement requirements"} & \text{otherwise} \end{array}$$

$LRFD_{9.7.3.2} = \text{"OK, distribution reinforcement requirements"}$

C7. Summary of Reinforcement Provided

Transverse reinforcing

Bar size $bar = "5"$

Top spacing $spacing_{neg} = 6.0 \cdot in$

Bottom spacing $spacing_{pos} = 6.0 \cdot in$

Shrinkage and temperature reinforcing

Bar size $bar_{st} = "5"$

Bottom spacing $bar_{spa.st} = 8.0 \cdot in$

$LRFD_{5.7.10.8}$ = "OK, minimum shrinkage and temperature requirements"

Longitudinal Distribution reinforcing

Bar size $bar_{dist} = "5"$

Bottom spacing $bar_{spa.dist} = 8.0 \cdot in$

$LRFD_{9.7.3.2}$ = "OK, distribution reinforcement requirements"

D. Median Barrier Reinforcement

Per SDG 4.2.4.B, the minimum transverse top slab reinforcing at the barrier may be provided per the table in Section 4.2.4.B if the minimum slab depth is provided. For the 32" F-shape median used for this project, the minimum slab depth is 8" and the minimum transverse reinforcing is:

$A_{s.med.barrier} := 0.4 \frac{in^2}{ft}$ top and bottom of slab

The reinforcing provided in the top of the slab is..

$$A_{s.prov.top} := \frac{A_{bar}}{spacing_{neg}} = 0.62 \cdot \frac{in^2}{ft}$$

The reinforcing provided in the bottom of the slab is...

$$A_{s.prov.bot} := \frac{A_{bar}}{spacing_{pos}} = 0.62 \cdot \frac{in^2}{ft}$$

$$Median\_Barrier := if\left(A_{s.prov.top} > A_{s.med.barrier} \wedge A_{s.prov.bot} > A_{s.med.barrier}, "OK", "NG"\right)$$

$Median\_Barrier = "OK"$

SUPERSTRUCTURE DESIGN

Deck Overhang Design

References

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\206DeckTraditional.xmcd(R)

Description

This section provides the overhang deck design.

Page Contents

FDOT Criteria

Deck Slab Design [SDG 4.2.4]

The deck overhang shall be designed using the traditional design method. For the deck overhang design and median barriers, the minimum transverse top slab reinforcing may be provided without further analysis where the indicated minimum slab depths are provided in **[SDG 4.2.4B]** and the total deck overhang is 6 feet or less.

Deck slab designed by the traditional design method:

$A_{s.TL4} := 0.8in^2$ per foot of overhang slab

Superstructure Components - Traffic Railings [SDG 6.7]

All new traffic railing barriers shall satisfy LRFD Chapter 13, TL-4 criteria.

A. Input Variables

Beam top flange width............................. $b_{tf} = 48 \cdot in$

Thickness of slab................................... $t_{slab} = 8 \cdot in$

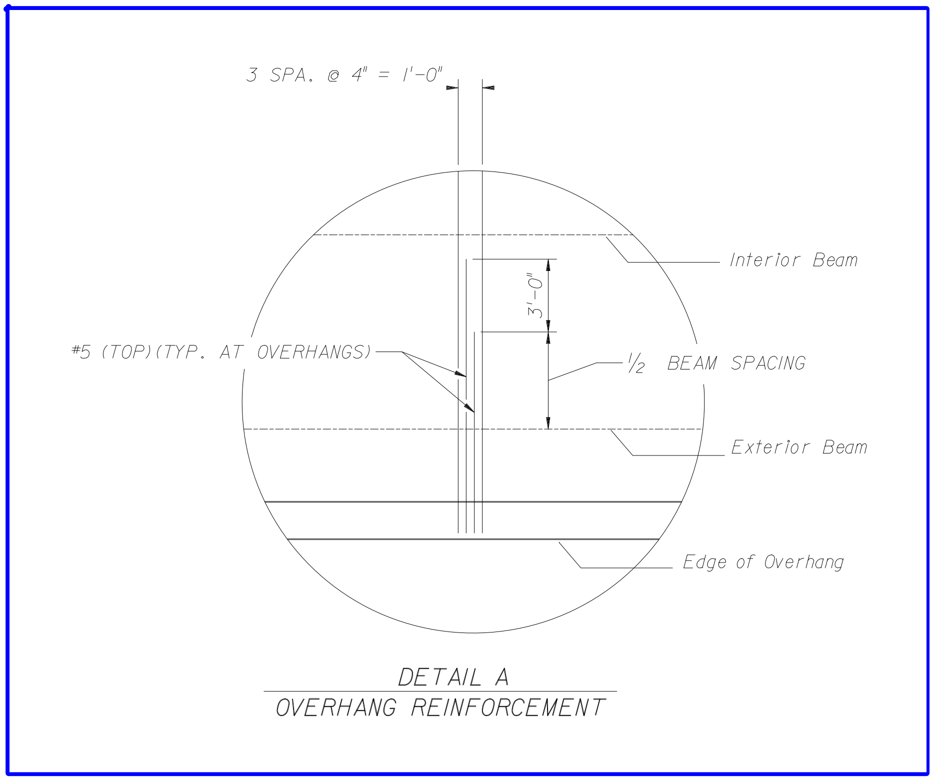
Milling surface thickness......................... $t_{mill} = 0.5 \cdot in$

Deck overhang....................................... $Overhang = 4.54\,ft$

Dynamic load allowance......................... $IM = 1.33$

Design width of overhang........................ $b_{overhang} := 1ft$

Proposed reinforcement detail..................



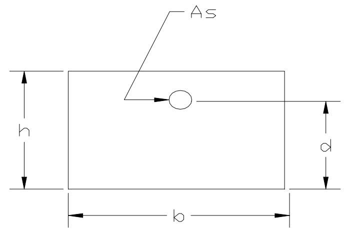
B. Deck Overhang Reinforcement

B1. Negative Moment Region - Reinforcement Requirements [SDG 4.2.4B]

Reinforcement required for the extreme event limit states

$A_{s.TL4} = 0.8 \cdot in^2$ per foot of deck overhang

Using variables defined in this example,



$f_{c.slab} = 4.5 \cdot ksi$ $\qquad t_{slab} = h$

$f_y = 60 \cdot ksi$

$\phi = 0.9$

$t_{slab} = 8 \cdot in$

$b = 12 \cdot in$ $\qquad b := b_{slab}$

Initial assumption for area of steel required

Size of bar...................................... $bar = "5"$

Proposed bar spacing...................... $spacing := 4 \cdot in$

Bar area... $A_{bar} = 0.310 \cdot in^2$

Bar diameter.................................. $dia = 0.625 \cdot in$

Area of steel provided per foot of slab...... $A_{s.overhang} := \dfrac{A_{bar} \cdot 1ft}{spacing} = 0.93 \cdot in^2$

B2. Limits for Reinforcement [LRFD 5.7.3.3]

Area of steel provided............................ $A_s := A_{s.overhang} = 0.93 \cdot in^2$

Minimum Reinforcement

The minimum reinforcement requirements ensure the moment capacity provided is at least 1.2 times greater than the cracking moment.

Modulus of Rupture.............................. $f_r := 0.24 \cdot \sqrt{f_{c.slab} \cdot ksi} = 0.51 \cdot ksi$ **[SDG 1.4.1.B]**

Distance from the extreme tensile fiber to the neutral axis of the composite section... $y := \frac{t_{slab}}{2} = 4 \cdot in$

Moment of inertia for the section............ $I_{slab} := \frac{1}{12} \cdot b \cdot t_{slab}^{3} = 512 \cdot in^{4}$

Section modulus.................................... $S := \frac{I_{slab}}{y} = 128 \cdot in^{3}$

Flexural cracking variability factor........... $\gamma_1 := 1.6$

Ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement....................................... $\gamma_3 := 0.62$

Cracking moment.................................. $M_{cr} := \gamma_3 \cdot \left[\left(\gamma_1 \cdot f_r\right) \cdot S\right] = 5.39 \cdot kip \cdot ft$

Minimum distance to reinforcing steel...... $d_s := \min\left(d_{s.neg}, d_{s.pos}\right) = 5.69 \cdot in$

Minimum reinforcement required............ $A_{min} := \frac{\frac{M_{cr}}{\phi}}{f_y \cdot \left[d_s - \frac{1}{2}\left(\frac{A_s \cdot f_y}{0.85 \cdot f_{c.slab} \cdot b}\right)\right]} = 0.24 \cdot in^{2}$

Required area of steel for minimum reinforcement should not be less than $A_s \cdot 133\%$ or A_{min} .... $A_{s.req} := \min\left(A_{s.TL4} \cdot 133 \cdot \%, A_{min}\right) = 0.24 \cdot in^{2}$

Maximum bar spacing for mimimum reinforcement....................................... $spacing_{max} := \frac{b}{\left(\frac{A_{s.req}}{A_{bar}}\right)} = 15.78 \cdot in$

The bar spacing should be less than the maximum bar spacing for minimum reinforcement

$LRFD_{5.7.3.3.2} :=$ | "OK, minimum reinforcement requirements are satisfied" if $spacing \le spacing_{max}$
| "NG, section is under-reinforced, so redesign!" otherwise

$LRFD_{5.7.3.3.2} =$ "OK, minimum reinforcement requirements are satisfied"

B3. Shrinkage and Temperature Reinforcement [LRFD 5.10.8]

Gross area of section.............................. $A_g := b_{overhang} \cdot t_{slab} = 96 \cdot in^2$

Minimum area of shrinkage and temperature reinforcement...................... $A_{ST} := \min\left[\max\left[0.11\frac{in^2}{ft}, \frac{1.3 \cdot \frac{kip}{in \cdot ft} \cdot A_g}{2\left(b_{overhang} + t_{slab}\right) \cdot f_y}\right], 0.6\frac{in^2}{ft}\right] = 0.11 \cdot \frac{in^2}{ft}$

Maximum spacing for shrinkage and temperature reinforcement....................... $spacing_{ST} := \min\left(\frac{A_{bar}}{A_{ST}}, 3 \cdot t_{slab}, 18 \cdot in\right) = 18 \cdot in$

The bar spacing should be less than the maximum spacing for shrinkage and temperature reinforcement

$LRFD_{5.7.10.8} :=$ | "OK, minimum shrinkage and temperature requirements" if $spacing \le spacing_{ST}$
| "NG, minimum shrinkage and temperature requirements" otherwise

$LRFD_{5.7.10.8} =$ "OK, minimum shrinkage and temperature requirements"

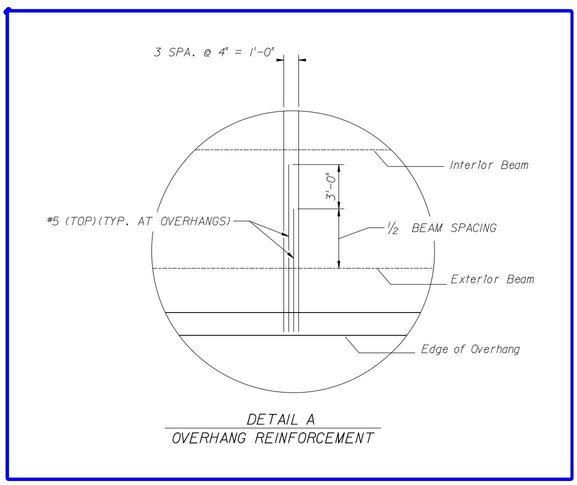
B4. Summary

Size of bar

bar = "5"

Proposed bar spacing

$spacing = 4 \cdot in$



$LRFD_{5.7.3.3.2} =$ "OK, minimum reinforcement requirements are satisfied"

$LRFD_{5.7.10.8} =$ "OK, minimum shrinkage and temperature requirements"

SUPERSTRUCTURE DESIGN

Creep and Shrinkage Parameters

References

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\207DeckCantilever.xmcd(R)

Description

This section provides the creep and shrinkage factors as per the **LRFD 5.4.2.3.2 and 5.4.2.3.3.**

Page Contents

A. Input Variables

A1. Time Dependent Variables

Relative humidity.................................... $H = 75$

Age (days) of concrete when load is applied.. $T_0 = 1$

Age (days) of concrete deck when section becomes composite............................... $T_1 = 120$

Age (days) used to determine long term losses... $T_2 = 10000$

A2. Transformed Properties

Required thickness of deck slab............... $t_{slab} = 8 \cdot in$

Effective slab width for interior beam....... $b_{eff.interior} = 120.0 \cdot in$

Effective slab width for exterior beam...... $b_{eff.exterior} = 139.4 \cdot in$

Superstructure beam type........................ BeamTypeTog = "FIB-36"

A3. Volume to Surface Area Ratios (Notional Thickness)

The volume and surface area are calculated for 1 ft. of length. The surface area only includes the area exposed to atmospheric drying. The volume and surface area of the deck are analyzed using the effective slab width for the interior beam.

Effective slab width.............................. $b_{eff} := b_{eff.interior}$

BeamTypeTog := BeamType

$$PCBeams := \begin{pmatrix} \text{"ERROR"} & \text{"FIB-36"} & \text{"FIB-45"} & \text{"FIB-54"} & \text{"FIB-63"} & \text{"FIB-72"} & \text{"FIB-78"} \\ \text{"A"} & 807 & 870 & 933 & 996 & 1059 & 1101 \\ \text{"b.tf"} & 48 & 48 & 48 & 48 & 48 & 48 \end{pmatrix}$$

$$\text{output}(\text{beamprops}, \text{type}) := \begin{cases} \text{beamprops}^{\langle 1\rangle} & \text{if type} = \text{"FIB-36"} \\ \text{beamprops}^{\langle 2\rangle} & \text{if type} = \text{"FIB-45"} \\ \text{beamprops}^{\langle 3\rangle} & \text{if type} = \text{"FIB-54"} \\ \text{beamprops}^{\langle 4\rangle} & \text{if type} = \text{"FIB-63"} \\ \text{beamprops}^{\langle 5\rangle} & \text{if type} = \text{"FIB-72"} \\ \text{beamprops}^{\langle 6\rangle} & \text{if type} = \text{"FIB-78"} \end{cases}$$

$$\text{BeamType} := \begin{cases} \text{FIB36} & \text{if BeamType} = \text{"FIB-36"} \\ \text{FIB45} & \text{if BeamType} = \text{"FIB-45"} \\ \text{FIB54} & \text{if BeamType} = \text{"FIB-54"} \\ \text{FIB63} & \text{if BeamType} = \text{"FIB-63"} \\ \text{FIB72} & \text{if BeamType} = \text{"FIB-72"} \\ \text{FIB78} & \text{if BeamType} = \text{"FIB-78"} \end{cases}$$

$n := \text{rows}(\text{BeamType})$

$n = 15$

$ii := 0 .. n$

$$\text{Surface}(\text{BeamType}) := \sum_{ii=0}^{n-2} \left[\left(\text{BeamType}_{ii+1,0} - \text{BeamType}_{ii,0}\right)^2 + \left(\text{BeamType}_{ii+1,1} - \text{BeamType}_{ii,1}\right)^2\right]^{0.5}$$

$\text{Surface}_{beam} := \text{Surface}(\text{BeamType}) \cdot (1 \cdot \text{ft})$

$$\text{output}(\text{PCBeams}, \text{BeamTypeTog}) = \begin{pmatrix} \text{"FIB-36"} \\ 807 \\ 48 \end{pmatrix}$$

$A_{nc} := \text{output}(\text{PCBeams}, \text{BeamTypeTog})_1 \cdot \text{in}^2$

$b_{tf} := \text{output}(\text{PCBeams}, \text{BeamTypeTog})_2 \cdot \text{in}$

$\text{Volume}_{beam} := A_{nc} \cdot 1 \cdot \text{ft}$

$\text{Surface}_{deck} := \left[2 \cdot \left(b_{eff}\right) - b_{tf}\right] \cdot (1 \cdot \text{ft})$

$\text{Volume}_{deck} := b_{eff} \cdot t_{slab} \cdot (1 \cdot \text{ft})$

$\text{Volume} := \text{Volume}_{beam} + \text{Volume}_{deck}$

$\text{Surface} := \text{Surface}_{beam} + \text{Surface}_{deck} - b_{tf} \cdot 1\text{ft}$

Volume of beam....................................	$Volume_{beam} = 5.6 \cdot ft^3$
Volume of deck.....................................	$Volume_{deck} = 6.7 \cdot ft^3$
Volume of composite section..................	$Volume = 12.3 \cdot ft^3$
Surface area of beam.............................	$Surface_{beam} = 17.5 \cdot ft^2$
Surface area of deck.............................	$Surface_{deck} = 16.0 \cdot ft^2$
Surface area of composite section..........	$Surface = 29.5 \cdot ft^2$

The shrinkage coefficient uses the notional thickness of the composite section..........

$$h_{o.SH} := \frac{Volume}{Surface} = 4.99 \cdot in$$

The creep coefficient uses the notional thickness of the non-composite section, since the forces responsible for creep are initially applied to the non-composite section..

$$h_{o.CR} := \frac{Volume_{beam}}{Surface_{beam}} = 3.84 \cdot in$$

B. Shrinkage Coefficient (LRFD 5.4.2.3.3)

Shrinkage can range from approximately zero for concrete continually immersed in water to greater than 0.0008 for concrete that is improperly cured. Several factors influence the shrinkage of concrete.

- Aggregate characteristics and proportions
- Average humidity at the bridge site
- W/C ratio
- Type of cure
- Volume to surface area ratio of member
- Duration of drying period

Shrinkage strain for both accelerated & moist-cured concretes without shrinkage-prone aggregates.................... $\varepsilon_{sh} = k_s \cdot k_{hs} \cdot k_f \cdot k_{td} \cdot 0.48 \cdot 10^{-3}$

Factor for the effect of concrete strength.. $k_f := \dfrac{5}{1 + \dfrac{f_{ci.beam}}{ksi}} = 0.71$

Factor for effects of the volume to surface ratio... $k_s := \max\left(1.45 - 0.13 \cdot \dfrac{h_{o.SH}}{in}, 1.0\right) = 1$

Time development factor....................... $k_{td} = \dfrac{t}{61 - 4 \cdot \dfrac{f_{ci.beam}}{ksi} + t}$

Factor for relative humidity.................... $k_{hs} := 2.00 - 0.014H = 0.95$

Using variables defined in this example,

Shrinkage strain.............................. $\varepsilon_{sh}(t) := k_s \cdot k_{hs} \cdot k_f \cdot k_{td_t} \cdot 0.48 \cdot 10^{-3}$

Shrinkage strain on composite section at Day $T_1 = 120$ .... $\varepsilon_{sh}(T_1) = 0.00025$

Shrinkage strain on composite section at Day $T_2 = 10000$ .... $\varepsilon_{sh}(T_2) = 0.00032$

Shrinkage strain on composite section from Day $T_1 = 120$ to Day $T_2 = 10000$ .... $\varepsilon_{SH} := \varepsilon_{sh}(T_2) - \varepsilon_{sh}(T_1) = 0.000076$

.

***Note**: Shrinkage and Creep [**LRFD 5.4.2.3**]*

Assumptions for shrinkage strain....... *0.0002 after 28 days*

0.0005 after one year

C. Creep Coefficient (LRFD 5.4.2.3.2)

Creep is influenced by the same factors as shrinkage and also by the following factors:

- Magnitude and duration of stress
- Maturity of concrete at loading
- Temperature of concrete

For typical temperature ranges in bridges, temperature is not a factor in estimating creep.

Concrete shortening due to creep generally ranges from 0.5 to 4.0 times the initial elastic shortening, depending primarily on concrete maturity at loading.

Creep Coefficient $\Psi(t, t_i) = 1.9 \cdot k_s \cdot k_{hc} \cdot k_f \cdot k_{td} \cdot t_i^{-0.118}$

Factor for effect of concrete strength...... $k_f := \dfrac{5}{1 + \left(\dfrac{f_{ci.beam}}{ksi}\right)} = 0.71$

Factor for the effect of the volume-to-surface ratio of the component............... $k_s := \max\left(1.45 - 0.13 \cdot \dfrac{h_{o.CR}}{in}, 1.0\right) = 1$

Humidity factor for creep....................... $k_{hc} := 1.56 - 0.008 \cdot H = 0.96$

Using variables defined in this example,

Creep coefficient............................ $\Psi(t, t_i) := 1.9 \cdot k_s \cdot k_{hc} \cdot k_f \cdot k_{td_t} \cdot t_i^{-0.118}$

Creep coefficient on non-composite section from Day $T_0 = 1$ to Day $T_1 = 120$ .... $\psi_{cr1} := \Psi(T_1, T_0) = 1$

Creep coefficient on non-composite section from Day $T_0 = 1$ to Day $T_2 = 10000$ .... $\psi_{cr2} := \Psi(T_2, T_0) = 1.3$

Creep factor at Day T_1 .... $C_{120} := 1 + \psi_{cr1} = 2$

Creep factor at Day T_2 .... $C_{10000} := 1 + \psi_{cr2} = 2.3$

Expansion Joint Design

References

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\208CreepShrinkage.xmcd(R)

Description

This section provides the design of the bridge expansion joints.

Page Contents

LRFD Criteria

Uniform Temperature [3.12.2]

Superseded by SDG 2.7.1 and SDG 6.3.

Shrinkage and Creep [5.4.2.3]

Movement and Loads - General [14.4.1]

Bridge Joints [14.5]

FDOT Criteria

Uniform Temperature - Joints and Bearings [SDG 2.7.1]

Delete LRFD [3.12.2] and substitute in lieu thereof SDG Chapter 6.

Expansion Joints [SDG 6.4]

A. Input Variables

A1. Bridge Geometry

Overall bridge length.............................. $L_{bridge} = 180\ \text{ft}$

Bridge design span length....................... $L_{span} = 90\ \text{ft}$

Skew angle... $Skew = -20 \cdot deg$

A2. Temperature Movement [SDG 6.3]

Temperature Range by Superstructure Material				
Superstructure Material	**Temperature Range** (Degrees Fahrenheit)			
	Mean	High	Low	Range
Concrete Only	70	105	35	70
Concrete Deck on Steel Girder	70	110	30	80
Steel Only	70	120	30	90

The temperature values for "Concrete Only" in the preceding table apply to this example.

Temperature mean.......................... $t_{mean} = 70 \cdot °F$

Temperature high............................ $t_{high} = 105 \cdot °F$

Temperature low............................. $t_{low} = 35 \cdot °F$

Temperature rise............................. $\Delta t_{rise} := t_{high} - t_{mean} = 35 \cdot °F$

Temperature fall.............................. $\Delta t_{fall} := t_{mean} - t_{low} = 35 \cdot °F$

Coefficient of thermal expansion **[LRFD 5.4.2.2]** for normal weight concrete......... $\alpha_t = 6 \times 10^{-6} \cdot \frac{1}{°F}$

A3. Expansion Joints [SDG 6.4]

Expansion Joint Type	**Maximum Open Width "W" (measured in the direction of travel at deck surface)**
Hot Poured or Poured Joint without Backer Rod	3/4-inch
Poured Joint with Backer Rod	3-inches
Armored Elastomeric Strip Seal (Single gap)	Per LRFD [14.5.3.2]
Modular Joint (Multiple modular gaps)	Per LRFD [14.5.3.2]
Finger Joint	Per LRFD [14.5.3.2]

For new construction, use only the joint types listed in the preceding table. A typical joint for most prestressed beam bridges is the poured joint with backer rod..

Proposed joint width at 70º F (per FDOT Instructions for Design Standards Index 21110) .... $W := 2 \cdot in$

Maximum joint width....................... $W_{max} := 3 \cdot in$

Minimum joint width........................ $W_{min} := 0.5 \cdot W$

A4. Movement [SDG 6.4.2]

Temperature

The movement along the beam due to temperature should be resolved along the axis of the expansion joint or skew.

Displacements normal to skew at top of bents

Temperature rise............................. $\Delta z_{TempR} := \alpha_t \cdot \Delta t_{rise} \cdot \cos(|Skew|) \cdot L_{span} = 0.21 \cdot in$

Temperature Fall............................. $\Delta z_{TempF} := \alpha_t \cdot \Delta t_{fall} \cdot \cos(|Skew|) \cdot L_{span} = 0.21 \cdot in$

Displacements parallel to skew at top of bents

Temperature rise............................. $\Delta x_{TempR} := \alpha_t \cdot \Delta t_{rise} \cdot \sin(|Skew|) \cdot L_{span} = 0.08 \cdot in$

Temperature Fall............................. $\Delta x_{TempF} := \alpha_t \cdot \Delta t_{fall} \cdot (\sin(|Skew|) \cdot L_{span}) = 0.08 \cdot in$

For poured joint with backer rod, displacements parallel to the skew are not significant in most joint designs. For this example, these displacements are ignored.

Creep and Shrinkage

The following assumptions are used in this design example:

- Creep and Shrinkage prior to day 120 (casting of deck) is neglected for the expansion joint design.
- Creep [**LRFD 5.4.2.3**] is not considered at this time. After day 120, all beams are assumed to creep towards their centers. The slab will offer some restraint to this movement of the beam. The beam and slab interaction, combined with forces not being applied to the center of gravity for the composite section, is likely to produce longitudinal movements and rotations. For most prestressed beams designed as simple spans for dead and live load, these joint movements due to creep are ignored.

Shrinkage after day 120 is calculated using **LRFD 5.4.2.3**.

Creep strain.. $\varepsilon_{CR} := 0.$

Shrinkage strain.................................... $\varepsilon_{SH} = 0.00008$

Strain due to creep and shrinkage............ $\varepsilon_{CS} := \varepsilon_{CR} + \varepsilon_{SH} = 0.000076$

The movement along the beam due to creep and shrinkage should be resolved along the axis of the expansion joint or skew.

Displacements normal to skew at top of bents.. $\Delta z_{CS} := \varepsilon_{CS} \cdot \cos(|Skew|) \cdot L_{span} = 0.08 \cdot in$

Displacements parallel to skew at top of bents.. $\Delta x_{CS} := \varepsilon_{CS} \cdot \sin(|Skew|) \cdot L_{span} = 0.03 \cdot in$

For poured joint with backer rod, displacements parallel to the skew are not significant in most joint designs. For this example, these displacements are ignored.

B. Expansion Joint Design

For prestressed concrete structures, the movement is based on the greater of two cases:

- Movement from the combination of temperature fall, creep, and shrinkage
- Movement from factored effects of temperature

B1. Movement from Creep, Shrinkage and Temperature (SDG 6.4.2)

The combination of creep, shrinkage, and temperature fall tends to "open" the expansion joint.

Movement from the combination of temperature fall, creep, and shrinkage....... $\Delta z_{Temperature.Fall} = \Delta z_{temperature.fall} + \Delta z_{creep.shrinkage}$

Using variables defined in this example..... $\Delta_{CST} := \Delta z_{CS} + \Delta z_{TempF} = 0.29 \cdot in$

Joint width from opening caused by creep, shrinkage, and temperature...................... $W_{CSTopen} := W + \Delta_{CST} = 2.29 \cdot in$

The joint width from opening should not exceed the maximum joint width.

$$CST_{Jt\_Open} := \begin{cases} \text{"OK, joint width does not exceed maximum joint width"} & \text{if } W_{CSTopen} \le W_{max} \\ \text{"NG, joint width exceeds maximum joint width"} & \text{otherwise} \end{cases}$$

CST_{Jt_Open} = "OK, joint width does not exceed maximum joint width"

B2. Movement from Temperature (SDG 6.4.2)

Movement from factored effects of temperature rise

$$\Delta z_{rise.or.fall} = \gamma_{TU} \cdot \Delta z_{temperature.rise.or.fall}$$

Using variables defined in this example,

Joint width from opening caused by factored temperature fall......................... $W_{Topen} := W + \gamma_{TU} \cdot \Delta z_{TempF} = 2.26 \cdot in$

Joint width from closing caused by factored temperature rise......................... $W_{Tclose} := W - \gamma_{TU} \cdot \Delta z_{TempR} = 1.74 \cdot in$

The joint width from opening should not exceed the maximum joint width.

$$\text{Temperature}_{Jt\_Open} := \left| \begin{array}{ll} \text{"OK, joint width does not exceed maximum joint width"} & \text{if } W_{Topen} \le W_{max} \\ \text{"NG, joint width exceeds maximum joint width"} & \text{otherwise} \end{array} \right.$$

$$\text{Temperature}_{Jt\_Open} = \text{"OK, joint width does not exceed maximum joint width"}$$

The joint width from closing should not be less than the minimum joint width.

$$\text{Temperature}_{Jt\_Close} := \left| \begin{array}{ll} \text{"OK, joint width is not less than minimum joint width"} & \text{if } W_{Tclose} \ge W_{min} \\ \text{"NG, joint width exceeds minimum joint width"} & \text{otherwise} \end{array} \right.$$

$$\text{Temperature}_{Jt\_Close} = \text{"OK, joint width is not less than minimum joint width"}$$

B3. Temperature Adjustment for Field Placement of Joint

For field temperatures other than 70º F, a temperature adjustment is provided. The adjustment is used during construction to obtain the desired joint width..................

$$T_{Adj} := \frac{\Delta z_{TempR}}{\Delta t_{rise}} = 0.0061 \cdot \frac{in}{°F}$$

B4. Bearing Design Movement/Strain

For the bearing pad design, the following strain due to temperature, creep and shrinkage will be utilized.......................

$$\varepsilon_{CST} := \left(\varepsilon_{CR} + \varepsilon_{SH} + \alpha_t \cdot \Delta t_{fall}\right) = 0.000286$$

C. Design Summary

Joint width at 70º.................................. $W = 2 \cdot in$

Joint width from opening caused by creep, shrinkage, and temperature...................... $W_{CSTopen} = 2.29 \cdot in$

CST_{Jt_Open} = "OK, joint width does not exceed maximum joint width" .... $W_{max} = 3 \cdot in$

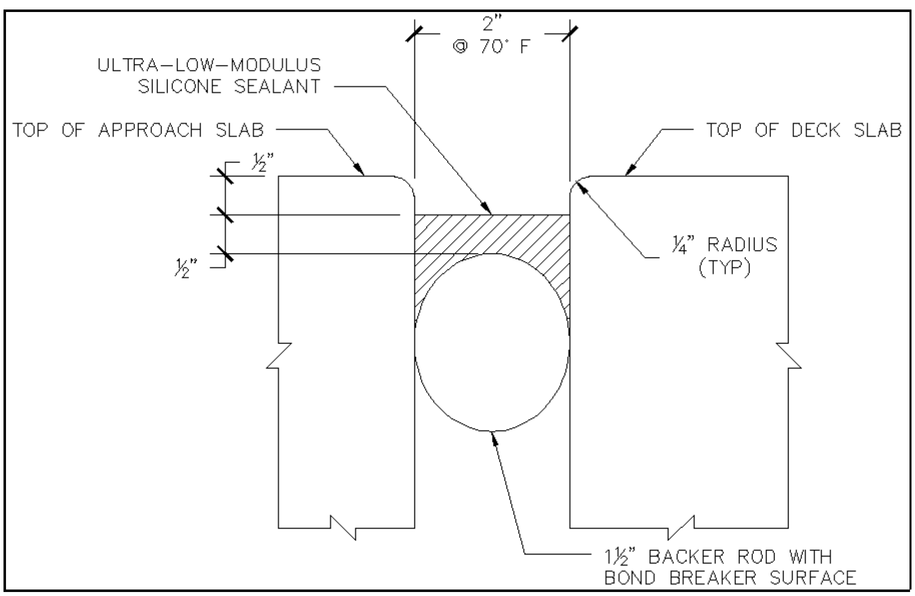
Joint width from opening caused by factored temperature.............................. $W_{Topen} = 2.26 \cdot in$

$Temperature_{Jt\_Open}$ = "OK, joint width does not exceed maximum joint width" .... $W_{max} = 3 \cdot in$

Joint width from closing caused by factored temperature.............................. $W_{Tclose} = 1.74 \cdot in$

$Temperature_{Jt\_Close}$ = "OK, joint width is not less than minimum joint width" .... $W_{min} = 1 \cdot in$

Adjustment for field temperatures other than 70º.. $T_{Adj} = 0.0061 \cdot \frac{in}{°F}$



EXPANSION JOINT DETAIL

Defined Units

Composite Neoprene Bearing Pad Design

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\209ExpansionJoint.xmcd(R)

Description

This section provides the design of the bridge composite neoprene bearing pad. Only the interior beam at End bent 1 bearing pad is designed within this file.

For the design of bearing pads for any other beam type (exterior beam) and location (at pier), design is similar to methodology shown in this file. This may be modified by changing input values in section A3 of these calculations.

Page Contents

LRFD Criteria

Force Effects Due to Superimposed Deformations [LRFD 3.12]

Modified by SDG 2.7 and SDG 6.3.

Minimum Support Length Requirements [LRFD 4.7.4.4]

FDOT Criteria

Seismic Provisions - General [SDG 2.3.1]

Simple span concrete beam bridges supported on elastomeric bearings are exempt from seismic design. Design for minimum seismic support length only.

Vessel Collision - Design Methodology - Damage Permitted [SDG 2.11.4]

Ship impact on bearings is not considered in this example.

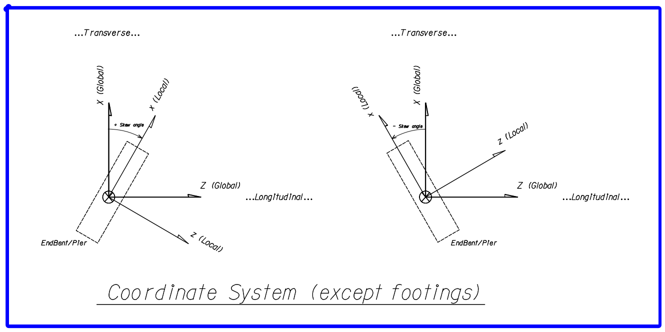
Temperature Movement [SDG 6.3]

Bearings [SDG 6.5]

Specifies design of Composite neoprene bearing pads in accordance with LRFD Method B [LRFD 14.7.5].

A. Input Variables

A1. Bridge Geometry



Bridge design span length........................ $L_{span} = 90\,ft$

Skew angle.. $Skew = -20 \cdot deg$

The bearing pad will be placed in alignment with the beam, so for the purposes of bearing pad selection, $Skew_{BP} := 0deg$.

Beam grade... $Beam_{Grade} = 0.15 \cdot \%$

A2. Bearing Design Movement/Strain

For the bearing pad design, the following strain due to temperature, creep and shrinkage will be utilized......................... $\varepsilon_{CST} = 0.00029$

Shear deformation................................. $\Delta_s := \varepsilon_{CST} \cdot L_{span} = 0.31 \cdot in$

A3. Bearing Pad Design Loads

The design of the interior and exterior beams follow the same procedures and concept as outlined in this design example. In order to minimize the calculations, only one beam type will be evaluated. Flexibility to evaluate an interior or exterior beam is given by changing the input values chosen below (see input options note).

DC dead loads...................................... $R_{DC} := V_{DC.BeamInt}(Support) = 97.42 \cdot kip$

***Note**: Input options......*

$$\begin{pmatrix} V_{DC.BeamInt}(Support) \\ V_{DC.BeamExt}(Support) \\ V_{DW.BeamInt}(Support) \\ V_{DW.BeamExt}(Support) \\ \theta_{LL.Int} \\ \theta_{LL.Ext} \\ R_{LL.Int} \\ R_{LL.Ext} \end{pmatrix}$$

DW dead loads..................................... $R_{DW} := V_{DW.BeamInt}(Support) = 0 \cdot kip$

DC Dead Load Rotation .... $\theta_{DC} := \theta_{DC.BeamInt} = 0.81 \cdot deg$

DW Dead Load Rotation .... $\theta_{DW} := \theta_{DW.BeamInt} = 0 \cdot deg$

Live Load Rotation .... $\theta_{LL} := \theta_{LL.Int} = 0.19 \cdot deg$

Live Load Reaction................................ $R_{LL} := R_{LL.Int} = 93.56 \cdot kip$

Service I Limit State Design Loads.

$Service1 = 1.0 \cdot DC + 1.0 \cdot DW + 1.0 \cdot LL$

Total Dead Load Bearing Design Load....... $R_{BrgDL} := 1.0 \cdot R_{DC} + 1.0 \cdot R_{DW} = 97.42 \cdot kip$

Live Load Bearing Design Load................ $R_{BrgLL} := 1.0 \cdot R_{LL} = 93.56 \cdot kip$

Dead Load Design Rotation..................... $\theta_{DL} := \theta_{DC} + \theta_{DW} = 0.81 \cdot deg$

Live Load Design Rotation....................... $\theta_{LL} = 0.19141 \cdot deg$

B. Composite Bearing Pad Design

B1. Bearing Pad Selection

The Instructions for Design Standard Index 20510 contains the following table:

LIMITING PARAMETERS FOR COMPOSITE ELASTOMERIC BEARING PADS USED WITH FDOT STANDARD FLORIDA-I BEAMS					
Pad Type	**Maximum Service Live Load (kips)**	**Maximum Service Dead Load (LL = Actual Service Live Load)**	**Skew Angle (degrees)**	**Maximum Shear Deflection (in)**	**Shear Modulus, G (Psi)**
D	135	DL=147+1.75(135-LL)	0 - 5	0.75	110
	110	DL=120+1.75(110-LL)	0 - 15		
E	150	DL=233+1.75(150-LL)	0 - 5	0.75	110
	110	DL=113+1.75(110-LL)	0 - 20		
F	150	DL=290+1.75(150-LL)	0 - 5	1	110
	120	DL=139+1.75(120-LL)	0 - 30		
G	145	DL=230+1.75(145-LL)	0 - 30	1	150
	95	DL=98+1.75(95-LL)	0 - 45		
H	180	DL=268+1.75(180-LL)	0 - 35	1.25	150
	135	DL=230+1.75(135-LL)	0 - 45		
J	145	DL=227+1.75(145-LL)	0 - 45	1.5	150
K	200	DL=383+1.75(200-LL)	0 - 45	1.5	150

Based on the table, bearing pad type D will be selected with the following limitations:

$$R_{LL.All} := 135kip$$

$$R_{DL.All} := 147kip + 1.75\left(135kip - R_{BrgLL}\right) = 219.52 \cdot kip$$

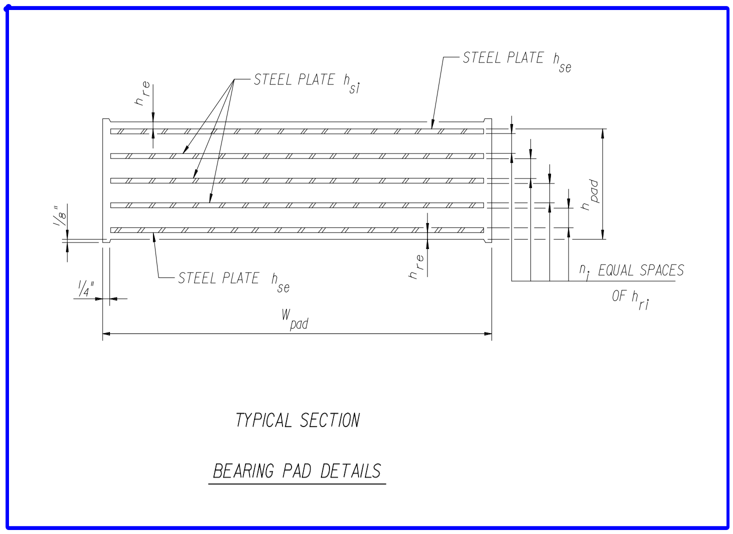
$$Skew_{All} := 5deg$$

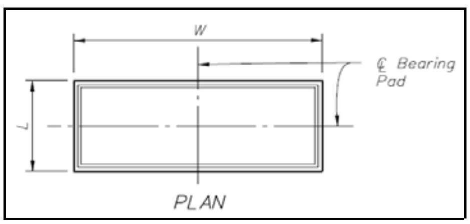
$$\Delta_{S.All} := 0.75in$$

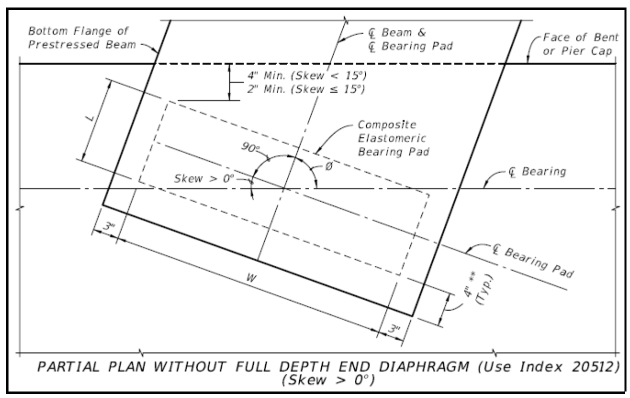
B2. Minimum Support Length [LRFD 4.7.4.4]

Minimum support length........................ $N := \left(8 + 0.02 \cdot \frac{L_{span}}{ft} + .08 \cdot \frac{z_{sup}}{ft}\right) \cdot \left[1 + .000125 \cdot \left(\frac{Skew}{deg}\right)^2\right] \cdot in = 12.01 \cdot i$

C. Design Summary







C1. Bearing Pad Dimensions

Based on Design Standard Index 20510, the bearing pad D dimensions are:

Length of bearing pad............................ $L_{pad} := 8 \cdot in$

Width of bearing pad............................. $W_{pad} := 32 \cdot in$

Height of bearing pad............................. $h_{pad} := 1.90625 \cdot in$

Number of external elastomer layers......... $n_e := 2$

Number of internal elastomer layers......... $n_i := 2$

Support length provided.......................... $N_{prov} := J + \frac{1}{2} \cdot L_{pad} + \text{if}(|Skew| < 15deg, 4in, 2in) = 14 \cdot in$

C2. Design Checks

$Live\_Load := if(R_{LL.All} > R_{BrgLL}, "OK", "NG")$ $Live\_Load = "OK"$

$Dead\_Load := if(R_{DL.All} > R_{BrgDL}, "OK", "NG")$ $Dead\_Load = "OK"$

$Skew\_Chk := if(Skew_{All} > Skew_{BP}, "OK", "NG")$ $Skew\_Chk = "OK"$

$Shear_{All} := if(\Delta_{S.All} > \Delta_s, "OK", "NG")$ $Shear_{All} = "OK"$

$Support\_Length := if(N_{prov} > N, "OK", "NG")$ $Support\_Length = "OK"$

SUPERSTRUCTURE DESIGN

Beam Stability

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\210BearingPad.xmcd(R)

Description

These are calculations for the Lateral Stability of Precast Concrete Bridge Girders during construction. The number of intermediate bracing points (0-6) represents any intermediate bracing that is to be present between the points of bearing. A value of zero represents no intermediate bracing points between the bearing points.

Page Contents

A. Input Variables

A1. Bridge Geometry

Bridge design span length........................ $L_{span} = 90\,ft$

Skew angle between bearing pad and girder.. $Skew_{brg.pad} := 0$

Number of intermediate bracing points (from 0 to 6).. $n_b := 1$

Sweep tolerance................................... $tol_S := \dfrac{\frac{1}{8}in}{10ft}$

Initial imperfection of bracing.................. $e_b := .25in$

A2. Girder Properties

Girder type.. $BeamType := "FIB\text{-}36"$

Unit weight of concrete.......................... $\gamma_{conc} = 0.15\cdot\dfrac{kip}{ft^3}$

Unit weight of concrete for deck pour...... $w_{cd} := \gamma_{conc} = 0.15\cdot\dfrac{kip}{ft^3}$

Concrete strength.................................. $f_{c.beam} = 8.5\cdot ksi$

Beam spacing....................................... $BeamSpacing = 10\,ft$

Number of beams in cross-section........... $N_{beams} = 9$

Overhang length.................................... $Overhang = 4.54\,ft$

Deflection of deck limit at edge of cantilever... $\delta_{max} := 0.25in$ (*0.25 in recommended value*)

Deck thickness..................................... $t_{slab.total} := t_{slab} + t_{mill} = 8.5\cdot in$

Unbraced length of beam........................ $L_b := \dfrac{L_{span}}{n_b + 1} = 45\,ft$

Girder height.. $h_{nc} = 3\,ft$

Top flange width................................... $b_{tf} = 4\,ft$

Bottom flange width...............................	$b_{bf} = 3.17\,ft$
Modulus of Elasticity.............................	$E_{c.beam} = 4.78 \times 10^3 \cdot ksi$
Shear Modulus.....................................	$G_{shear} := 0.416667 \cdot E_{c.beam} = 1992.1 \cdot ksi$
Area of concrete...................................	$A_{nc} = 807 \cdot in^2$
Moment of Inertia, about x-axis..............	$I_{nc} = 127545 \cdot in^4$
Moment of Inertia, about y-axis..............	$I_y = 81131 \cdot in^4$
Distance from CG to top of beam...........	$yt_{nc} = 19.51 \cdot in$
Distance from CG to bottom of beam.......	$yb_{nc} = 16.49 \cdot in$
Torsional constant................................	$Jx = 28654 \cdot in^4$
Section Modulii.....................................	$S_{tnc} = 6537 \cdot in^3$
	$S_{bnc} = 7735 \cdot in^3$
	$S_{yt} := \frac{2 \cdot I_y}{b_{tf}} = 3380 \cdot in^3$
	$S_{yb} := \frac{2 \cdot I_y}{b_{bf}} = 4270 \cdot in^3$
Self-weight of beam...............................	$w := w_{BeamInt} = 0.84 \cdot klf$
Self-weight of slab...............................	$w_d := t_{slab.total} \cdot w_{cd} = 106.25 \cdot psf$
Effective prestressing force (may assume all losses have occurred).........................	$F_{pe} := A_{ps.midspan} \cdot \left(f_{pj} - \Delta f_{pT}\right) = 1508.6 \cdot kip$
Eccentricity of prestressing.....................	$e_{cg.nc.tr} = 11.51 \cdot in$

A3. Bearing Pad Properties

Bearing pad plan dimensions.................... $W_{pad} = 32 \cdot in$

$L_{pad} = 8 \cdot in$

Thickness of internal elastomer layer........ $h_{ri} := 0.5in$

Number of interior layers of elastomer...... $n_i = 2$

Elastomer Shear Modulus........................ $G_{bp} := 93.5psi$ *(For bearing pads, use 0.85G in calculations because material is not homogeneous)*

Tilt angle of support.............................. $\alpha := .01$ *(0.01 recommended value)*

Distance from bottom of beam to roll axis (half bearing pad thickness)..................... $h_r := \dfrac{h_{pad}}{2} = 0.95 \cdot in$

A4. Loads

Basic wind speed (mph).......................... $V_B := V \cdot mph = 150 \cdot mph$

Wind speed factor for construction inactive wind speed................................. $R_E := 0.6$ $V := V_B \cdot R_E = 90 \cdot mph$

Construction active wind speed (20 mph recommended)...................................... $V_E := 20mph$

Construction wind load factor.................. $\gamma := 1.25$

Gust effect factor.................................. $G = 0.85$

Pressure coefficient, single girder............. $C_{pg} := 2.2$

Pressure coefficient, entire bridge section.. $C_p := C_{p.sup}$

Bridge height, measured to mid-height of beam.. $Height := z_{sup} = 20.5\,ft$

Velocity pressure exposure coefficient...... $K_z := K_{z.sup} = 0.91$

Construction active wind load for single girder.. $w_{wE} := 0.00256 \cdot K_z \cdot G \cdot C_{pg} \cdot \left(\frac{V_E}{mph}\right)^2 \cdot psf = 1.74 \cdot psf$

Construction inactive wind load for single girder.. $w_w := 0.00256 \cdot K_z \cdot G \cdot C_{pg} \cdot \left(\frac{V}{mph}\right)^2 \cdot psf = 35.16 \cdot psf$

Construction active wind load for entire bridge section.. $w_{wD} := 0.00256 \cdot K_z \cdot G \cdot C_p \cdot \left(\frac{V_E}{mph}\right)^2 \cdot psf = 0.87 \cdot psf$

Weight of build up................................ $w_b := 50plf$

Weight of forms................................... $w_f := 20psf$ (*20 psf recommended*)

Live loads during deck pour.................... $w_l := 20psf$

$P_l := 75plf$

(*Use 20 psf and 75 plf at edge of overhang per AASHTO Guide Design Specifications for Temporary Works)*

Total weight of finishing machine........... $w_{fm} := 20kip$ *(recommend 10 kips for bridge widths less than 45 feet and 20 kips otherwise)*

Wheel location of finishing machine in relation to edge of overhang, positive is to exterior of overhang edge, negative is to interior of overhang edge.................... $d_{fm} := 2.5in$ *(2.5 in. recommended)*

A5. Lateral Deflection and Eccentricity of Girder Center of Gravity

Maximum lateral deflection of uncracked section.. $z_o := \frac{w \cdot L_{span}^4}{120 \cdot E_{c.beam} \cdot I_y} = 2.05 \cdot in$

Eccentricity due to sweep...................... $e_S := \min(1.5in, L_{span} \cdot tol_S) \cdot \frac{2}{3} = 0.75 \cdot in$

Eccentricity due to construction inactive wind speed.. $e_w := \frac{w_w \cdot h_{nc} \cdot L_{span}^4}{120 \cdot E_{c.beam} \cdot I_y} = 0.26 \cdot in$

Eccentricity due to wind loading at construction active wind speed girder only $e_{wE} := \frac{w_{wE} \cdot h_{nc} \cdot L_{span}^4}{120 \cdot E_{c.beam} \cdot I_y} = 0.01 \cdot in$

Eccentricity due to wind loading at construction active wind speed entire bridge section.......................................

$$e_{wD} := \frac{w_{wD} \cdot h_{nc} \cdot L_{span}^{4}}{120 \cdot E_{c.beam} \cdot I_y} = 0.0063 \cdot in$$

A6. Bearing Pad Rotational Stiffness

$$b\_a := (.5 \;\; .6 \;\; .7 \;\; .75 \;\; .8 \;\; .9 \;\; 1 \;\; 1.2 \;\; 1.4 \;\; 2 \;\; 4 \;\; 10 \;\; 1000)$$

$$C := (136.7 \;\; 116.7 \;\; 104.4 \;\; 100 \;\; 96.2 \;\; 90.4 \;\; 86.2 \;\; 80.4 \;\; 76.7 \;\; 70.8 \;\; 64.9 \;\; 61.9 \;\; 60)$$

$$C' := \text{linterp}\left(b\_a^{T}, C^{T}, \frac{L_{pad}}{W_{pad}}\right) = 186.7$$

Effect of skew on stiffness (coefficient)...

$$Ang := (0 \;\; 15 \;\; 30 \;\; 45 \;\; 60)$$

$$Stiffness := (.8883 \;\; .5922 \;\; .4666 \;\; .3948 \;\; .323)$$

Bearing pad rotational stiffness.........

$$K_{\theta} := \text{linterp}\left(Ang^{T}, Stiffness^{T}, \frac{Skew}{deg}\right) \cdot \frac{G_{bp} \cdot W_{pad}^{5} \cdot L_{pad}}{C' \cdot n_i \cdot h_{ri}^{3}} = 689966 \cdot \frac{kip \cdot in}{rad}$$

Coefficient for reaction at bracing based on number of brace points, intermediate (i), end (e)...

$$k_{vi} := \begin{pmatrix} 0 \\ 1.25 \\ 1.1 \\ 1.143 \\ 1.132 \\ 1.135 \\ 1.134 \end{pmatrix} \quad k_{ve} := \begin{pmatrix} .5 \\ .375 \\ .4 \\ .393 \\ .395 \\ .395 \\ .395 \end{pmatrix}$$

$$K_{vi} := k_{vi_{n_b}} = 1.25$$

$$K_{ve} := k_{ve_{n_b}} = 0.38$$

$$k_m := \begin{pmatrix} .12513 & .12513 & .12513 & .12513 & .12513 & .12513 & .12513 & .12513 & .12513 & .12513 & .12513 \\ .07818 & .05212 & .03905 & .03128 & .02874 & .02697 & .02569 & .02472 & .02395 & .02333 & .02281 \\ .06396 & .04357 & .03337 & .02725 & .02317 & .02026 & .01808 & .01637 & .01501 & .01391 & .01344 \\ .06481 & .04321 & .0324 & .02592 & .02181 & .01899 & .01689 & .01526 & .01395 & .01289 & .01199 \\ .06349 & .04294 & .03267 & .02651 & .02239 & .01946 & .01726 & .01554 & .01417 & .01306 & .01212 \\ .06377 & .04251 & .03189 & .02551 & .02136 & .01847 & .0163 & .01462 & .01327 & .01216 & .01125 \\ .06298 & .04227 & .0319 & .02569 & .02155 & .01858 & .01636 & .01464 & .01326 & .01213 & .01119 \end{pmatrix}$$

Coefficient for bending moment in girder based on number of brace points.............

$$K_M := k_{m_{n_b, N_{beams}-2}} = 0.02472$$

B. Stability Calculations

B1. Calculations of Bending Moments

Unfactored vertical load during deck placement for ext. beam (not including finishing machine)................................

$$w_{D.ext} := w + w_b + P_1 + \left(w_d + w_1\right)\cdot(.5\cdot BeamSpacing + Overhang) + w_f\cdot\left(.5\cdot BeamSpacing + Overhang - b_{tf}\right) = 2.28\cdot klf$$

$$w_{D.int} := w + w_b + \left(w_d + w_1\right)\cdot BeamSpacing + w_f\cdot\left(BeamSpacing - b_{tf}\right) = 2.27\cdot klf$$

Strength I torsional distributed overhang moment during deck placement

$$M_c := \left[1.25\cdot\left(w_d + w_f\right) + 1.5\cdot w_1\right]\cdot\left(Overhang - .5\cdot b_{tf}\right)\cdot\left[.5\cdot b_{tf} + .5\cdot\left(Overhang - .5\cdot b_{tf}\right)\right] + 1.5\cdot P_1\cdot Overhang = 2.07\cdot\frac{kip\cdot ft}{ft}$$

Strength I torsional finishing machine moment..

$$M_{fm} := 1.5\cdot 0.5\cdot w_{fm}\cdot\left(Overhang + d_{fm}\right) = 71.25\cdot kip\cdot ft$$

Lateral moment due to construction inactive wind speed...............................

$$M_w := K_M\cdot w_w\cdot h_{nc}\cdot L_{span}^{\ 2} = 21.12\cdot kip\cdot ft$$

Vertical moment due to girder self-weight

$$M_g := \frac{w\cdot L_{span}^{\ 2}}{8} = 851.13\cdot kip\cdot ft$$

Lateral moment due to construction active wind speed, braced condition..................

$$M_{wE} := K_M\cdot w_{wE}\cdot h_{nc}\cdot L_{span}^{\ 2} = 1.04\cdot kip\cdot ft$$

Lateral moment due to construction active wind speed, unbraced condition..............

$$M_{wE.u} := .125\cdot w_{wE}\cdot h_{nc}\cdot L_{span}^{\ 2} = 5.27\cdot kip\cdot ft$$

Vertical moment due to self-weight and construction loads during deck placement, exterior girder......................................

$$M_{gD} := \frac{w_{D.ext}\cdot L_{span}^{\ 2} + w_{fm}\cdot L_{span}}{8} = 2534.61\cdot kip\cdot ft$$

B2. Service Stress Check for Girder Placement (Prior to beam bracing)

Camber (approximate)........................... $\delta_c := \dfrac{L_{span}^2 \cdot \left(F_{pe} \cdot e_{cg.nc.tr} - \dfrac{5w \cdot L_{span}^2}{48}\right) \cdot 2}{8 \cdot E_{c.beam} \cdot I_{nc}} = 4.23 \cdot in$

Distance from center of gravity to roll axis.. $y := yb_{nc} + h_r + \delta_c \cdot \dfrac{2}{3} = 20.26 \cdot in$

Elastic rotational spring constant (sum of 2 bearing pads).................................... $K_\theta = 6.9 \times 10^5 \cdot \dfrac{kip \cdot in}{rad}$

Radius of stability................................ $r := \dfrac{K_\theta}{w \cdot L_{span}} = 759.98\ ft$

Stress at top of beam, tension................. $f_{ttE} := -\dfrac{F_{pe}}{A_{nc}} + \dfrac{F_{pe} \cdot e_{cg.nc.tr}}{S_{tnc}} - \dfrac{M_g}{S_{tnc}} + \dfrac{M_{wE.u}}{S_{yt}} = -0.76 \cdot ksi$

Stress at top of beam, compression......... $f_{tcE} := -\dfrac{F_{pe}}{A_{nc}} + \dfrac{F_{pe} \cdot e_{cg.nc.tr}}{S_{tnc}} - \dfrac{M_g}{S_{tnc}} - \dfrac{M_{wE.u}}{S_{yt}} = -0.8 \cdot ksi$

Compression check......................... $Ck_{E.t.comp} := \text{if}\left(f_{tcE} \le 6 \cdot \sqrt{\dfrac{f_{c.beam}}{psi}} \cdot psi \wedge f_{tcE} \ge -0.6 \cdot f_{c.beam}, 1, 0\right) = 1$

Tension check................................ $Ck_{E.t.tens} := \text{if}\left(f_{ttE} \le 6 \cdot \sqrt{\dfrac{f_{c.beam}}{psi}} \cdot psi \wedge f_{ttE} \ge -0.6 \cdot f_{c.beam}, 1, 0\right) = 1$

Stress at bottom of beam, tension........... $f_{btE} := -\dfrac{F_{pe}}{A_{nc}} - \dfrac{F_{pe} \cdot e_{cg.nc.tr}}{S_{bnc}} + \dfrac{M_g}{S_{bnc}} + \dfrac{M_{wE.u}}{S_{yb}} = -2.78 \cdot ksi$

Stress at bottom of beam, compression.... $f_{bcE} := -\dfrac{F_{pe}}{A_{nc}} - \dfrac{F_{pe} \cdot e_{cg.nc.tr}}{S_{bnc}} + \dfrac{M_g}{S_{bnc}} - \dfrac{M_{wE.u}}{S_{yb}} = -2.81 \cdot ksi$

Compression check........................... $Ck_{E.b.comp} := \text{if}\left(f_{bcE} \le 6 \cdot \sqrt{\dfrac{f_{c.beam}}{psi}} \cdot psi \wedge f_{bcE} \ge -0.6 \cdot f_{c.beam}, 1, 0\right) = 1$

Tension check.................................. $Ck_{E.b.tens} := \text{if}\left(f_{btE} \le 6 \cdot \sqrt{\dfrac{f_{c.beam}}{psi}} \cdot psi \wedge f_{btE} \ge -0.6 \cdot f_{c.beam}, 1, 0\right) = 1$

Check for stress at girder placement........

$$Ck_{stress.plcmnt} := \text{if}\left(\min\left(Ck_{E.t.comp}, Ck_{E.t.tens}, Ck_{E.b.comp}, Ck_{E.b.tens}\right) = 1, \text{"OK"}, \text{"Not OK"}\right)$$

$$Ck_{stress.plcmnt} = \text{"OK"}$$

B3. Roll Stability Check for Girder Placement (Prior to beam bracing)

Modulus of rupture................................ $f_r := 0.24 \cdot \sqrt{\frac{f_{c.beam}}{ksi}} \cdot ksi = 0.7 \cdot ksi$

Lateral cracking moment........................ $M_{lat} := \min\left[\frac{(f_r - f_{ttE}) \cdot I_y}{\left(\frac{b_{tf}}{2}\right)}, \frac{(f_r - f_{btE}) \cdot I_y}{\left(\frac{b_{bf}}{2}\right)}\right] = 410.5 \cdot kip \cdot ft$

Rotation angle at cracking...................... $\theta_{cr} := \frac{M_{lat}}{M_g} = 27.64 \cdot deg$

Rotation angle at failure..........................

$$\theta_f := \min\left[0.4, \frac{5 \cdot z_o \cdot \alpha + \left[(5 \cdot z_o \cdot \alpha)^2 + 10 \cdot z_o \cdot \left(e_S + e_{wE} + \alpha \cdot z_o + 2.5 \cdot e_{wE} \cdot \alpha + y \cdot \alpha + \frac{w_{wE} \cdot h_{nc}^2}{2 \cdot w}\right)\right]^{.5}}{5 \cdot z_o}\right] = 22.92 \cdot deg$$

Final rotation.. $\theta := \frac{\alpha \cdot r + e_S + e_{wE} + \frac{w_{wE} \cdot h_{nc}^2}{2 \cdot w}}{r - y - z_o} = 0.58 \cdot deg$

Factor of safety for cracking (Unbraced beam).. $FS_{cr} := \frac{r \cdot (\theta_{cr} - \alpha)}{z_o \cdot \theta_{cr} + e_S + e_{wE} + y \cdot \theta_{cr} + \frac{w_{wE} \cdot h_{nc}^2}{2 \cdot w}} = 370.2$

Factor of safety for failure (Unbraced beam)

$$FS_f := \frac{r \cdot (\theta_f - \alpha)}{z_o \cdot (1 + 2.5 \cdot \theta_f) \cdot \theta_f + e_S + e_{wE} \cdot (1 + 2.5 \cdot \theta_f) + y \cdot \theta_f + \frac{w_{wE} \cdot h_{nc}^2}{2 \cdot w}} = 334.6$$

Check for stability at girder placement......

$$Ck_{stab.plcmnt} := \text{if}\left[(\theta \ge 0) \wedge (FS_{cr} \ge 1) \wedge (FS_f \ge 1.5), \text{"OK"}, \text{"Not OK"}\right]$$

$$Ck_{stab.plcmnt} = \text{"OK"}$$

B4. Service Stress Check for Braced Beam (Prior to deck placement)

Stress at top of beam, tension.................

$$f_{tt} := -\frac{F_{pe}}{A_{nc}} + \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{tnc}} - \frac{M_g}{S_{tnc}} + \frac{M_w}{S_{yt}} = -0.7 \cdot ksi$$

Stress at top of beam, compression..........

$$f_{tc} := -\frac{F_{pe}}{A_{nc}} + \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{tnc}} - \frac{M_g}{S_{tnc}} - \frac{M_w}{S_{yt}} = -0.85 \cdot ksi$$

Compression check...........................

$$Ck_{B.t.comp} := \text{if}\left(f_{tc} \le 6 \cdot \sqrt{\frac{f_{c.beam}}{psi}} \cdot psi \wedge f_{tc} \ge -0.6 \cdot f_{c.beam}, 1, 0\right) = 1$$

Tension check.................................

$$Ck_{B.t.tens} := \text{if}\left[f_{tt} \le 6 \cdot \sqrt{\frac{\left(f_{c.beam}\right)}{psi}} \cdot psi \wedge f_{tt} \ge -0.6 \cdot f_{c.beam}, 1, 0\right] = 1$$

Stress at bottom of beam, tension...........

$$f_{bt} := -\frac{F_{pe}}{A_{nc}} - \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{bnc}} + \frac{M_g}{S_{bnc}} + \frac{M_w}{S_{yb}} = -2.73 \cdot ksi$$

Stress at bottom of beam, compression....

$$f_{bc} := -\frac{F_{pe}}{A_{nc}} - \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{bnc}} + \frac{M_g}{S_{bnc}} - \frac{M_w}{S_{yb}} = -2.85 \cdot ksi$$

Compression check...........................

$$Ck_{B.b.comp} := \text{if}\left(f_{bc} \le 6 \cdot \sqrt{\frac{f_{c.beam}}{psi}} \cdot psi \wedge f_{bc} \ge -0.6 \cdot f_{c.beam}, 1, 0\right) = 1$$

Tension check.................................

$$Ck_{B.b.tens} := \text{if}\left(f_{bt} \le 6 \cdot \sqrt{\frac{f_{c.beam}}{psi}} \cdot psi \wedge f_{bt} \ge -0.6 \cdot f_{c.beam}, 1, 0\right) = 1$$

Check for stress at braced condition........

$$Ck_{stress.braced} := \text{if}\left(\min\left(Ck_{B.t.comp}, Ck_{B.t.tens}, Ck_{B.b.comp}, Ck_{B.b.tens}\right) = 1, \text{"OK"}, \text{"Not OK"}\right)$$

$$Ck_{stress.braced} = \text{"OK"}$$

B5. Roll Stability Check for Braced Beam (Prior to deck placement)

Initial rotation.......................................

$$\theta_i := \frac{\alpha \cdot r + e_S}{r - y - z_o} + \frac{\min\left(e_b, e_w\right)}{y} = 1.29 \cdot deg$$

Maximum torque between bracing points..

$$T_B := w \cdot L_{span} \cdot e_w = 1.62 \cdot kip \cdot ft$$

Twist due to torque................................ $\phi_B := \frac{T_B \cdot .5 \cdot L_b}{G_{shear} \cdot Jx} = 0.0053 \cdot deg$

Total rotation.. $\theta_w := \theta_i + \phi_B = 1.29 \cdot deg$

Rotation limits....................................... $\theta_{w.max} := \min(\theta_{cr}, 5 \cdot deg) = 5 \cdot deg$

Wind load rotation check......................... $FS_{\theta w} := \frac{\theta_{w.max}}{\theta_w} = 3.87$

$Ck_{stab.braced} := \text{if}(FS_{\theta w} \ge 1, \text{"OK"}, \text{"Not OK"})$

$Ck_{stab.braced} = \text{"OK"}$

B6. Service Stress Check for Braced Beam (During deck placement)

Stress at top of beam, tension.................. $f_{ttD} := -\frac{F_{pe}}{A_{nc}} + \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{tnc}} - \frac{M_{gD}}{S_{tnc}} + \frac{M_{wE}}{S_{yt}} = -3.86 \cdot ksi$

Stress at top of beam, compression.......... $f_{tcD} := -\frac{F_{pe}}{A_{nc}} + \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{tnc}} - \frac{M_{gD}}{S_{tnc}} - \frac{M_{wE}}{S_{yt}} = -3.87 \cdot ksi$

Compression check........................... $Ck_{D.t.comp} := \text{if}\left(f_{tcD} \le 6 \cdot \sqrt{\frac{f_{c.beam}}{psi}} \cdot psi \wedge f_{tcD} \ge -0.6 \cdot f_{c.beam}, 1, 0\right) = 1$

Tension check................................. $Ck_{D.t.tens} := \text{if}\left(f_{ttD} \le 6 \cdot \sqrt{\frac{f_{c.beam}}{psi}} \cdot psi \wedge f_{ttD} \ge -0.6 \cdot f_{c.beam}, 1, 0\right) = 1$

Stress at bottom of beam, tension............ $f_{btD} := -\frac{F_{pe}}{A_{nc}} - \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{bnc}} + \frac{M_{gD}}{S_{bnc}} + \frac{M_{wE}}{S_{yb}} = -0.18 \cdot ksi$

Stress at bottom of beam, compression.... $f_{bcD} := -\frac{F_{pe}}{A_{nc}} - \frac{F_{pe} \cdot e_{cg.nc.tr}}{S_{bnc}} + \frac{M_{gD}}{S_{bnc}} - \frac{M_{wE}}{S_{yb}} = -0.18 \cdot ksi$

Compression check...............................

$Ck_{D.b.comp} := \text{if}\left(f_{bcD} \le 6 \cdot \sqrt{\frac{f_{c.beam}}{psi}} \cdot psi \wedge f_{bcD} \ge -0.6 \cdot f_{c.beam}, 1, 0\right) = 1$

Tension check................................. $Ck_{D.b.tens} := \text{if}\left(f_{btD} \le 6\cdot\sqrt{\frac{f_{c.beam}}{psi}}\cdot psi \wedge f_{btD} \ge -0.6\cdot f_{c.beam}, 1, 0\right) = 1$

Check for stresses at deck placement condition

$$Ck_{stress.deck} := \text{if}\left(\min\left(Ck_{D.t.comp}, Ck_{D.t.tens}, Ck_{D.b.comp}, Ck_{D.b.tens}\right) = 1, \text{"OK"}, \text{"Not OK"}\right)$$

$$Ck_{stress.deck} = \text{"OK"}$$

B7. Roll Stability Check during Deck Placement

Lateral cracking moment....................... $M_{latD} := \min\left[\frac{(f_r - f_{ttD})\cdot I_y}{.5\cdot b_{tf}}, \frac{(f_r - f_{btD})\cdot I_y}{.5\cdot b_{bf}}\right] = 312.49\cdot kip\cdot ft$

Rotation angle at cracking...................... $\theta_{crD} := \frac{M_{latD}}{M_{gD}} = 7.06\cdot deg$

Initial rotation...................................... $\theta_{i.D} := \frac{\alpha\cdot r + e_S}{r - y - z_o} + \frac{\min(e_b, e_{wD})}{y} = 0.6\cdot deg$

Torque due to construction live loads....... $T_D := (.5\cdot w_{fm} + P_1\cdot L_b)\cdot(Overhang + d_{fm}) = 63.53\cdot kip\cdot ft$

Twist due to construction live loads......... $\phi_D := \frac{T_D\cdot .5\cdot L_b}{G_{shear}\cdot Jx} = 0.21\cdot deg$

Deflection at cantilever due to twist.......... $\delta_D := Overhang\cdot\tan(\phi_D) = 0.2\cdot in$

Total rotation....................................... $\theta_D := \theta_{i.D} + \phi_D = 0.8\cdot deg$

Rotation limits...................................... $\theta_{D.max} := \min(\theta_{crD}, 5\cdot deg) = 5\cdot deg$

Deck placement rotation check................ $Ck_{stab.deck} := \text{if}\left(\delta_D \le \delta_{max} \wedge \theta_D \le \theta_{D.max}, \text{"OK"}, \text{"Not OK"}\right)$

$$Ck_{stab.deck} = \text{"OK"}$$

C. Verify Bracing Requirements

C1. Bracing Requirements

Factored horizontal force at each beam end and anchor brace, at midheight of beam....

$$F_e := w_w \cdot \gamma \cdot h_{nc} \cdot L_b \cdot K_{ve} = 2.22 \cdot kip$$

Factored horizontal bracing force at each intermediate span brace (if present), at midheight of beam.................................

$$F_i := \text{if}\left(n_b = 0, \text{"N/A"}, w_w \cdot \gamma \cdot h_{nc} \cdot L_b \cdot K_{vi}\right) = 7.42 \cdot kip$$

Factored overturning force at each beam end and anchor brace, at top of beam

$$M_e := M_{fm} + M_c \cdot L_b \cdot K_{ve} + w_{wD} \cdot \gamma \cdot h_{nc} \cdot L_b \cdot K_{ve} \cdot .5 \cdot h_{nc} - .9 \cdot w \cdot L_b \cdot \left[\frac{b_{bf}}{2} - \left(z_o \cdot \theta_{i.D} + e_S \cdots + \min\left(e_b, e_{wD}\right) + y \cdot \theta_{i.D}\right)\right] \cdot K_{ve} = 87.1 \cdot kip \cdot f$$

Factored overturning force at each intermediate span brace (if present), at top of beam

$$M_i := M_{fm} + M_c \cdot L_b \cdot K_{vi} + w_{wD} \cdot \gamma \cdot h_{nc} \cdot L_b \cdot K_{vi} \cdot .5 \cdot h_{nc} - .9 \cdot w \cdot L_b \cdot \left[\frac{b_{bf}}{2} - \left(z_o \cdot \theta_{i.D} + e_S \cdots + \min\left(e_b, e_{wD}\right) + y \cdot \theta_{i.D}\right)\right] \cdot K_{vi} = 124.2 \cdot kip \cdot f$$

C2. Verification of Bracing Adequacy

Stress checks.......................................

$Ck_{stress.plcmnt} = \text{"OK"}$

$Ck_{stress.braced} = \text{"OK"}$

$Ck_{stress.deck} = \text{"OK"}$

Stability checks.....................................

$Ck_{stab.plcmnt} = \text{"OK"}$

$Ck_{stab.braced} = \text{"OK"}$

$Ck_{stab.deck} = \text{"OK"}$

(*If* $Ck_{stab.plcmnt}$ *is Not OK," the girder must be braced prior to crane release.*)

D. Summary

D1. Temporary Bracing Variables

Maximum unbraced length...................... $L_b = 45\,ft$

Factored horizontal force at each beam end and anchor brace, at midheight of beam.... $F_e = 2.22 \cdot kip$

Factored horizontal bracing force at each intermediate span brace (if present), at midheight of beam................................. $F_i = 7.42 \cdot kip$

Factored overturning force at each beam end and anchor brace, at top of beam....... $M_e = 87.14 \cdot kip \cdot ft$

Factored overturning force at each intermediate span brace (if present), at top of beam... $M_i = 124.22 \cdot kip \cdot ft$

Dead Loads

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\211BeamStability.xmcd(R)

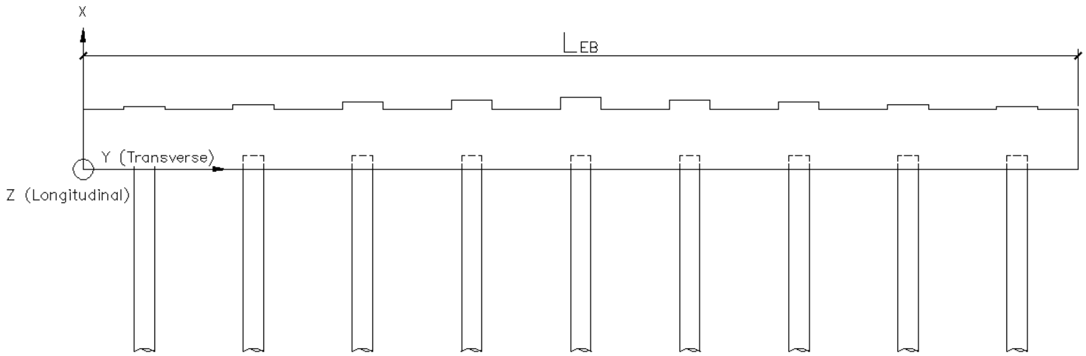
Description

This section provides the design dead loads applied to the substructure from the superstructure. The self-weight of the substructure is generated by the analysis program for the substructure model.

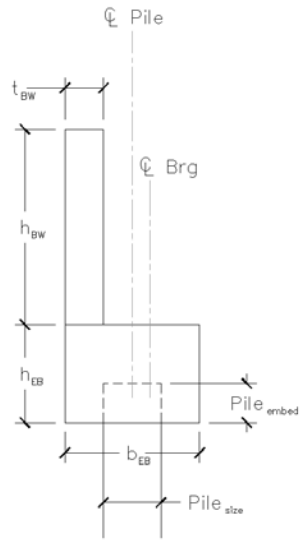
Page Contents

A. General Criteria

A1. End Bent Geometry



*(**Note**: End bent back wall not shown)*



Depth of end bent cap.......... $h_{EB} = 2.5\,ft$

Width of end bent cap.......... $b_{EB} = 3.5\,ft$

Length of end bent cap......... $L_{EB} = 88\,ft$

Height of back wall.............. $h_{BW} = 3.6\,ft$

Backwall design width.......... $L_{BW} = 1\,ft$

Thickness of back wall.......... $t_{BW} = 1\,ft$

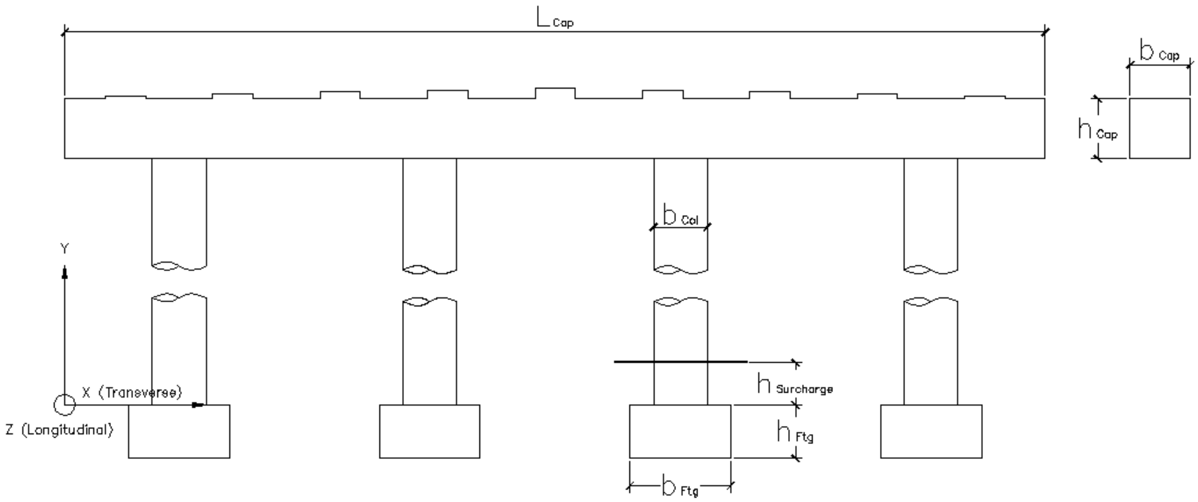
Pile Geometry:

Pile Embedment Depth.......... $Pile_{embed} = 1\,ft$

Pile Size.............................. $Pile_{size} = 18 \cdot in$

A2. Pier Geometry

A model of the substructure has been created utilizing RISA. The model will have the loads applied at the pedestals from the superstructure. In addition, it will generate it's own self-weight based on the following member properties of the pier:



Depth of pier cap................. $h_{Cap} = 4.5\,ft$

Width of pier cap................. $b_{Cap} = 4.5\,ft$

Length of pier cap................ $L_{Cap} = 88\,ft$

Height of pier column........... $h_{Col} = 14\,ft$

Column diameter.................. $b_{Col} = 4\,ft$

Number of columns............ $n_{Col} = 4$

Surcharge.......................... $h_{Surcharge} = 2\,ft$

Height of footing.................. $h_{Ftg} = 4\,ft$

Width of footing................... $b_{Ftg} = 7.5\,ft$

Length of footing.................. $L_{Ftg} = 7.5\,ft$

B. Dead Loads (DC, DW)

B1. Beam Dead Loads

The dead loads of the superstructure (moment and shears) were previously computed utilizing the beam design length, $L_{design} = 87.750\,ft$ (see section 2.01 Dead Loads). For reactions on the pier, the reactions should be computed based on the span length, $L_{span} = 90.0\,ft$. Conservatively, we will adjust the loads as follows:

Modification factors for span loads......... $\kappa_1 := \dfrac{L_{span}}{L_{design}} = 1.03$

DC load at end bent for interior beam........ $P_{DC.BeamInt} := \kappa_1 \cdot V_{DC.BeamInt}(Support) = 99.92 \cdot kip$

DC load at end bent for exterior beam....... $P_{DC.BeamExt} := \kappa_1 \cdot V_{DC.BeamExt}(Support) = 95.02 \cdot kip$

DW load at end bent for interior beam....... $P_{DW.BeamInt} := \kappa_1 \cdot V_{DW.BeamInt}(Support) = 0 \cdot kip$

DW load at end bent for exterior beam...... $P_{DW.BeamExt} := \kappa_1 \cdot V_{DW.BeamExt}(Support) = 0 \cdot kip$

B2. End Bent Dead loads

DC load at end bent for interior beam........ $P_{DC.EndbentInt} := P_{DC.BeamInt} = 99.92 \cdot kip$

DC load at end bent for exterior beam....... $P_{DC.EndbentExt} := P_{DC.BeamExt} = 95.02 \cdot kip$

DW load at end bent for interior beam....... $P_{DW.EndbentInt} := P_{DW.BeamInt} = 0 \cdot kip$

DW load at end bent for exterior beam...... $P_{DW.EndbentExt} := P_{DW.BeamExt} = 0 \cdot kip$

B3. Pier Dead Loads

Dead load at pier for interior beam........... $P_{DC.PierInt} := 2\left(P_{DC.BeamInt}\right) = 199.83 \cdot kip$

Dead load at pier for exterior beam.......... $P_{DC.PierExt} := 2\left(P_{DC.BeamExt}\right) = 190.05 \cdot kip$

Dead load at pier for interior beam.......... $P_{DW.PierInt} := 2\left(P_{DW.BeamInt}\right) = 0 \cdot kip$

Dead load at pier for exterior beam.......... $P_{DW.PierExt} := 2\left(P_{DW.BeamExt}\right) = 0 \cdot kip$

B4. End Bent and Pier Dead Load Summary

End Bent Beam Reactions

UNFACTORED BEAM REACTIONS AT END BENTS						
	DC Loads (kip)			DW Loads (kip)		
Beam	x	y	z	x	y	z
1	0.0	-95.0	0.0	0.0	0.0	0.0
2	0.0	-99.9	0.0	0.0	0.0	0.0
3	0.0	-99.9	0.0	0.0	0.0	0.0
4	0.0	-99.9	0.0	0.0	0.0	0.0
5	0.0	-99.9	0.0	0.0	0.0	0.0
6	0.0	-99.9	0.0	0.0	0.0	0.0
7	0.0	-99.9	0.0	0.0	0.0	0.0
8	0.0	-99.9	0.0	0.0	0.0	0.0
9	0.0	-95.0	0.0	0.0	0.0	0.0

Pier Beam Reactions

UNFACTORED BEAM REACTIONS AT PIER						
	DC Loads (kip)			DW Loads (kip)		
Beam	x	y	z	x	y	z
1	0.0	-190.0	0.0	0.0	0.0	0.0
2	0.0	-199.8	0.0	0.0	0.0	0.0
3	0.0	-199.8	0.0	0.0	0.0	0.0
4	0.0	-199.8	0.0	0.0	0.0	0.0
5	0.0	-199.8	0.0	0.0	0.0	0.0
6	0.0	-199.8	0.0	0.0	0.0	0.0
7	0.0	-199.8	0.0	0.0	0.0	0.0
8	0.0	-199.8	0.0	0.0	0.0	0.0
9	0.0	-190.0	0.0	0.0	0.0	0.0

SUBSTRUCTURE DESIGN

Pier Cap Live Load Analysis

References

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\301SubDeadLoad.xmcd(R)

Description

This section provides the pier cap design live load.

Page Contents

A. Input Variables

A1. Shear: Skewed Modification Factor [LRFD 4.6.2.2.3c]

Skew modification factor for shear **shall** be applied to the exterior beam at the obtuse corner (θ>90 deg) and to all beams in a multibeam bridge, whereas $g_{v.Skew} = 1.064$.

A2. Maximum Live Load Reaction at Intermediate Pier - Two HL-93 Vehicles

The live load reaction (including impact and skew modification factors) is applied on the deck as two wheel-line loads.

HL-93 Line Load, including truck and lane $HL93 := \frac{R_{LLIs}}{2} \cdot g_{v.Skew} = 78.5 \cdot kip$

The HL-93 line load can be placed within 2' of the overhang and median barriers.

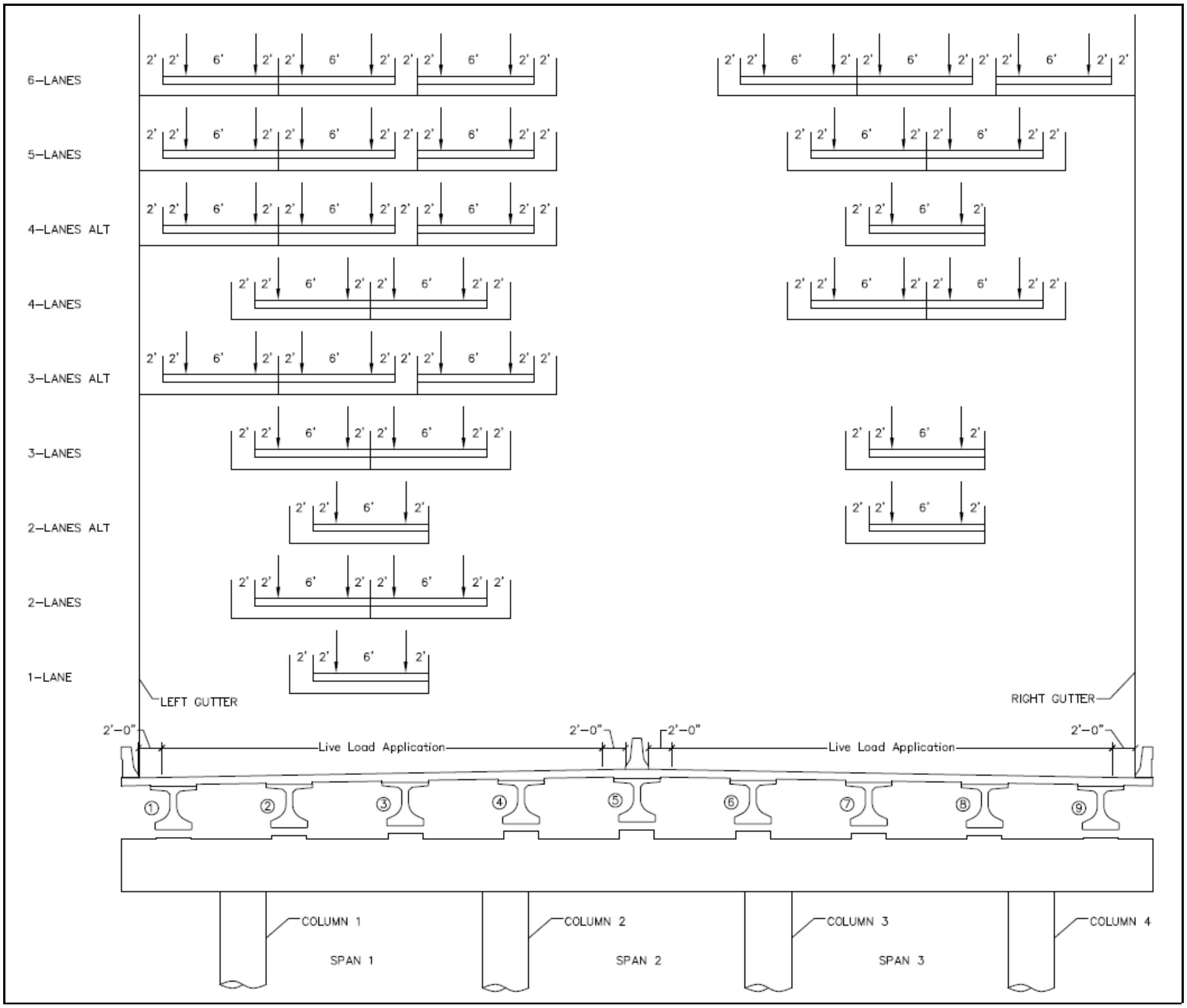
A3. HL-93 Vehicle Placement

HL-93 vehicles, comprising of HL-93 wheel-line loads and lane loads, should be placed on the deck to maximize the moments in the pier cap. Note that for the maximum cap moments, live load may be placed on both sides of the roadway. Utilizing our engineering judgement, it is possible to have up to six lanes of HL-93 vehicles at a single time. However, note that for the calculation of the braking forces, vehicles in only one roadway were utilized since the braking forces would be counter productive or in opposite directions.

Depending on the number of design lanes, a multiple presence factor (LRFD Table 3.6.1.1.2-1) is applied to the HL-93 wheel line loads and lane load.

$$Lanes := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix} \qquad MPF := \begin{pmatrix} 1.2 \\ 1.0 \\ 0.85 \\ 0.65 \\ 0.65 \\ 0.65 \end{pmatrix} \qquad HL93\_Line\_Load := HL93 \cdot MPF$$

$$HL93\_Line\_Load = \begin{pmatrix} 94.2 \\ 78.5 \\ 66.725 \\ 51.025 \\ 51.025 \\ 51.025 \end{pmatrix} \cdot kip$$


6-LANES
5-LANES
4-LANES ALT
4-LANES
3-LANES ALT
3-LANES
2-LANES ALT
2-LANES
1-LANE
LEFT GUTTER
RIGHT GUTTER
2'-0"
Live Load Application
COLUMN 1
COLUMN 2
COLUMN 3
COLUMN 4
SPAN 1
SPAN 2
SPAN 3

SUBSTRUCTURE DESIGN

Pier Cap Design Loads

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\302SubLiveLoad.xmcd(R)

Description

This section provides the design parameters necessary for the substructure pier cap design. The loads calculated in this file are only from the superstructure. Substructure self-weight, wind on substructure and uniform temperature on substructure can be generated by the substructure analysis model/program chosen by the user.

For this design example, RISA was chosen as the analysis model/program.

Page Contents

LRFD Criteria

STRENGTH I - Basic load combination relating to the normal vehicular use of the bridge without wind.

$WA = 0$ Water load and stream pressure are not applicable.

$FR = 0$ No friction forces.

TU Uniform temperature load effects on the pier will be generated by the substructure analysis model.

$$Strength1 = 1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL + 1.75 \cdot BR + 0.50 \cdot (TU + CR + SH)$$

STRENGTH II - Load combination relating to the use of the bridge by Owner-specified special design vehicles, evaluation permit vehicles, or both without wind.

"Permit vehicles are not evaluated in this design example"

STRENGTH III - Load combination relating to the bridge exposed to wind velocity exceeding 55 MPH.

"Applicable to pier column design but not to substructure pier cap design"

STRENGTH IV - Load combination relating to very high dead load to live load force effect ratios.

"Not applicable for the substructure design in this design example"

STRENGTH V - Load combination relating to normal vehicular use of the bridge with wind of 55 MPH velocity.

$$Strength5 = 1.25 \cdot DC + 1.50 \cdot DW + 1.35 \cdot LL + 1.35 \cdot BR + 1.3 \cdot WS + 1.0 \cdot WL \ldots + 0.50 \cdot (TU + CR + SH)$$

EXTREME EVENT I - Load combination including earthquake.

"Not applicable for this simple span prestressed beam bridge design example"

EXTREME EVENT II - Load combination relating to ice load, collision by vessels and vehicles, and certain hydraulic events.

"Not applicable for the substructure design in this design example"

SERVICE I - Load combination relating to the normal operational use of the bridge with a 55 MPH wind and all loads taken at their nominal values.

$$Service1 = 1.0 \cdot DC + 1.0 \cdot DW + 1.0 \cdot LL + 1.0 \cdot BR + 1.0WS + 1.0 \cdot WL + 1.0 \cdot (TU + CR + SH)$$

SERVICE II - Load combination intended to control yielding of steel structures and slip of slip-critical connections due to vehicular live load.

"Not applicable for this simple span prestressed beam bridge design example"

SERVICE III -	Load combination for longitudinal analysis relating to tension in prestressed concrete superstructures with the objective of crack control.
	"Not applicable for the substructure design in this design example"
SERVICE IV -	Load combination relating only to tension in prestressed concrete columns with the objective of crack control.
	"Not applicable for the substructure design in this design example"
FATIGUE I -	Fatigue and fracture load combination related to infinite load-induced fatigue life.
	"Not applicable for the substructure design in this design example"
FATIGUE II -	Fatigue and fracture load combination relating to finite load-induced fatigue life.
	"Not applicable for the substructure design in this design example"

A. General Criteria

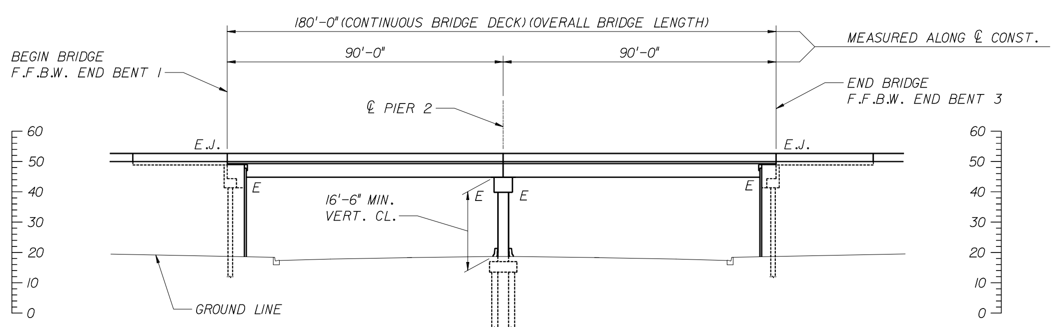
A1. Bearing Design Movement/Strain

Strain due to temperature, creep and shrinkage.. $\varepsilon_{CST} = 0.00029$ *(**Note**: See Sect. 2.09.B4 - Bearing Design Movement/Strain)*

A2. Pier Dead Load Summary

UNFACTORED BEAM REACTIONS AT PIER						
	DC Loads (kip)			DW Loads (kip)		
Beam	**x**	**y**	**z**	**x**	**y**	**z**
1	0.0	-190.0	0.0	0.0	0.0	0.0
2	0.0	-199.8	0.0	0.0	0.0	0.0
3	0.0	-199.8	0.0	0.0	0.0	0.0
4	0.0	-199.8	0.0	0.0	0.0	0.0
5	0.0	-199.8	0.0	0.0	0.0	0.0
6	0.0	-199.8	0.0	0.0	0.0	0.0
7	0.0	-199.8	0.0	0.0	0.0	0.0
8	0.0	-199.8	0.0	0.0	0.0	0.0
9	0.0	-190.0	0.0	0.0	0.0	0.0

A3. Center of Movement



ELEVATION

By inspection, the center of movement will be the intermediate pier.................... $L_0 := L_{span} = 90\,\text{ft}$

B. Lateral Load Analysis

B1. Centrifugal Force: CE [LRFD 3.6.3]

Since the design speed is not specified, it will be conservatively taken as the maximum specified in the AASHTO publication, A Policy on Geometric Design of Highways and Streets.

Design Speed....................................... $V_{design} := 70mph$

Factor per LRFD 3.6.3.......................... $f := \frac{4}{3}$

Horizontal radius................................... $R := 3800ft$

Centrifugal factor................................. $C := \frac{f \cdot V_{design}^2}{g \cdot R}$

$C = 0.11$

Centrifugal force.................................. $P_c := C \cdot \frac{R_{trucks} \cdot IM}{2} \cdot g_{v.Skew} = 6.5 \cdot kip$

$HL93 + P_c = 85.01 \cdot kip$

$HL93 - P_c = 72 \cdot kip$

B2. Braking Force: BR [LRFD 3.6.4]

The braking force should be taken as the greater of:

25% of axle weight for design truck / tandem

5% of design truck / tandem and lane

The number of lanes for braking force calculations depends on future expectations of the bridge. For this example, the bridge is not expected to become one-directional in the future, and future widening is expected to occur to the outside. From this information, the number of lanes is

$N_{lanes} = 3$

The multiple presence factor (LRFD Table 3.6.1.1.2-1) should be taken into account..

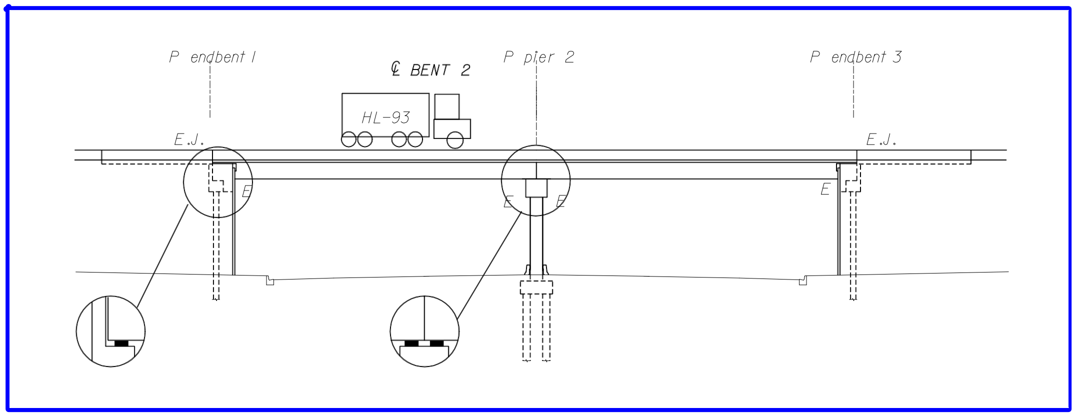
$$MPF := \begin{cases} 1.2 & \text{if } N_{lanes} = 1 \\ 1.0 & \text{if } N_{lanes} = 2 \\ 0.85 & \text{if } N_{lanes} = 3 \\ 0.65 & \text{otherwise} \end{cases} = 0.85$$

Braking force as 25% of axle weight for design truck / tandem............................ $BR_{Force.1} := 25\% \cdot (72 \cdot kip) \cdot N_{lanes} \cdot MPF = 45.9 \cdot kip$

Braking force as 5% of axle weight for design truck / tandem and lane................. $BR_{Force.2} := 5\% \cdot \left(72 \cdot kip + w_L \cdot L_{bridge}\right) \cdot N_{lanes} \cdot MPF = 23.87 \cdot kip$

Governing braking force......................... $BR_{Force} := \max\left(BR_{Force.1}, BR_{Force.2}\right) = 45.9 \cdot kip$

Distribution of Braking Forces to Pier



The same bearing pads are provided at the pier and end bent to distribute the braking forces. The braking force transferred to the pier or end bents is a function of the bearing pad and pier column stiffnesses. For this example, (1) the pier column stiffnesses are ignored, (2) the deck is continuous over pier 2 and expansion joints are provided only at the end bents.

Braking force at pier.............................. $BR_{Pier} = BR_{Force} \cdot \left(K_{Pier}\right)$

where... $$K_{Pier} = \frac{N_{pads.pier} \cdot K_{pad}}{\sum \left(N_{pads.pier} + N_{pads.endbent}\right) \cdot K_{pad}}$$

Simplifying and using variables defined in this example,

pier stiffness can be calculated as....... $$K_{Pier} := \frac{2 \cdot N_{beams}}{(1 + 2 + 1) \cdot N_{beams}} = 0.5$$

corresponding braking force............. $BR_{Pier} := BR_{Force} \cdot \left(K_{Pier}\right) = 22.95 \cdot kip$

Since the bridge superstructure is very stiff in the longitudinal direction, the braking forces are assumed to be equally distributed to the beams under the respective roadway.

$beams := 5$

Braking force at pier per beam................. $$BR_{Pier} := \frac{BR_{Pier}}{beams} = 4.59 \cdot kip$$

Adjustments for Skew

The braking force is transferred to the pier by the bearing pads. The braking forces need to be resolved along the direction of the skew for design of the pier substructure.

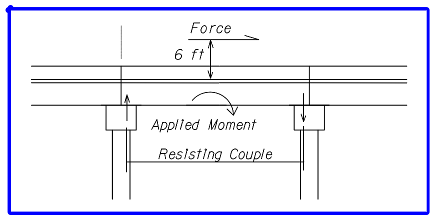
Braking force perpendicular (z-direction) to the pier per beam.................................. $BR_{z.Pier} := BR_{Pier} \cdot \cos(Skew) = 4.31 \cdot kip$

Braking force parallel (x-direction) to the pier per beam.. $BR_{x.Pier} := BR_{Pier} \cdot \sin(Skew) = -1.57 \cdot kip$

Adjustments for Braking Force Loads Applied 6' above Deck

The longitudinal moment induced by braking forces over a pier is resisted by the moment arm. Conservatively, assume the braking occurs over one span only, then the result is an uplift reaction on the downstation end bent or pier and a downward reaction at the upstation end bent or pier. In this example, the braking is assumed to occur in span 1 and the eccentricity of the downward load with the bearing and centerline of pier eccentricities is ignored.

Moment arm from top of bearing pad to location of applied load........................... $M_{arm} := 6ft + h = 9.75\,ft$



Braking force in pier (y-direction), vertical

$$BR_{y.Pier} := \frac{-BR_{Pier} \cdot M_{arm}}{L_{span}} = -0.5 \cdot kip$$

Only the downward component of this force is considered. Typically, the vertical forces (uplift) are small and can be ignored.

BRAKING FORCES AT PIER			
	BR Loads (kip)		
Beam	x	y	z
1	-1.6	-0.5	4.3
2	-1.6	-0.5	4.3
3	-1.6	-0.5	4.3
4	-1.6	-0.5	4.3
5	-1.6	-0.5	4.3
6	0.0	0.0	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0

B3. Creep, Shrinkage, and Temperature Forces

The forces transferred from the superstructure to the substructure due to temperature, creep, and shrinkage are influenced by the shear displacements in the bearing pad. In this example, only temperature and shrinkage effects are considered. Creep is ignored, since this example assumes the beams will creep towards their center and the composite deck will offer some restraint.

Displacements at top of pier due to temperature, creep, and shrinkage...........

$$\Delta_{Pier2} := \left(L_0 - x_{dist_1}\right) \cdot \varepsilon_{CST} = 0 \cdot in$$

where $\varepsilon_{CST} = 0.00029$

Since the bridge has two equal spans and fairly constant pier stiffnesses, the center of movement is the intermediate pier. The center of movement has no displacements, so the pier has no displacements.

Shear force transferred through each bearing pad due to creep, shrinkage, and temperature...

$$CST_{Pier} := \frac{G_{bp} \cdot L_{pad} \cdot W_{pad} \cdot \Delta_{Pier2}}{h_{pad}} = 0 \cdot kip$$

This force needs to be resolved along the direction of the skew...

Shear force perpendicular (z-direction) to the pier per beam.................................

$$CST_{z.Pier} := CST_{Pier} \cdot \cos(Skew) = 0 \cdot kip$$

Shear force parallel (x-direction) to the pier per beam...

$$CST_{x.Pier} := CST_{Pier} \cdot \sin(Skew) = 0 \cdot kip$$

Summary of beam reactions at the pier due to creep, shrinkage and temperature...

CREEP, SHRINKAGE, TEMPERATURE FORCES AT PIER

	CR, SH, TU Loads (kip)		
Beam	**x**	**y**	**z**
1	0.0	0.0	0.0
2	0.0	0.0	0.0
3	0.0	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.0	0.0
6	0.0	0.0	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0

Note:
Shrinkage and temperature effects from the pier substructure can be calculated within the substructure model / analysis program. These values are only from the superstructure.

B4. Wind Pressure on Structure: WS

The wind loads are applied to the superstructure and substructure.

Loads from Superstructure [SDG 2.4], Strength III and Service IV Limit States

The wind pressure on the superstructure consists of lateral (x-direction) and longitudinal (z-direction) components.

Height above ground that the wind pressure is applied.. $z_{sup} = 20.5\,ft$

Design wind pressure .... $P_{z.super} := P_{z.sup.StrIII.ServIV} \cdot ksf = 48.83 \cdot psf$

For prestressed beam bridges, the following wind pressures factors are given to account for the angle of attack [SDG Table 2.4.1-3]

$$Wind_{skew} := \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{Factor} := \begin{array}{cc} \underline{x} & \underline{z} \\ \left(\begin{array}{cc} 1.0 & 0.0 \\ 0.88 & 0.12 \\ 0.82 & 0.24 \\ 0.66 & 0.32 \\ 0.34 & 0.38 \end{array}\right) \end{array} \quad [Global]$$

Wind pressures based on angle of attack are as follows.. $Wind_{super} := P_{z.super} \cdot Wind_{Factor}$

$$Wind_{skew} = \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{super} = \begin{array}{cc} \underline{x} & \underline{z} \\ \left(\begin{array}{cc} 48.83 & 0 \\ 42.97 & 5.86 \\ 40.04 & 11.72 \\ 32.23 & 15.63 \\ 16.6 & 18.56 \end{array}\right) \end{array} \cdot psf \quad [Global]$$

The exposed superstructure area influences the wind forces that are transferred to the supporting substructure. Tributary areas are used to determine the exposed superstructure area.

Exposed superstructure area at Pier 2....... $A_{Super} := L_{span} \cdot (h + 2.667ft) = 577.53\,ft^2$

Forces due to wind applied to the superstructure...................................... $WS_{Super.Pier} := Wind_{super} \cdot A_{Super}$

$$WS_{Super.Pier} = \begin{array}{cc} \underline{x} & \underline{z} \\ \left(\begin{array}{cc} 28.2 & 0.0 \\ 24.8 & 3.4 \\ 23.1 & 6.8 \\ 18.6 & 9.0 \\ 9.6 & 10.7 \end{array}\right) \end{array} \cdot kip \quad [Global]$$

Case 1 - Skew Angle of Wind = 0 degrees

Maximum transverse force...................... $F_{WS.x.case1} := WS_{Super.Pier_{0,0}} = 28.2 \cdot kip$

Maximum longitudinal force.................... $F_{WS.z.case1} := WS_{Super.Pier_{0,1}} = 0 \cdot kip$

The forces due to wind need to be resolved along the direction of the skew.

Force perpendicular (z-direction) to the pier........... $WS_{z.Pier.case1} := \left|F_{WS.z.case1} \cdot \cos(Skew)\right| + \left|F_{WS.x.case1} \cdot \sin(Skew)\right| = 9.65 \cdot kip$

Force parallel (x-direction) to the pier....... $WS_{x.Pier.case1} := \left|F_{WS.z.case1} \cdot \sin(Skew)\right| + \left|F_{WS.x.case1} \cdot \cos(Skew)\right| = 26.5 \cdot kip$

Case 2 - Skew Angle of Wind = 60 degrees

Maximum transverse force...................... $F_{WS.x.case2} := WS_{Super.Pier_{4,0}} = 9.59 \cdot kip$

Maximum longitudinal force.................... $F_{WS.z.case2} := WS_{Super.Pier_{4,1}} = 10.72 \cdot kip$

The forces due to wind need to be resolved along the direction of the skew.

Force perpendicular (z-direction) to the pier........... $WS_{z.Pier.case2} := \left|F_{WS.z.case2} \cdot \cos(Skew)\right| + \left|F_{WS.x.case2} \cdot \sin(Skew)\right| = 13.35 \cdot kip$

Force parallel (x-direction) to the pier....... $WS_{x.Pier.case2} := \left|F_{WS.z.case2} \cdot \sin(Skew)\right| + \left|F_{WS.x.case2} \cdot \cos(Skew)\right| = 12.68 \cdot kip$

A conservative approach is taken to minimize the analysis required. The maximum transverse and longitudinal forces are used in the following calculations.

Force perpendicular (z-direction) to the pier....................................... $WS_{z.Pier.StrIII.ServIV} := \max\left(WS_{z.Pier.case1}, WS_{z.Pier.case2}\right) = 13.35 \cdot k$

Force parallel (x-direction) to the pier....... $WS_{x.Pier.StrIII.ServIV} := \max\left(WS_{x.Pier.case1}, WS_{x.Pier.case2}\right) = 26.5 \cdot ki$

Loads from Superstructure [SDG 2.4], Strength V and Service I Limit States

The wind pressure on the superstructure consists of lateral (x-direction) and longitudinal (z-direction) components.

Height above ground that the wind pressure is applied.. $z_{sup} = 20.5\,ft$

Design wind pressure .... $P_{z.super} := P_{z.sup.StrV.ServI} \cdot ksf = 0.01 \cdot ksf$

For prestressed beam bridges, the following wind pressures factors are given to account for the angle of attack [SDG Table 2.4.1-3]

$$Wind_{skew} = \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{Factor} = \begin{array}{cc} \underline{x} & \underline{z} \\ \begin{pmatrix} 1 & 0 \\ 0.88 & 0.12 \\ 0.82 & 0.24 \\ 0.66 & 0.32 \\ 0.34 & 0.38 \end{pmatrix} & \end{array} \quad \text{[Global]}$$

Wind pressures based on angle of attack are as follows.. $Wind_{super} := P_{z.super} \cdot Wind_{Factor}$

$$Wind_{skew} = \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{super} = \begin{array}{cc} \underline{x} & \underline{z} \\ \begin{pmatrix} 10.63 & 0 \\ 9.36 & 1.28 \\ 8.72 & 2.55 \\ 7.02 & 3.4 \\ 3.62 & 4.04 \end{pmatrix} & \end{array} \cdot psf \quad \text{[Global]}$$

The exposed superstructure area influences the wind forces that are transferred to the supporting substructure. Tributary areas are used to determine the exposed superstructure area.

Exposed superstructure area at Pier 2....... $A_{Super} := L_{span} \cdot (h + 2.667ft) = 577.53\,ft^2$

Forces due to wind applied to the superstructure...................................... $WS_{Super.Pier} := Wind_{super} \cdot A_{Super}$

$$WS_{Super.Pier} = \begin{array}{cc} \underline{x} & \underline{z} \\ \begin{pmatrix} 6.1 & 0.0 \\ 5.4 & 0.7 \\ 5.0 & 1.5 \\ 4.1 & 2.0 \\ 2.1 & 2.3 \end{pmatrix} & \end{array} \cdot kip \quad \text{[Global]}$$

Case 1 - Skew Angle of Wind = 0 degrees

Maximum transverse force...................... $F_{WS.x.case1} := WS_{Super.Pier_{0,0}} = 6.14 \cdot kip$

Maximum longitudinal force.................... $F_{WS.z.case1} := WS_{Super.Pier_{0,1}} = 0 \cdot kip$

The forces due to wind need to be resolved along the direction of the skew.

Force perpendicular (z-direction) to the pier.......... $WS_{z.Pier.case1} := \left|F_{WS.z.case1} \cdot \cos(Skew)\right| + \left|F_{WS.x.case1} \cdot \sin(Skew)\right| = 2.1 \cdot kip$

Force parallel (x-direction) to the pier.. $WS_{x.Pier.case1} := \left|F_{WS.z.case1} \cdot \sin(Skew)\right| + \left|F_{WS.x.case1} \cdot \cos(Skew)\right| = 5.77 \cdot kip$

Case 2 - Skew Angle of Wind = 60 degrees

Maximum transverse force...................... $F_{WS.x.case2} := WS_{Super.Pier_{4,0}} = 2.09 \cdot kip$

Maximum longitudinal force.................... $F_{WS.z.case2} := WS_{Super.Pier_{4,1}} = 2.33 \cdot kip$

The forces due to wind need to be resolved along the direction of the skew.

Force perpendicular (z-direction) to the pier....... $WS_{z.Pier.case2} := \left|F_{WS.z.case2} \cdot \cos(Skew)\right| + \left|F_{WS.x.case2} \cdot \sin(Skew)\right| = 2.91 \cdot kip$

Force parallel (x-direction) to the pier.. $WS_{x.Pier.case2} := \left|F_{WS.z.case2} \cdot \sin(Skew)\right| + \left|F_{WS.x.case2} \cdot \cos(Skew)\right| = 2.76 \cdot kip$

A conservative approach is taken to minimize the analysis required. The maximum transverse and longitudinal forces are used in the following calculations.

Force perpendicular (z-direction) to the pier.. $WS_{z.Pier.StrV.ServI} := \max\left(WS_{z.Pier.case1}, WS_{z.Pier.case2}\right) = 2.91 \cdot ki$

Force parallel (x-direction) to the pier....... $WS_{x.Pier.StrV.ServI} := \max\left(WS_{x.Pier.case1}, WS_{x.Pier.case2}\right) = 5.77 \cdot k$

WIND ON STRUCTURE FORCES AT PIER						
	Strength III, Service IV **WS Loads (kip)**			Strength V, Service I **WS Loads (kip)**		
Beam	**x**	**y**	**z**	**x**	**y**	**z**
1	2.9	0.0	1.5	0.6	0	0.3
2	2.9	0.0	1.5	0.6	0	0.3
3	2.9	0.0	1.5	0.6	0	0.3
4	2.9	0.0	1.5	0.6	0	0.3
5	2.9	0.0	1.5	0.6	0	0.3
6	2.9	0.0	1.5	0.6	0	0.3
7	2.9	0.0	1.5	0.6	0	0.3
8	2.9	0.0	1.5	0.6	0	0.3
9	2.9	0.0	1.5	0.6	0	0.3

Loads on Substructure [SDG 2.4], Strength III and Service IV Limit States

Design wind pressure .... $P_{z.sub} := P_{z.sub.StrIII.ServIV} \cdot ksf = 66.59 \cdot psf$

For prestressed beam bridges, the following wind pressures factors are given in the SDG..

$$Wind_{skew} = \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{Factor} = \begin{pmatrix} 1 & 0 \\ 0.88 & 0.12 \\ 0.82 & 0.24 \\ 0.66 & 0.32 \\ 0.34 & 0.38 \end{pmatrix}$$

For prestressed beam bridges, the following wind pressures are given in the SDG.........

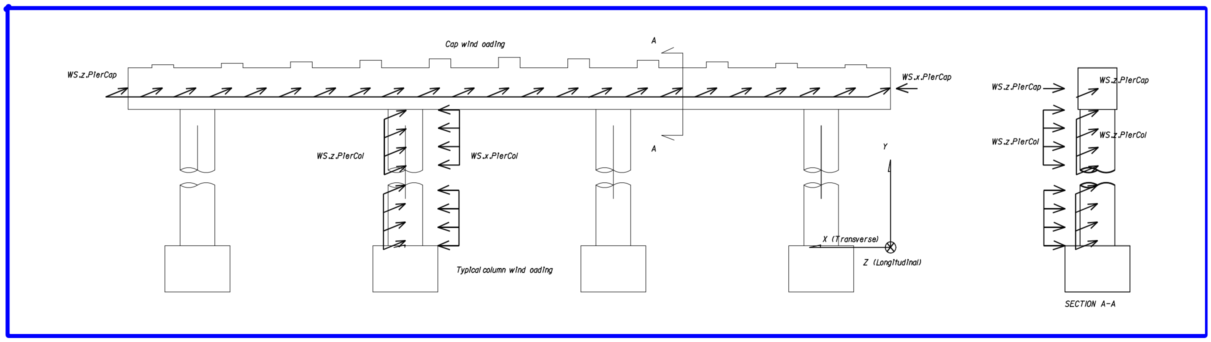
$$Wind_{sub} := P_{z.sub} \cdot Wind_{Factor}$$

$$Wind_{skew} = \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{sub} = \begin{array}{c} \underline{x} \quad \underline{z} \\ \begin{pmatrix} 66.59 & 0 \\ 58.6 & 7.99 \\ 54.6 & 15.98 \\ 43.95 & 21.31 \\ 22.64 & 25.3 \end{pmatrix} \end{array} \cdot psf \qquad \text{[Global]}$$

General equation for wind forces applied to the substructure....................................

$$WS_{Force} = \left(Wind_{Pressure}\right) \cdot \left(Exposed\ Area_{Substructure}\right) \cdot \left(Skew_{Adjustment}\right)$$

For modeling purposes in this example, the following information summarizes the placement of wind forces on the substructure.



The longitudinal (z-direction) wind load on the pier cap is applied as a line load along the front of the cap.

$$WS_{z.PierCap.StrIIIServIV} := Wind_{sub_{4,1}} \cdot h_{Cap} \cdot \cos(Skew) - Wind_{sub_{0,0}} \cdot h_{Cap} \cdot \sin(Skew) = 0.21 \cdot klf$$

The transverse (x-direction) wind load on the pier cap is applied as a point load on the end of the cap.

$$WS_{x.PierCap.StrIIIServIV} := Wind_{sub_{0,0}} \cdot \left(b_{Cap} \cdot h_{Cap}\right) \cdot \cos(Skew) + Wind_{sub_{4,1}} \cdot \left(L_{Cap} \cdot h_{Cap}\right) \cdot \sin(Skew) = -2.16 \cdot kip$$

The longitudinal (z-direction) wind load on the column is applied as a line load on the exposed column height.

$$WS_{z.PierCol.StrIIIServIV} := Wind_{sub_{4,1}} \cdot b_{Col} \cdot \cos(Skew) - Wind_{sub_{0,0}} \cdot b_{Col} \cdot \sin(Skew) = 0.19 \cdot klf$$

The transverse (x-direction) wind load on the column is applied as a line load on the exposed column height.

$$WS_{x.PierCol.StrIIIServIV} := Wind_{sub_{0,0}} \cdot b_{Col} \cdot \cos(Skew) + Wind_{sub_{4,1}} \cdot b_{Col} \cdot \sin(Skew) = 0.22 \cdot klf$$

Loads on Substructure [SDG 2.4], Strength V and Service I Limit States

Design wind pressure .... $P_{z.sub} := P_{z.sub.StrV.ServI} \cdot ksf = 14.5 \cdot psf$

For prestressed beam bridges, the following wind pressures factors are given in the SDG..

$$Wind_{skew} = \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{Factor} = \begin{pmatrix} 1 & 0 \\ 0.88 & 0.12 \\ 0.82 & 0.24 \\ 0.66 & 0.32 \\ 0.34 & 0.38 \end{pmatrix}$$

For prestressed beam bridges, the following wind pressures are given in the SDG.........

$Wind_{sub} := P_{z.sub} \cdot Wind_{Factor}$

[Global]

$$Wind_{skew} = \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{sub} = \begin{array}{c} \begin{array}{cc} \underline{x} & \underline{z} \end{array} \\ \begin{pmatrix} 14.5 & 0 \\ 12.76 & 1.74 \\ 11.89 & 3.48 \\ 9.57 & 4.64 \\ 4.93 & 5.51 \end{pmatrix} \end{array} \cdot psf$$

General equation for wind forces applied to the substructure....................................

$$WS_{Force} = (Wind_{Pressure}) \cdot (Exposed\ Area_{Substructure}) \cdot (Skew_{Adjustment})$$

The longitudinal (z-direction) wind load on the pier cap is applied as a line load along the front of the cap.

$$WS_{z.PierCap.StrVServI} := Wind_{sub_{4,1}} \cdot h_{Cap} \cdot \cos(Skew) - Wind_{sub_{0,0}} \cdot h_{Cap} \cdot \sin(Skew) = 0.05 \cdot klf$$

The transverse (x-direction) wind load on the pier cap is applied as a point load on the end of the cap.

$$WS_{x.PierCap.StrVServI} := Wind_{sub_{0,0}} \cdot (b_{Cap} \cdot h_{Cap}) \cdot \cos(Skew) + Wind_{sub_{4,1}} \cdot (L_{Cap} \cdot h_{Cap}) \cdot \sin(Skew) = -0.47 \cdot kip$$

The longitudinal (z-direction) wind load on the column is applied as a line load on the exposed column height.

$$WS_{z.PierCol.StrVServI} := Wind_{sub_{4,1}} \cdot b_{Col} \cdot \cos(Skew) - Wind_{sub_{0,0}} \cdot b_{Col} \cdot \sin(Skew) = 0.04 \cdot klf$$

The transverse (x-direction) wind load on the column is applied as a line load on the exposed column height.

$$WS_{x.PierCol.StrVServI} := Wind_{sub_{0,0}} \cdot b_{Col} \cdot \cos(Skew) + Wind_{sub_{4,1}} \cdot b_{Col} \cdot \sin(Skew) = 0.05 \cdot klf$$

B5. Wind Pressure on Vehicles [LRFD 3.8.1.3]

The LRFD specifies that wind load should be applied to vehicles on the bridge..........

$$Skew_{wind} := \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{LRFD} := \begin{array}{c} \begin{array}{cc} \underline{x} & \underline{z} \end{array} \\ \begin{pmatrix} .100 & 0 \\ .088 & .012 \\ .082 & .024 \\ .066 & .032 \\ .034 & .038 \end{pmatrix} \end{array} \cdot \frac{kip}{ft}$$

Height above ground for wind pressure on vehicles..

$$Z_2 := h_{Col} - h_{Surcharge} + 6ft + h_{Cap} + h = 26.25\ ft$$

The wind forces on vehicles are transmitted to Pier 2 of the substructure using tributary lengths...........................

$L_{Pier} := L_{span} = 90\,ft$

Forces due to wind on vehicles applied to the superstructure.................................

$WL_{Super.Pier} := Wind_{LRFD} \cdot L_{Pier}$

$$WL_{Super.Pier} = \begin{array}{cc} \underline{x} & \underline{z} \\ \end{array} \begin{pmatrix} 9.0 & 0.0 \\ 7.9 & 1.1 \\ 7.4 & 2.2 \\ 5.9 & 2.9 \\ 3.1 & 3.4 \end{pmatrix} \cdot kip$$

A conservative approach is taken to minimize the analysis required. The maximum transverse and longitudinal forces are used in the following calculations.

Maximum transverse force.....................

$F_{WL.x} := WL_{Super.Pier_{0,0}} = 9 \cdot kip$

Maximum longitudinal force...................

$F_{WL.z} := WL_{Super.Pier_{4,1}} = 3.42 \cdot kip$

The forces due to wind need to be resolved along the direction of the skew.

Force perpendicular (z-direction) to the pier..

$WL_{z.Pier} := \left|F_{WL.z} \cdot \cos(Skew)\right| + \left|F_{WL.x} \cdot \sin(Skew)\right| = 6.29 \cdot kip$

Force perpendicular (z-direction) to the pier per beam..

$WL_{z.Beam} := \dfrac{WL_{z.Pier}}{N_{beams}} = 0.7 \cdot kip$

Force parallel (x-direction) to the cap........

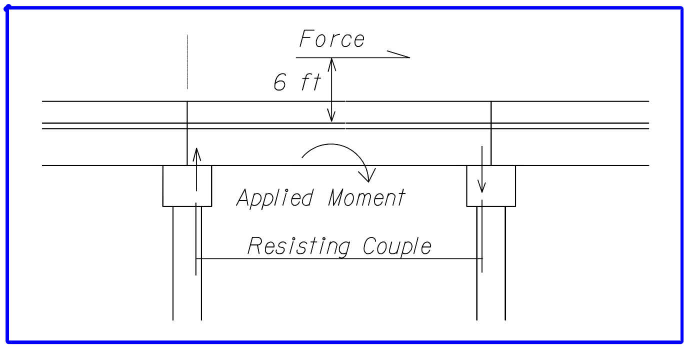
$WL_{x.Pier} := \left|F_{WL.z} \cdot \sin(Skew)\right| + \left|F_{WL.x} \cdot \cos(Skew)\right| = 9.63 \cdot kip$

Force parallel (x-direction) to the cap per beam...

$WL_{x.Beam} := \dfrac{WL_{x.Pier}}{N_{beams}} = 1.07 \cdot kip$

Longitudinal Adjustments for Wind on Vehicles

The longitudinal moment is resisted by the moment arm (similar to braking forces).



Moment arm from top of bearing pad to location of applied load........................... $M_{arm} = 9.750\ \text{ft}$ $\quad$ *($M_{arm} = h + 6\ ft$)*

Vertical force in pier due to wind pressure on vehicle per beam (+/-)........................ $WL_{y.Beam} := \dfrac{-WL_{z.Beam} \cdot M_{arm}}{L_{span}} = -0.08 \cdot \text{kip}$

For this design example, this component of the load is **ignored**.

Transverse Adjustments for Wind on Vehicles

Using the principles of the lever rule for transverse distribution of live load on beams, the wind on live can be distributed similarly. It assumes that the wind acting on the live load will cause the vehicle to tilt over. Using the lever rule, the tilting effect of the vehicle is resisted by up and down reactions on the beams assuming the deck to act as a simple span between beams.

Moment arm from top of bearing pad to location of applied load........................... $M_{arm} = 9.750\ \text{ft}$

Vertical reaction on one beam on pier from transverse wind pressure on vehicles (+/-) $WL_{y.Beam} := \dfrac{-WL_{x.Pier} \cdot M_{arm}}{BeamSpacing} = -9.39 \cdot \text{kip}$

Since this load can occur at any beam location, conservatively apply this load to all beams

WIND ON LIVE LOAD FORCES AT PIER			
	WL Loads (kip)		
Beam	**x**	**y**	**z**
1	1.1	9.4	0.7
2	1.1	-9.4	0.7
3	1.1	9.4	0.7
4	1.1	-9.4	0.7
5	1.1	9.4	0.7
6	1.1	-9.4	0.7
7	1.1	9.4	0.7
8	1.1	-9.4	0.7
9	1.1	0.0	0.7

C. Design Limit States

The design loads for strength I, strength V, and service I limit states are summarized in this section. For each limit state, maximum positive moment in the cap, maximum negative moment in the cap, and maximum shear are presented.

These reactions are from the superstructure **only**, acting on the substructure. In the RISA analysis model, include the following loads:

- DC: self-weight of the substructure, include pier cap and columns
- TU: a temperature increase and fall on the pier substructure utilizing the following parameters:

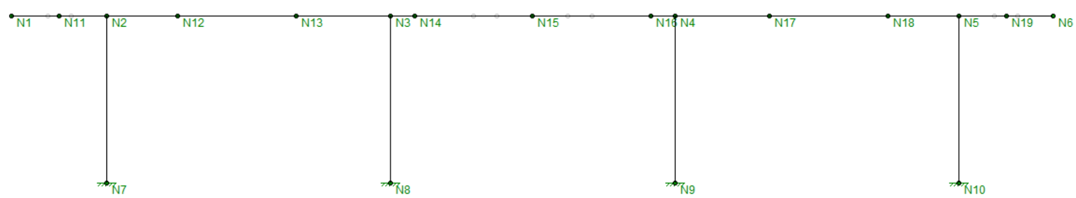
coefficient of expansion $\alpha_t = 6 \times 10^{-6} \cdot \frac{1}{°F}$

temperature change $temperature_{increase} = temperature_{fall} = 35 \cdot °F$

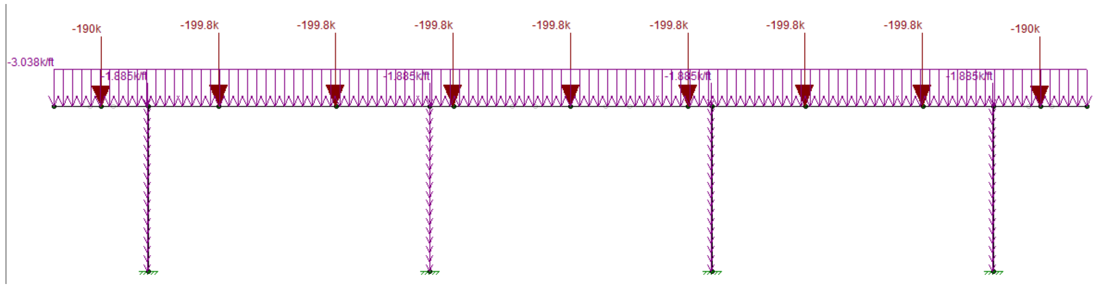
Two load cases would be required for temperature with a positive and negative strain being inputed.

Fixity of the pier was provided at the bottom of the columns.

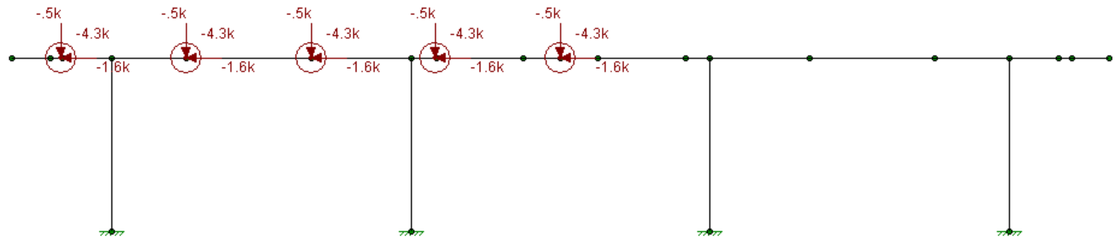
Substructure Model



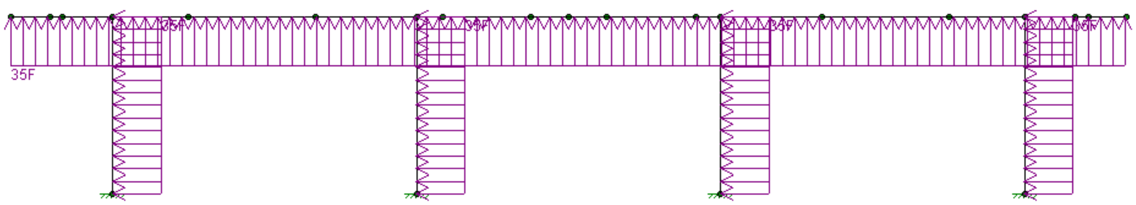
Forces applied directly to the analysis model: DC Load Case



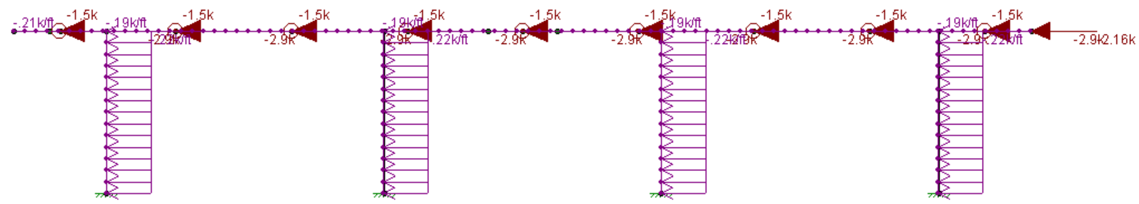
Forces applied directly to the analysis model: BR Load Case



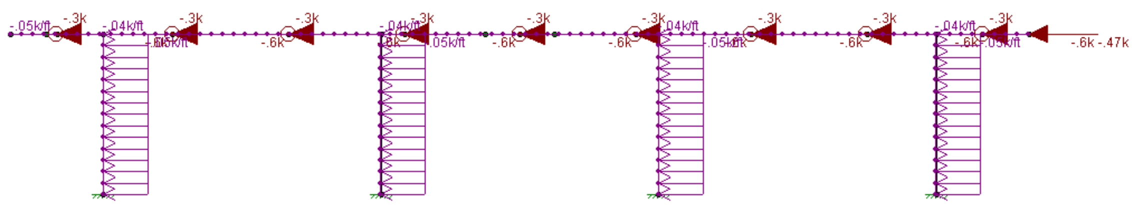
Forces applied directly to the analysis model: CR/SH/TU Load Case



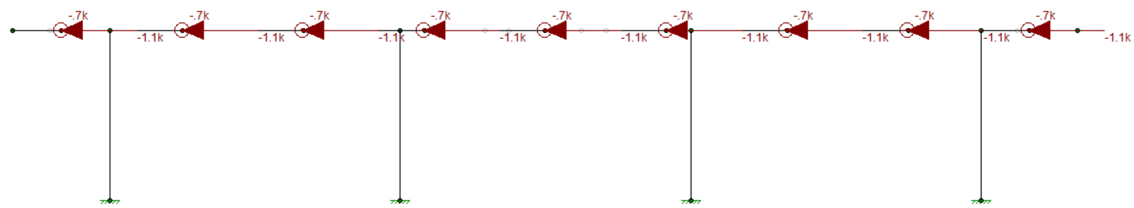
Forces applied directly to the analysis model: WS (Strength III/Service IV)



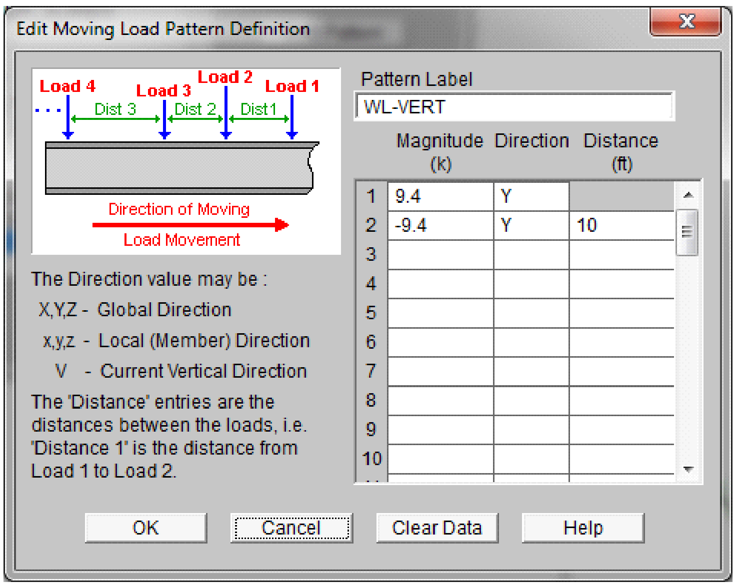
Forces applied directly to the analysis model: WS (Strength V/Service I)



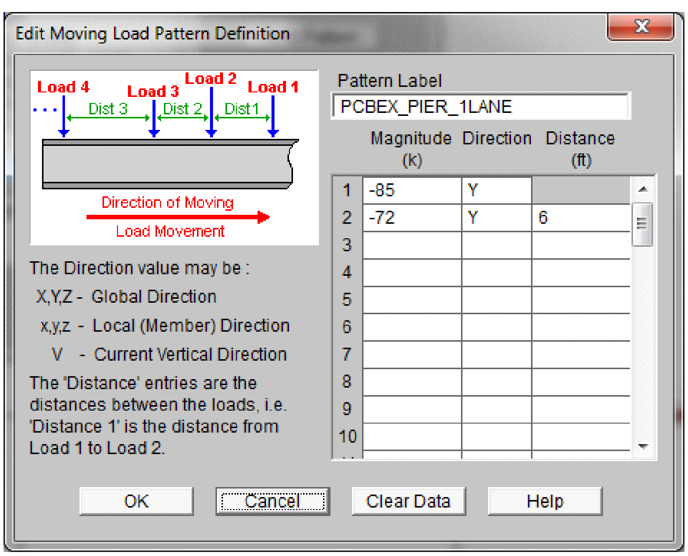
Forces applied directly to the analysis model: WL



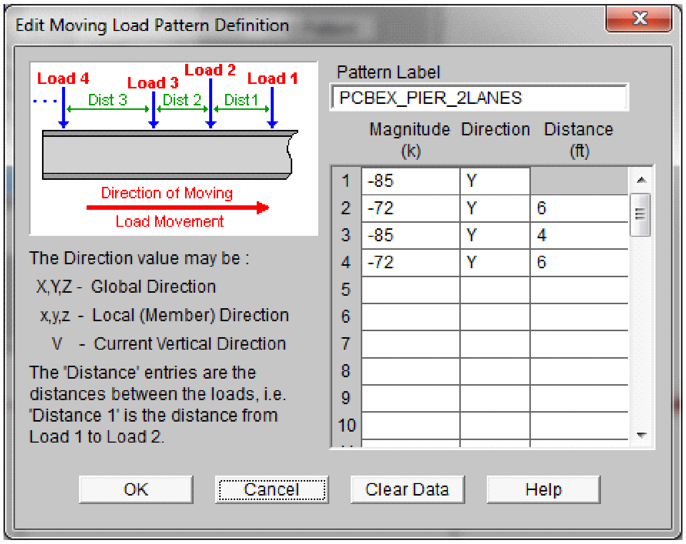
Moving Load Pattern: WL Vertical



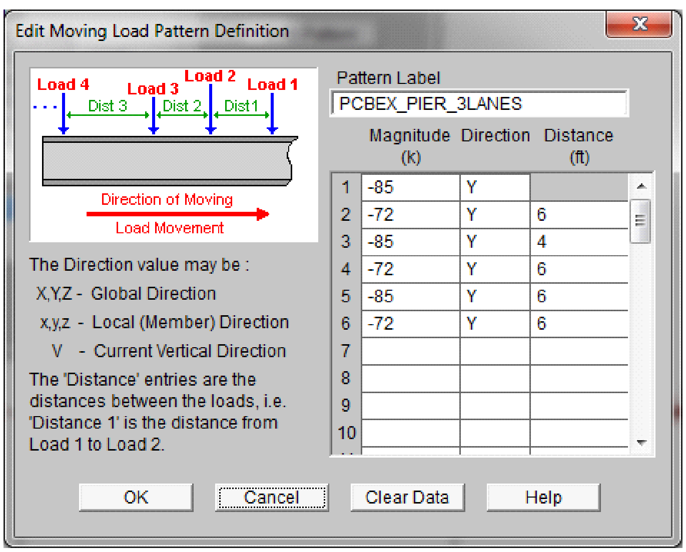
Moving Load Pattern: One Lane



Moving Load Pattern: Two Lanes



Moving Load Pattern: Three Lanes



Moving Load Cases:

	T...	Pattern	Incre...	Bot...	1st J...	2nd ...	3rd J...	4th J...	5th J...	6th J...	7th J...	8th J...	9th J...	10th ...
1	M1	PCBEX_PIER_1LANE	1	☐	L1A	L1B								
2	M2	PCBEX_PIER_2LANES	1	☐	L2A	L2B								
3	M3	PCBEX_PIER_3LANES	1	☐	L2A	L2B								
4	M4	PCBEX_PIER_1LANE	1	☐	R1A	R1B								
5	M5	PCBEX_PIER_2LANES	1	☐	R2A	R2B								
6	M6	PCBEX_PIER_3LANES	1	☐	R2A	R2B								
7	M7	WL-VERT	1	☑	L1A	R2B								

Basic Load Cases:

	BLC Description	Category	X Gravity	Y Gravity	Z Gravity	Joint	Point	Distrib...	Area (...	Surfac...
1	DC Load	None				9		1		
2	DW Load	None								
3	BR Load	None				15				
4	CR/SH/TU Loads	None						1		
5	WS StrIII/ServIV	None				18				
6	WS StrV/ServI	None				18				
7	WL	None				18				

Load Combinations:

- All applied loads in the substructure analysis model should be multiplied by the appropriate load factor values and combined with the limit state loads calculated in this file for the final results. The multiple presence factor is included with the moving load case factors.

	Description	Sol...	PD...	SR...	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor
1	StrI-1Lane +TU	☑			1	1.25	3	2.1	4	.5	M1	2.1								
2	StrI-1Lane -TU	☑			1	1.25	3	2.1	4	-.5	M1	2.1								
3	StrI-2Lanes(2) +TU	☑			1	1.25	3	1.75	4	.5	M2	1.75								
4	StrI-2Lanes(2) -TU	☑			1	1.25	3	1.75	4	-.5	M2	1.75								
5	StrI-2Lanes(1+1) +TU	☑			1	1.25	3	1.75	4	.5	M1	1.75	M4	1.75						
6	StrI-2Lanes(1+1) -TU	☑			1	1.25	3	1.75	4	-.5	M1	1.75	M4	1.75						
7	StrI-3Lanes(3) +TU	☑			1	1.25	3	1.488	4	.5	M3	1.488								
8	StrI-3Lanes(3) -TU	☑			1	1.25	3	1.488	4	-.5	M3	1.488								
9	StrI-3Lanes(2+1) +TU	☑			1	1.25	3	1.488	4	.5	M2	1.488	M4	1.488						
10	StrI-3Lanes(2+1) -TU	☑			1	1.25	3	1.488	4	-.5	M2	1.488	M4	1.488						
11	StrI-4Lanes(3+1) +TU	☑			1	1.25	3	1.138	4	.5	M3	1.138	M4	1.138						
12	StrI-4Lanes(3+1) -TU	☑			1	1.25	3	1.138	4	-.5	M3	1.138	M4	1.138						
13	StrI-4Lanes(2+2) +TU	☑			1	1.25	3	1.138	4	.5	M2	1.138	M5	1.138						
14	StrI-4Lanes(2+2) -TU	☑			1	1.25	3	1.138	4	-.5	M2	1.138	M5	1.138						
15	StrI-5Lanes +TU	☑			1	1.25	3	1.138	4	.5	M3	1.138	M5	1.138						
16	StrI-5Lanes -TU	☑			1	1.25	3	1.138	4	-.5	M3	1.138	M5	1.138						
17	StrI-6Lanes +TU	☑			1	1.25	3	1.138	4	.5	M3	1.138	M6	1.138						
18	StrI-6Lanes -TU	☑			1	1.25	3	1.138	4	-.5	M3	1.138	M6	1.138						
19	StrIII-1Lane +TU	☑			1	1.25	4	.5	5	1.4										
20	StrIII-1Lane -TU	☑			1	1.25	4	-.5	5	1.4										
21	StrIII-2Lanes(2) +TU	☑			1	1.25	4	.5	5	1.4										
22	StrIII-2Lanes(2) -TU	☑			1	1.25	4	-.5	5	1.4										
23	StrIII-2Lanes(1+1) +TU	☑			1	1.25	4	.5	5	1.4										
24	StrIII-2Lanes(1+1) -TU	☑			1	1.25	4	-.5	5	1.4										
25	StrIII-3Lanes(3) +TU	☑			1	1.25	4	.5	5	1.4										
26	StrIII-3Lanes(3) -TU	☑			1	1.25	4	-.5	5	1.4										
27	StrIII-3Lanes(2+1) +TU	☑			1	1.25	4	.5	5	1.4										
28	StrIII-3Lanes(2+1) -TU	☑			1	1.25	4	-.5	5	1.4										
29	StrIII-4Lanes(3+1) +TU	☑			1	1.25	4	.5	5	1.4										
30	StrIII-4Lanes(3+1) -TU	☑			1	1.25	4	-.5	5	1.4										
31	StrIII-4Lanes(2+2) +TU	☑			1	1.25	4	.5	5	1.4										
32	StrIII-4Lanes(2+2) -TU	☑			1	1.25	4	-.5	5	1.4										
33	StrIII-5Lanes +TU	☑			1	1.25	4	.5	5	1.4										
34	StrIII-5Lanes -TU	☑			1	1.25	4	-.5	5	1.4										
35	StrIII-6Lanes +TU	☑			1	1.25	4	.5	5	1.4										
36	StrIII-6Lanes -TU	☑			1	1.25	4	-.5	5	1.4										

Load Combinations

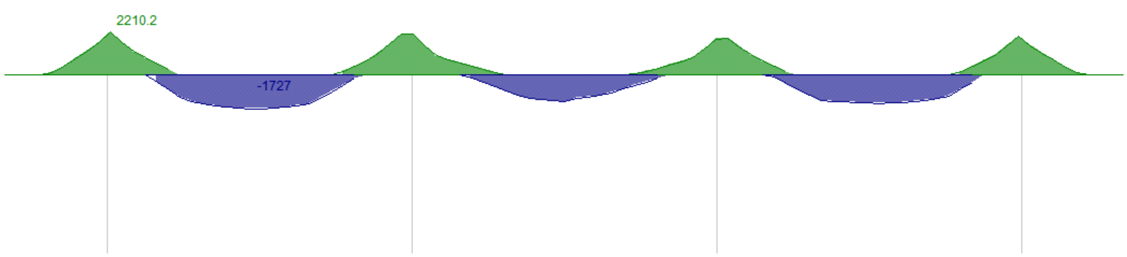
Combinations | Design

	Description	Sol...	PD...	SR...	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor
37	StrV-1Lane +TU	☐			1	1.25	3	1.62	4	.5	6	1.3	7	1	M7	1	M1	1.62		
38	StrV-1Lane -TU	☐			1	1.25	3	1.62	4	-.5	6	1.3	7	1	M7	1	M1	1.62		
39	StrV-2Lanes(2) +TU	☐			1	1.25	3	1.35	4	.5	6	1.3	7	1	M7	1	M2	1.35		
40	StrV-2Lanes(2) -TU	☐			1	1.25	3	1.35	4	-.5	6	1.3	7	1	M7	1	M2	1.35		
41	StrV-2lanes(1+1) +TU	☐			1	1.25	3	1.35	4	.5	6	1.3	7	1	M7	1	M1	1.35	M4	1.35
42	StrV-2lanes(1+1) -TU	☐			1	1.25	3	1.35	4	-.5	6	1.3	7	1	M7	1	M1	1.35	M4	1.35
43	StrV-3Lanes(3) +TU	☐			1	1.25	3	1.15	4	.5	6	1.3	7	1	M7	1	M3	1.15		
44	StrV-3Lanes(3) -TU	☐			1	1.25	3	1.15	4	-.5	6	1.3	7	1	M7	1	M3	1.15		
45	StrV-3Lanes(2+1) +TU	☐			1	1.25	3	1.15	4	.5	6	1.3	7	1	M7	1	M2	1.15	M4	1.15
46	StrV-3Lanes(2+1) -TU	☐			1	1.25	3	1.15	4	-.5	6	1.3	7	1	M7	1	M2	1.15	M4	1.15
47	StrV-4Lanes(3+1) +TU	☐			1	1.25	3	.88	4	.5	6	1.3	7	1	M7	1	M3	.88	M4	.88
48	StrV-4Lanes(3+1) -TU	☐			1	1.25	3	.88	4	-.5	6	1.3	7	1	M7	1	M3	.88	M4	.88
49	StrV-4Lanes(2+2) +TU	☐			1	1.25	3	.88	4	.5	6	1.3	7	1	M7	1	M2	.88	M5	.88
50	StrV-4Lanes(2+2) -TU	☐			1	1.25	3	.88	4	-.5	6	1.3	7	1	M7	1	M2	.88	M5	.88
51	StrV-5Lanes +TU	☐			1	1.25	3	.88	4	.5	6	1.3	7	1	M7	1	M3	.88	M5	.88
52	StrV-5Lanes -TU	☐			1	1.25	3	.88	4	-.5	6	1.3	7	1	M7	1	M3	.88	M5	.88
53	StrV-6Lanes +TU	☐			1	1.25	3	.88	4	.5	6	1.3	7	1	M7	1	M3	.88	M6	.88
54	StrV-6Lanes -TU	☐			1	1.25	3	.88	4	-.5	6	1.3	7	1	M7	1	M3	.88	M6	.88
55	ServI-1Lane +TU	☐			1	1	3	1.2	4	1	6	1	7	1	M7	1	M1	1.2		
56	ServI-1Lane -TU	☐			1	1	3	1.2	4	-1	6	1	7	1	M7	1	M1	1.2		
57	ServI-2Lanes(2) +TU	☐			1	1	3	1	4	1	6	1	7	1	M7	1	M2	1		
58	ServI-2Lanes(2) -TU	☐			1	1	3	1	4	-1	6	1	7	1	M7	1	M2	1		
59	ServI-2Lanes(1+1) +TU	☐			1	1	3	1	4	1	6	1	7	1	M7	1	M1	1	M4	1
60	ServI-2Lanes(1+1) -TU	☐			1	1	3	1	4	-1	6	1	7	1	M7	1	M1	1	M4	1
61	ServI-3Lanes(3) +TU	☐			1	1	3	.85	4	1	6	1	7	1	M7	1	M3	.85		
62	ServI-3Lanes(3) -TU	☐			1	1	3	.85	4	-1	6	1	7	1	M7	1	M3	.85		
63	ServI-3Lanes(2+1) +TU	☐			1	1	3	.85	4	1	6	1	7	1	M7	1	M2	.85	M4	.85
64	ServI-3Lanes(2+1) -TU	☐			1	1	3	.85	4	-1	6	1	7	1	M7	1	M2	.85	M4	.85
65	ServI-4Lanes(3+1) +TU	☐			1	1	3	.65	4	1	6	1	7	1	M7	1	M3	.65	M4	.65
66	ServI-4Lanes(3+1) -TU	☐			1	1	3	.65	4	-1	6	1	7	1	M7	1	M3	.65	M4	.65
67	ServI-4Lanes(2+2) +TU	☐			1	1	3	.65	4	1	6	1	7	1	M7	1	M2	.65	M5	.65
68	ServI-4Lanes(2+2) -TU	☐			1	1	3	.65	4	-1	6	1	7	1	M7	1	M2	.65	M5	.65
69	ServI-5Lanes +TU	☐			1	1	3	.65	4	1	6	1	7	1	M7	1	M3	.65	M5	.65
70	ServI-5Lanes -TU	☐			1	1	3	.65	4	-1	6	1	7	1	M7	1	M3	.65	M5	.65
71	ServI-6Lanes +TU	☐			1	1	3	.65	4	1	6	1	7	1	M7	1	M3	.65	M6	.65
72	ServI-6Lanes -TU	☐			1	1	3	.65	4	-1	6	1	7	1	M7	1	M3	.65	M6	.65

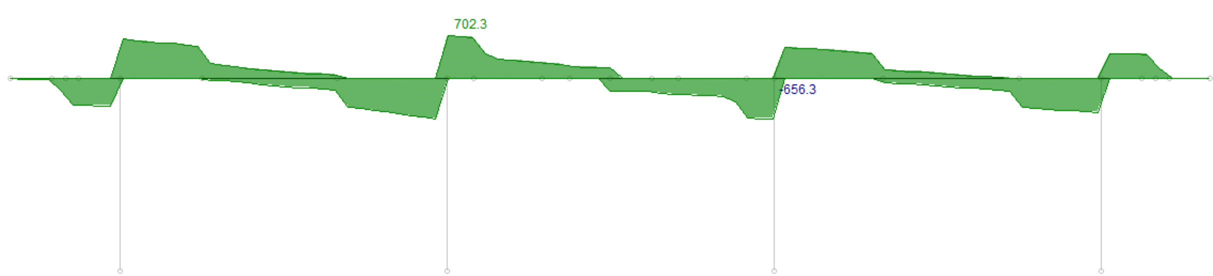
C1. Strength I Limit State

$$\text{Strength1} = 1.25 \cdot DC + 1.5 \cdot DW + 1.75 \cdot LL + 1.75BR + 0.50 \cdot (TU + CR + SH)$$

Beam Moment:



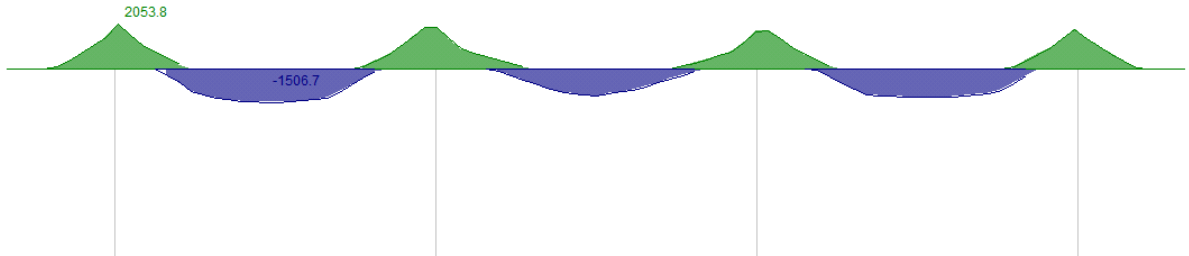
Beam Shear:



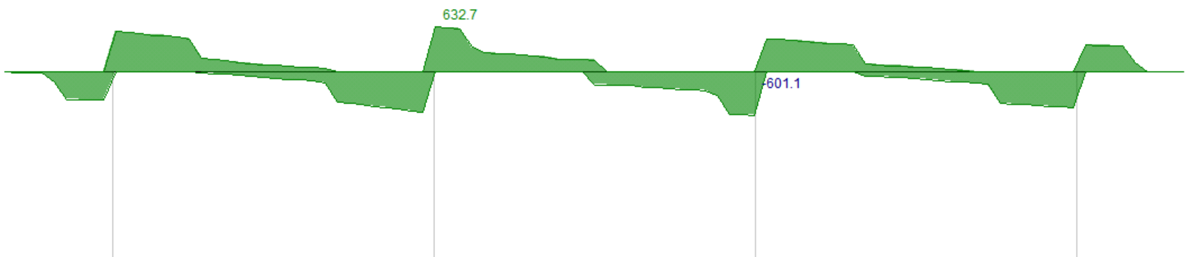
C2. Strength V Limit State

Strength5 $= 1.25 \cdot DC + 1.50 \cdot DW + 1.35 \cdot LL + 1.35 \cdot BR + 1.30 \cdot WS + 1.0 \cdot WL + 0.50 \cdot (TU + CR + SH)$

Beam Moment:



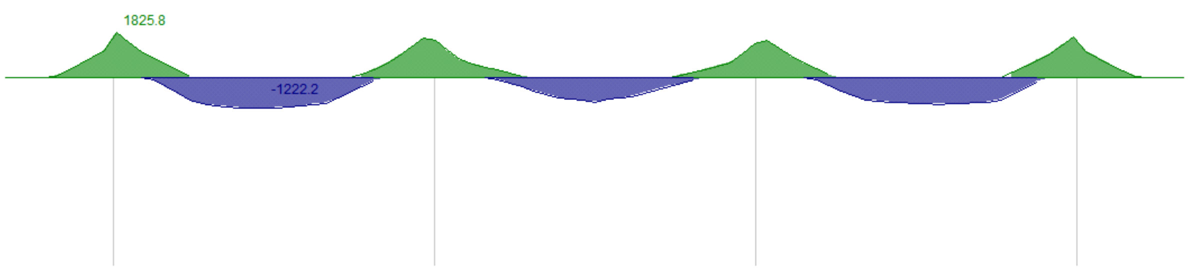
Beam Shear:



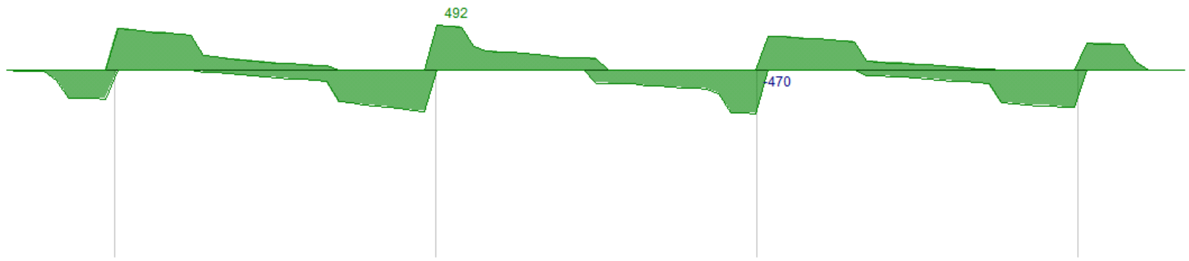
C3. Service I Limit State

Service1 $= 1.0 \cdot DC + 1.0 \cdot DW + 1.0 \cdot LL + 1.0 \cdot BR + 1.0WS + 1.0 \cdot WL + 1.0 \cdot CST$

Beam Moment:



Beam Shear:



C4. Summary of Results

		Strength I		Strength III		Strength V		Service I	
		Positive	Negative	Positive	Negative	Positive	Negative	Positive	Negative
Beam	Moment (kip-ft)	1727	-2210.2	-	-	1506.7	-2053.8	1222.2	-1825.8
	Shear (kip)	656.3	-702.8	-	-	601.1	-632.7	470	-492

Pier Cap Design

References

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\303PierCapLoads.xmcd(R)

Description

This section provides the criteria for the pier cap design.

Page Contents

A. Input Variables

Material Properties

Unit weight of concrete........................... $\gamma_{conc} = 150 \cdot pcf$

Modulus of elasticity for reinforcing steel.. $E_s = 29000 \cdot ksi$

Yield strength for reinforcing steel.......... $f_y = 60 \cdot ksi$

Design Parameters

Resistance factor for flexure and tension of reinforced concrete............................... $\phi = 0.9$

Resistance factor for shear and torsion (normal weight concrete)........................ $\phi_v := 0.90$

Design Lanes

Current lane configurations show two striped lanes per roadway with a traffic median barrier separating the roadways. Using the roadway clear width between barriers, $Rdwy_{width}$, the number of design traffic lanes per roadway, N_{lanes}, can be calculated as:

Roadway clear width............................. $Rdwy_{width} = 42\,ft$

Number of design traffic lanes per roadway... $N_{lanes} = 3$

Florida DOT Concrete & Environment [SDG 1.4]

Concrete cover for substructure not in contact with water................................ $cover_{sub} = 3 \cdot in$

Concrete cover for substructure in contact with water or earth.................... $cover_{sub.earth} = 4 \cdot in$

Minimum 28-day compressive strength for cast-in-place substructure...................... $f_{c.sub} = 5.5 \cdot ksi$

Modulus of elasticity for cast-in-place substructure... $E_{c.sub} = 3846 \cdot ksi$

Environmental classification for substructure... $Environment_{sub} =$ "Moderately"

Note: Epoxy coated reinforcing not allowed on FDOT projects.

Pier Geometry

Height of pier cap.................................. $h_{Cap} = 4.5\,ft$

Width of pier cap.................................. $b_{Cap} = 4.5\,ft$

Length of pier cap................................. $L_{Cap} = 88\,ft$

Length of pier column............................ $h_{Col} = 14\,ft$

Column diameter................................... $b_{Col} = 4\,ft$

Number of columns.............................. $n_{Col} = 4$

Surchage (column section in ground)....... $h_{Surcharge} = 2\,ft$

Design Loads - Moments, Shears and Torques

Moment (-M) - Service.......................... $M_{Service1.neg} = -1825.8 \cdot ft \cdot kip$

Moment (-M) - Strength.......................... $M_{Strength1.neg} = -2210.2 \cdot ft \cdot kip$

Corresponding Shear (-M) - Strength........ $V_{Strength1.neg} = -702.8 \cdot kip$ ****** See Note 1***

Corresponding Torsion (-M) - Strength..... $T_{Strength1.neg} := -55.2 \cdot ft \cdot kip$

Moment (+M) - Service......................... $M_{Service1.pos} = 1222.2 \cdot ft \cdot kip$

Moment (+M) - Strength........................ $M_{Strength1.pos} = 1727 \cdot ft \cdot kip$

Corresponding Shear (+M) - Strength....... $V_{Strength1.pos} = 656.3 \cdot kip$

Corresponding Torsion (+M) - Strength.... $T_{Strength1.pos} := -55.2ft \cdot kip$

***Note 1**:*
The design for shear on this section utilized the corresponding shear due to moment (-M). By inspection, the loading for maximum shear is similar to the shear produced by the loading for maximum moment (-M) in the cap.

In a design, the engineer will need to make sure that the applied live load maximizes the shear in the cap for design.

B. Positive Moment Design

A few recommendations on bar size and spacing are available to minimize problems during construction.

- Use the same size and spacing of reinforcing for both the negative and positive moment regions. This prevents field errors whereas the top steel is mistakenly placed at the bottom or vice versa.
- If this arrangement is not possible, give preference to maintaining the same spacing between the top and bottom reinforcement. Same grid pattern allows the concrete vibrator to be more effective in reaching the full depth of the cap.

The design procedure consists of calculating the reinforcement required to satisfy the design moment, then checking this reinforcement against criteria for crack control, minimum reinforcement, maximum reinforcement, shrinkage and temperature reinforcement, and distribution of reinforcement. The procedure is the same for both positive and negative moment regions.

$$M_u := M_{Strength1.pos} = 1727 \cdot kip \cdot ft$$

Factored resistance

$$M_r = \phi \cdot M_n$$

Nominal flexural resistance

$$M_n = A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2}\right) + A_s \cdot f_s \cdot \left(d_s - \frac{a}{2}\right) - A'_s \cdot f'_s \cdot \left(d'_s - \frac{a}{2}\right) + 0.85 \cdot f'_c \cdot (b - b_w) \cdot h_f \cdot \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

For a rectangular, non-prestressed section,

$$M_n = A_s \cdot f_s \cdot \left(d_s - \frac{a}{2}\right)$$

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b}$$

Assume $f_s := f_y$ [LRFD 5.7.2.1]

B1. Positive Moment Region Design - Flexural Resistance [LRFD 5.7.3.2]

Using variables defined in this example...... $M_r = \phi \cdot A_{s.pos} \cdot f_s \cdot \left[d_s - \frac{1}{2} \cdot \left(\frac{A_{s.pos} \cdot f_y}{0.85 \cdot f_{c.sub} \cdot b}\right)\right]$

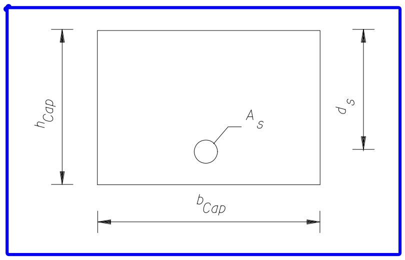
where $f_{c.sub} = 5.5 \cdot ksi$

$f_y = 60 \cdot ksi$

$\phi = 0.9$

$h_{Cap} = 54 \cdot in$

$b_{Cap} = 54 \cdot in$



Initial assumption for area of steel required

Number of bars................................ $n_{bar} := 9$

Size of bar...................................... $bar := "10"$

Note: if bar spacing is "-1", the spacing is less than 3", and a bigger bar size should be selected.

Bar area.. $A_{bar} = 1.270 \cdot in^2$

Bar diameter................................... $dia = 1.270 \cdot in$

Equivalent bar spacing....................... $bar_{spa} = 5.8 \cdot in$

Area of steel provided............................ $A_{s.pos} := n_{bar} \cdot A_{bar} = 11.43 \cdot in^2$

Distance from extreme compressive fiber to centroid of reinforcing steel (assuming a #6 stirrup).. $d_s := h_{Cap} - cover_{sub} - \dfrac{dia}{2} - \dfrac{3}{4} in = 49.6 \cdot in$

Stress block factor................................ $\beta_1 := \min\left[\max\left[0.85 - 0.05 \cdot \left(\dfrac{f_{c.sub} - 4ksi}{ksi}\right), 0.65\right], 0.85\right] = 0.78$

Distance between the neutral axis and compressive face.................................. $c := \dfrac{A_{s.pos} \cdot f_s}{0.85 \cdot f_{c.sub} \cdot \beta_1 \cdot b_{Cap}} = 3.51 \cdot in$

Solve the quadratic equation for the area of steel required.. Given $M_r = \phi \cdot A_s \cdot f_s \cdot \left[d_s - \dfrac{1}{2} \cdot \left(\dfrac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot b_{Cap}}\right)\right]$

Area of steel required............................. $A_{s.reqd} := Find(A_s) = 7.88 \cdot in^2$

The area of steel provided, $A_{s.pos} = 11.43 \cdot in^2$, should be greater than the area of steel required, $A_{s.reqd} = 7.88 \cdot in^2$. If not, decrease the spacing of the reinforcement. Once $A_{s.pos}$ is greater than $A_{s.reqd}$, the proposed reinforcing is adequate for the applied moments.

Moment capacity provided..................... $M_{r.pos} := \phi \cdot A_{s.pos} \cdot f_s \cdot \left[d_s - \dfrac{1}{2} \cdot \left(\dfrac{A_{s.pos} \cdot f_y}{0.85 \cdot f_{c.sub} \cdot b_{Cap}}\right)\right] = 2482.1 \cdot kip \cdot ft$

Check assumption that $f_s = f_y$................. $Check\_f_s := if\left(\dfrac{c}{d_s} < 0.6, "OK", "Not OK"\right)$

$Check\_f_s = "OK"$

B2. Limits for Reinforcement [LRFD 5.7.3.3]

Minimum Reinforcement

The minimum reinforcement requirements ensure the moment capacity provided is at least equal to the lesser of the cracking moment and 1.33 times the factored moment required by the applicable strength load combinations.

Modulus of Rupture.............................. $f_r := -0.24 \cdot \sqrt{f_{c.sub} \cdot ksi} = -562.8\ psi$ [SDG 1.4.1.B]

Section modulus of cap.......................... $S := \frac{b_{Cap} \cdot h_{Cap}^{2}}{6} = 26244 \cdot in^{3}$

Flexural cracking variability factor.......... $\gamma_1 := 1.6$

Ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement...................................... $\gamma_3 := 0.67$ (for ASTM A615, Grade 60 reinforcing steel, per SDG 1.4.1.C)

Cracking moment................................. $M_{cr} := \gamma_3 \cdot \gamma_1 \cdot f_r \cdot S = -1319.6 \cdot kip \cdot ft$

Required flexural resistance............ $M_{r.reqd} := \min\left(\left|M_{cr}\right|, 133 \cdot \% \cdot M_r\right) = 1319.6 \cdot kip \cdot ft$

Check that the capacity provided, $M_{r.pos} = 2482.1 \cdot ft \cdot kip$, exceeds minimum requirements, $M_{r.reqd} = 1319.6 \cdot ft \cdot kip$.

$$LRFD_{5.7.3.3.2} := \begin{cases} \text{"OK, minimum reinforcement for positive moment is satisfied"} & \text{if } M_{r.pos} \ge M_{r.reqd} \\ \text{"NG, reinforcement for positive moment is less than minimum"} & \text{otherwise} \end{cases}$$

$LRFD_{5.7.3.3.2}$ = "OK, minimum reinforcement for positive moment is satisfied"

B3. Crack Control by Distribution Reinforcement [LRFD 5.7.3.4]

Concrete is subjected to cracking. Limiting the width of expected cracks under service conditions increases the longevity of the structure. Potential cracks can be minimized through proper placement of the reinforcement. The check for crack control requires that the actual stress in the reinforcement should not exceed the service limit state stress (LRFD 5.7.3.4). The stress equations emphasize bar spacing rather than crack widths.

The maximum spacing of the mild steel reinforcement for control of cracking at the service limit state shall satisfy.........

$$s \le \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

where

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$

Exposure factor for Class 1 exposure condition.......................................

$$\gamma_e := 1.00$$ **[SDG 3.15.8]**

Overall thickness or depth of the component.............................

$$h_{Cap} = 4.5\,ft$$

Distance from extreme tension fiber to center of closest bar...............................

$$d_c := cover_{sub} + \frac{dia}{2} + \frac{3}{4}in = 4.38 \cdot in$$

$$\beta_s := 1 + \frac{d_c}{0.7(h_{Cap} - d_c)} = 1.13$$

The neutral axis of the section must be determined to determine the actual stress in the reinforcement. This process is iterative, so an initial assumption of the neutral axis must be made.

Guess value $x := 11.1 \cdot in$

Given $$\frac{1}{2} \cdot b_{Cap} \cdot x^2 = \frac{E_s}{E_{c.sub}} \cdot A_{s.pos} \cdot (d_s - x)$$

$$x_{na} := Find(x) = 11.09 \cdot in$$

Tensile force in the reinforcing steel due to service limit state moment......................

$$T_s := \frac{M_{Service1.pos}}{d_s - \frac{x_{na}}{3}} = 319.4 \cdot kip$$

Actual stress in the reinforcing steel due to service limit state moment.......................

$$f_{s.actual} := \frac{T_s}{A_{s.pos}} = 27.94 \cdot ksi$$

Required reinforcement spacing...........

$$s_{required} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{s.actual}} - 2 \cdot d_c = 13.47 \cdot in$$

Provided reinforcement spacing...........

$$bar_{spa} = 5.84 \cdot in$$

The required spacing of mild steel reinforcement in the layer closest to the tension face shall not be less than the reinforcement spacing provided due to the service limit state moment.

$$LRFD_{5.7.3.4} := \begin{array}{|l} \text{"OK, crack control for M is satisfied"} \quad \text{if } s_{required} \geq bar_{spa} \\ \text{"NG, crack control for M not satisfied, provide more reinforcement"} \quad \text{otherwise} \end{array}$$

$$LRFD_{5.7.3.4} = \text{"OK, crack control for M is satisfied"}$$

B4. Shrinkage and Temperature Reinforcement [LRFD 5.10.8]

Initial assumption for area of steel required

Size of bar...................................... $bar_{st} := \text{"6"}$

Spacing of bar................................ $bar_{spa.st} := 12 \cdot in$

Bar area.. $A_{bar} = 0.44 \cdot in^2$

Bar diameter................................... $dia = 0.750 \cdot in$

Minimum area of shrinkage and temperature reinforcement................

$$A_{shrink.temp} := \min\left[\begin{array}{c} \left(0.60 \dfrac{in^2}{ft}\right) \\ \max\left[\begin{array}{c} \left(0.11 \dfrac{in^2}{ft}\right) \\ \left[\dfrac{1.3 \cdot b_{Cap} \cdot h_{Cap} \cdot \dfrac{kip}{in \cdot ft}}{2\left(b_{Cap} + h_{Cap}\right) \cdot f_y}\right] \end{array}\right] \end{array}\right] = 0.29 \cdot \frac{in^2}{ft}$$

Maximum spacing of shrinkage and temperature reinforcement

$$spacing_{shrink.temp} := \begin{cases} \min\left(\dfrac{A_{bar}}{A_{shrink.temp}}, 12 \cdot in\right) & \text{if } \left(b_{Cap} > 36in\right) \cdot \left(h_{Cap} > 36in\right) \\ \min\left(\dfrac{A_{bar}}{A_{shrink.temp}}, 3 \cdot b_{Cap}, 3 \cdot h_{Cap}, 18in\right) & \text{otherwise} \end{cases} = 12 \cdot in$$

The bar spacing should be less than the maximum spacing for shrinkage and temperature reinforcement

$$LRFD_{5.7.10.8} := \begin{cases} \text{"OK, minimum shrinkage and temperature requirements"} & \text{if } bar_{spa.st} \le spacing_{shrink.temp} \\ \text{"NG, minimum shrinkage and temperature requirements"} & \text{otherwise} \end{cases}$$

$LRFD_{5.7.10.8} = \text{"OK, minimum shrinkage and temperature requirements"}$

B5. Mass Concrete Provisions [SDG 3.9]

Surface area of pier cap.......................... $Surface_{cap} := 2 \cdot b_{Cap} \cdot h_{Cap} + \left(2b_{Cap} + 2h_{Cap}\right) \cdot L_{Cap} = 1624.5\,ft^2$

Volume of pier cap................................ $Volume_{cap} := b_{Cap} \cdot h_{Cap} \cdot L_{Cap} = 1782 \cdot ft^3$

Mass concrete provisions apply if the volume to surface area ratio, $\dfrac{Volume_{cap}}{Surface_{cap}} = 1.1\,ft$, exceeds 1 ft and the minimum dimension exceeds 3 feet

$$SDG_{3.9} := \begin{array}{|l} \text{"Use mass concrete provisions"} \quad \text{if } \dfrac{Volume_{cap}}{Surface_{cap}} > 1.0 \cdot ft \wedge b_{Cap} > 3ft \wedge h_{Cap} > 3ft \\ \text{"Use regular concrete provisions"} \quad \text{otherwise} \end{array}$$

$SDG_{3.9}$ = "Use mass concrete provisions"

C. Negative Moment Design

$M_r := \left|M_{Strength1.neg}\right|$

$M_r = 2210.2 \cdot ft \cdot kip$

Factored resistance

$M_r = \phi \cdot M_n$

Nominal flexural resistance

$$M_n = A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2}\right) + A_s \cdot f_s \cdot \left(d_s - \frac{a}{2}\right) - A'_s \cdot f'_s \cdot \left(d'_s - \frac{a}{2}\right) + 0.85 \cdot f'_c \cdot \left(b - b_w\right) \cdot \beta_1 \cdot h_f \cdot \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

For a rectangular, non-prestressed section,

$$M_n = A_s \cdot f_s \cdot \left(d_s - \frac{a}{2}\right)$$

$$a = \frac{A_s \cdot f_y}{0.85 \cdot f'_c \cdot b}$$

Assume $f_s := f_y$ [LRFD 5.7.2.1]

C1. Negative Moment Region Design - Flexural Resistance [LRFD 5.7.3.2]

Using variables defined in this example,

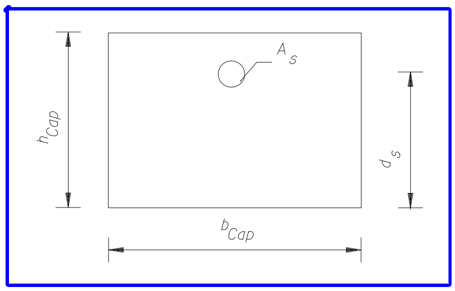
where $f_{c.sub} = 5.5 \cdot ksi$

$f_y = 60 \cdot ksi$

$\phi = 0.9$

$h_{Cap} = 54 \cdot in$

$b_{Cap} = 54 \cdot in$



Initial assumption for area of steel required

Number of bars................................ $n_{bar} := 9$

Size of bar...................................... bar := "10"

Note: if bar spacing is "-1", the spacing is less than 3", and a bigger bar size should be selected.

Bar area.. $A_{bar} = 1.270 \cdot in^2$

Bar diameter.................................... $dia = 1.270 \cdot in$

Equivalent bar spacing...................... $bar_{spa} = 5.8 \cdot in$

Area of steel provided............................ $A_{s.neg} := n_{bar} \cdot A_{bar} = 11.43 \cdot in^2$

Distance from extreme compressive fiber to centroid of reinforcing steel (assuming a #6 stirrup)... $d_s := h_{Cap} - cover_{sub} - \frac{dia}{2} - \frac{3}{4} in = 49.62 \cdot in$

Stress block factor................................ $\beta_1 := \min\left[\max\left[0.85 - 0.05 \cdot \left(\frac{f_{c.sub} - 4ksi}{ksi}\right), 0.65\right], 0.85\right] = 0.78$

Distance between the neutral axis and compressive face.................................. $c := \frac{A_{s.neg} \cdot f_s}{0.85 \cdot f_{c.sub} \cdot \beta_1 \cdot b_{Cap}} = 3.51 \cdot in$

Depth of equivalent stress block.............. $a := \beta_1 \cdot c = 2.72 \cdot in$

Solve the quadratic equation for the area of steel required....................................... Given $M_r = \phi \cdot A_s \cdot f_s \cdot \left[d_s - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot b_{Cap}}\right)\right]$

Area of steel required............................. $A_{s.reqd} := \mathrm{Find}(A_s) = 10.15 \cdot in^2$

The area of steel provided, $A_{s.neg} = 11.43 \cdot in^2$, should be greater than the area of steel required, $A_{s.reqd} = 10.15 \cdot in^2$. If not, decrease the spacing of the reinforcement. Once A_s is greater than $A_{s.reqd}$, the proposed reinforcing is adequate for the applied moments.

Moment capacity provided.................... $M_{r.neg} := \phi \cdot A_{s.neg} \cdot f_s \cdot \left(d_s - \frac{a}{2}\right) = 2482.1 \cdot kip \cdot ft$

Check assumption that $f_s = f_y$................ $Check\_f_s := \mathrm{if}\left(\frac{c}{d_s} < 0.6, \text{"OK"}, \text{"Not OK"}\right)$

$Check\_f_s = \text{"OK"}$

C2. Limits for Reinforcement [LRFD 5.7.3.3]

Minimum Reinforcement

The minimum reinforcement requirements ensure the moment capacity provided is at least equal to the lesser of the cracking moment and 1.33 times the factored moment required by the applicable strength load combinations.

Modulus of Rupture................................ $f_r := -0.24 \cdot \sqrt{f_{c.sub} \cdot ksi} = -562.85\, psi$ [SDG 1.4.1.B]

Section modulus of cap........................... $S := \dfrac{b_{Cap} \cdot h_{Cap}^{\;2}}{6} = 15.19 \cdot ft^3$

Flexural cracking variability factor........... $\gamma_1 := 1.6$

Ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement....................................... $\gamma_3 := 0.67$ (for ASTM A615, Grade 60 reinforcing steel, per SDG 1.4.1.C)

Cracking moment.................................. $M_{cr} := \gamma_1 \cdot \gamma_3 \cdot f_r \cdot S = -1319.6 \cdot kip \cdot ft$

Required flexural resistance............. $M_{r.reqd} := \min\left(\left|M_{cr}\right|, 133 \cdot \% \cdot M_r\right) = 1.32 \times 10^3 \cdot kip \cdot ft$

Check that the capacity provided, $M_{r.neg} = 2482.1 \cdot ft \cdot kip$, exceeds minimum requirements, $M_{r.reqd} = 1319.6 \cdot ft \cdot kip$.

$$LRFD_{5.7.3.3.2} := \begin{array}{|l} \text{"OK, minimum reinforcement for negative moment is satisfied"} \quad \text{if } M_{r.neg} \ge M_{r.reqd} \\ \text{"NG, reinforcement for negative moment is less than minimum"} \quad \text{otherwise} \end{array}$$

$LRFD_{5.7.3.3.2}$ = "OK, minimum reinforcement for negative moment is satisfied"

C3. Crack Control by Distribution Reinforcement [LRFD 5.7.3.4]

Concrete is subjected to cracking. Limiting the width of expected cracks under service conditions increases the longevity of the structure. Potential cracks can be minimized through proper placement of the reinforcement. The check for crack control requires that the actual stress in the reinforcement should not exceed the service limit state stress (LRFD 5.7.3.4). The stress equations emphasize bar spacing rather than crack widths.

The maximum spacing of the mild steel reinforcement for control of cracking at the service limit state shall satisfy.........

$$s \le \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

where

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$

Exposure factor for Class 1 exposure condition.......................................

$$\gamma_e := 1.00$$ **[SDG 3.15.8]**

Overall thickness or depth of the component.............................

$$h_{Cap} = 54 \cdot in$$

Distance from extreme tension fiber to center of closest bar..............................

$$d_c := cover_{sub} + \frac{dia}{2} + \frac{3}{4} in = 4.38 \cdot in$$

$$\beta_s := 1 + \frac{d_c}{0.7(h_{Cap} - d_c)} = 1.13$$

The neutral axis of the section must be determined to determine the actual stress in the reinforcement. This process is iterative, so an initial assumption of the neutral axis must be made.

Guess value $x := 11.1 \cdot in$

Given $$\frac{1}{2} \cdot b_{Cap} \cdot x^2 = \frac{E_s}{E_{c.sub}} \cdot A_{s.neg} \cdot (d_s - x)$$

$$x_{na} := Find(x) = 11.09 \cdot in$$

Tensile force in the reinforcing steel due to ser limit state moment......................

$$T_s := \frac{|M_{Service1.neg}|}{d_s - \frac{x_{na}}{3}} = 477.14 \cdot kip$$

Actual stress in the reinforcing steel due to service limit state moment.......................

$$f_{s.actual} := \frac{T_s}{A_{s.neg}} = 41.74 \cdot ksi$$

Required reinforcement spacing...........

$$s_{required} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{s.actual}} - 2 \cdot d_c = 6.12 \cdot in$$

Provided reinforcement spacing...........

$$bar_{spa} = 5.84 \cdot in$$

The required spacing of mild steel reinforcement in the layer closest to the tension face shall not be less than the reinforcement spacing provided due to the service limit state moment.

$$LRFD_{5.7.3.4} := \begin{array}{|l} \text{"OK, crack control for M is satisfied"} \quad \text{if } s_{required} \geq bar_{spa} \\ \text{"NG, crack control for M not satisfied, provide more reinforcement"} \quad \text{otherwise} \end{array}$$

$$LRFD_{5.7.3.4} = \text{"OK, crack control for M is satisfied"}$$

D. Shear and Torsion Design [LRFD 5.8]

D1. Check if Torsion Design is Required

$T_u := |T_{Strength1.neg}|$ $\qquad$ $V_u := |V_{Strength1.neg}|$

For normal weight concrete, torsional effects shall be investigated if................. $T_u > 0.25 \cdot \phi_v \cdot T_{cr}$

and.. $T_{cr} = 0.125 \cdot \sqrt{f_c} \cdot \frac{A_{cp}^2}{p_c} \cdot \sqrt{1 + \frac{f_{pc}}{0.125 \cdot \sqrt{f_c}}}$

Total area enclosed by outside perimeter of concrete cross section........................... $A_{cp} := h_{Cap} \cdot b_{Cap} = 20.25\ ft^2$

Length of outside perimeter of cross section.. $p_c := 2 \cdot (h_{Cap} + b_{Cap}) = 18\ ft$

Compressive stress in concrete after prestress losses have occured................. $f_{pc} := 0 \cdot psi$

Torsional cracking moment.................... $T_{cr} := 0.125 \cdot \sqrt{f_{c.sub} \cdot ksi} \cdot \frac{A_{cp}^2}{p_c} \cdot \sqrt{1 + \frac{f_{pc}}{0.125 \cdot \sqrt{f_{c.sub} \cdot ksi}}} = 961.7 \cdot kip \cdot ft$

$LRFD_{5.8.2} :=$ | "OK, torsion can be neglected" if $0.25 \cdot \phi_v \cdot T_{cr} \ge T_u$
"NG, torsion shall be investigated..." otherwise

$LRFD_{5.8.2} =$ "OK, torsion can be neglected"

D2. Determine Nominal Shear Resistance

Effective width of the section................. $b_v := b_{Cap} = 54 \cdot in$

Effective shear depth............................. $a := \frac{A_{s.pos} \cdot f_y}{0.85 \cdot f_{c.sub} \cdot b_{Cap}} = 2.72 \cdot in$

$d_v := \max\left(d_s - \frac{a}{2}, 0.9 \cdot d_s, 0.72 \cdot h_{Cap}\right) = 48.26 \cdot in$

Determination of β and θ **(LRFD 5.8.3.4)**

The pier cap is a non-prestressed concrete section not subjected to axial tension. Assuming it has at least the amount of transverse reinforcement specified in **LRFD 5.8.2.5** or an overall depth of less than 16 in, the Simplified Procedure for determining Shear Resistance per LRFD 5.8.3.4.1 may be used.

$$\beta := 2$$

$$\theta := 45 \cdot deg$$

Nominal shear resistance of concrete section... $V_c := 0.0316 \cdot \beta \cdot \sqrt{f_{c.sub} \cdot ksi} \cdot b_v \cdot d_v = 386.2 \cdot kip$

D3. Transverse Reinforcement

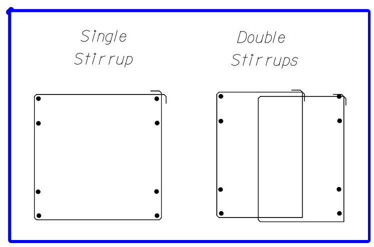
Because the overall depth is greater than 16 in, transverse reinforcement shall be provided in the pier cap according to **LRFD 5.8.2.5.**

$$A_v \geq .0316 \cdot \sqrt{f'_c} \cdot \frac{b_v \cdot s}{f_y}$$

The pier cap has no prestressing.

$$V_p := 0 \cdot kip$$

Stirrups



Size of stirrup bar ("4" "5" "6" "7")... bar := "6"

Number of stirrup bars ("single" "double")............................. n_{bar} := "double"

Area of shear reinforcement.................... $A_v = 1.760 \cdot in^2$

Diameter of shear reinforcement............. $dia = 0.750 \cdot in$

Nominal shear strength provided by shear reinforcement

$$V_n = V_c + V_p + V_s$$

where.. $V_n := \min\left(\frac{V_u}{\phi_v}, 0.25 \cdot f_{c.sub} \cdot b_v \cdot d_v + V_p\right) = 780.89 \cdot kip$

and... $V_s := V_n - V_c - V_p = 394.66 \cdot kip$

Spacing of stirrups

Minimum transverse reinforcement.......... $s_{min} := \dfrac{A_v \cdot f_y}{0.0316 \cdot b_v \cdot \sqrt{f_{c.sub} \cdot ksi}} = 26.39 \cdot in$

Transverse reinforcement required........... $s_{req} := \text{if}\left(V_s \le 0, s_{min}, \dfrac{A_v \cdot f_y \cdot d_v \cdot \cot(\theta)}{V_s}\right) = 12.91 \cdot in$

Minimum transverse reinforcement required... $s := \min(s_{min}, s_{req}) = 12.91 \cdot in$

Maximum transverse reinforcement spacing

$$s_{max} := \text{if}\left[\frac{V_u - \phi_v \cdot V_p}{\phi_v \cdot (b_v \cdot d_v)} < 0.125 \cdot f_{c.sub}, \min\left(\begin{pmatrix} 0.8 \cdot d_v \\ 24 \cdot in \end{pmatrix}\right), \min\left(\begin{pmatrix} 0.4 \cdot d_v \\ 12 \cdot in \end{pmatrix}\right)\right] = 24 \cdot in$$

Spacing of transverse reinforcement cannot exceed the following spacing........ $spacing := \text{floor}\left(\dfrac{\text{if}(s_{max} > s, s, s_{max})}{in}\right) \cdot in = 12 \cdot in$

LRFD 5.8.3.5 is applicable at the end bearing support areas. Therefore, this check is ignored.

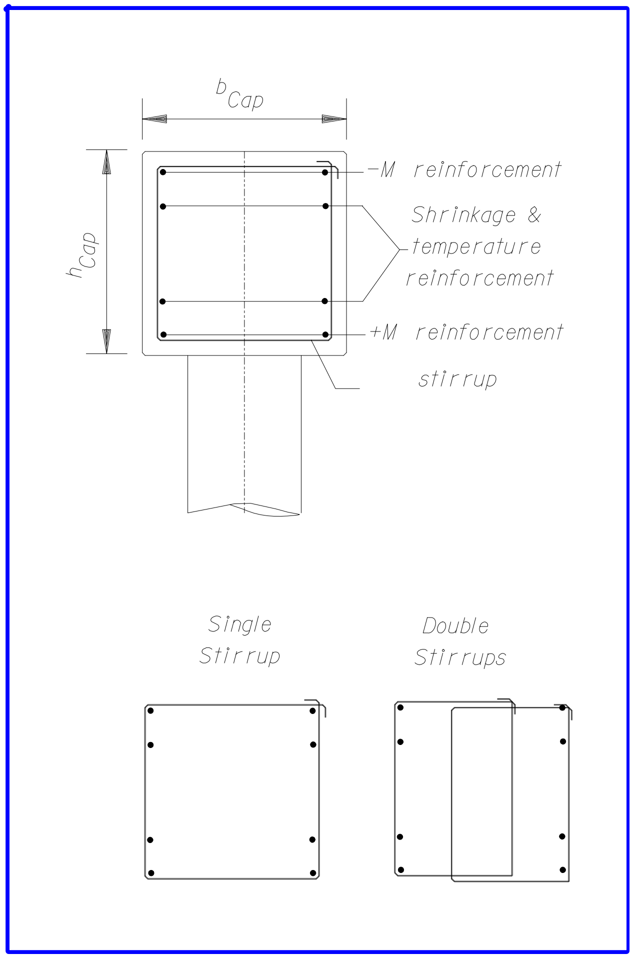
E. Summary of Reinforcement Provided

Negative moment (top) reinforcement

Bar size............ $bar_{negM} = "10"$

Number of bars.. $n_{bar.negM} = 9$

Bar spacing....... $bar_{spa.negM} = 5.8 \cdot in$



Positive moment (bottom) reinforcement

Bar size............ $bar_{posM} = "10"$

Number of bars.. $n_{bar.posM} = 9$

Bar spacing........ $bar_{spa.posM} = 5.8 \cdot in$

Transverse reinforcement

Bar size............ $bar = "6"$

Bar spacing........ $spacing = 12.0 \cdot in$

Type of stirrups. $n_{bar} = "double"$

Temperature and Shrinkage

Bar size............ $bar_{shrink.temp} = "6"$

Bar spacing....... $bar_{spa.st} = 12 \cdot in$

Pier Column Design Loads

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\304PierCapDesign.xmcd(R)

Description

This section provides the design parameters necessary for the substructure pier column design. Loads are given in Section 3.03: Pier Cap Design Loads. This section provides the results from analysis using RISA.

Page Contents

LRFD Criteria

STRENGTH I - Basic load combination relating to the normal vehicular use of the bridge without wind.

$WA = 0$ Water load and stream pressure are not applicable.

$FR = 0$ No friction forces.

TU Uniform temperature load effects on the pier will be generated by the substructure analysis model.

$$Strength1 = 1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL + 1.75 \cdot BR + 0.50 \cdot (TU + CR + SH)$$

STRENGTH II - Load combination relating to the use of the bridge by Owner-specified special design vehicles, evaluation permit vehicles, or both without wind.

"Permit vehicles are not evaluated in this design example"

STRENGTH III - Load combination relating to the bridge exposed to wind velocity exceeding 55 MPH.

$$Strength3 = 1.25 \cdot DC + 1.50 \cdot DW + 1.40 \cdot WS + 0.50 \cdot (TU + CR + SH)$$

STRENGTH IV - Load combination relating to very high dead load to live load force effect ratios.

"Not applicable for the substructure design in this design example"

STRENGTH V - Load combination relating to normal vehicular use of the bridge with wind of 55 MPH velocity.

$$Strength5 = 1.25 \cdot DC + 1.50 \cdot DW + 1.35 \cdot LL + 1.35 \cdot BR + 1.30 \cdot WS + 1.0 \cdot WL \ldots + 0.50 \cdot (TU + CR + SH)$$

EXTREME EVENT I - Load combination including earthquake.

"Not applicable for this simple span prestressed beam bridge design example"

EXTREME EVENT II - Load combination relating to ice load, collision by vessels and vehicles, and certain hydraulic events.

"Not applicable for the substructure design in this design example"

SERVICE I - Load combination relating to the normal operational use of the bridge with a 55 MPH wind and all loads taken at their nominal values.

"Not applicable for the pier column design in this design example"

SERVICE II - Load combination intended to control yielding of steel structures and slip of slip-critical connections due to vehicular live load.

"Not applicable for this simple span prestressed beam bridge design example"

SERVICE III - Load combination for longitudinal analysis relating to tension in prestressed concrete superstructures with the objective of crack control.

"Not applicable for the substructure design in this design example"

SERVICE IV - Load combination relating only to tension in prestressed concrete columns with the objective of crack control.

"Not applicable for the substructure design in this design example"

FATIGUE I - Fatigue and fracture load combination related to infinite load-induced fatigue life.

"Not applicable for the substructure design in this design example"

FATIGUE II - Fatigue and fracture load combination relating to finite load-induced fatigue life.

"Not applicable for the substructure design in this design example"

A. General Criteria

The following is a summary of all the loads previously calculated:

A1. Load Summary

- **Dead Loads -** Unfactored beam reactions at the pier for DC and DW loads

UNFACTORED BEAM REACTIONS AT PIER						
	DC Loads (kip)			DW Loads (kip)		
Beam	x	y	z	x	y	z
1	0.0	-190.0	0.0	0.0	0.0	0.0
2	0.0	-199.8	0.0	0.0	0.0	0.0
3	0.0	-199.8	0.0	0.0	0.0	0.0
4	0.0	-199.8	0.0	0.0	0.0	0.0
5	0.0	-199.8	0.0	0.0	0.0	0.0
6	0.0	-199.8	0.0	0.0	0.0	0.0
7	0.0	-199.8	0.0	0.0	0.0	0.0
8	0.0	-199.8	0.0	0.0	0.0	0.0
9	0.0	-190.0	0.0	0.0	0.0	0.0

- **Live load -**

  Live load analysis is done with RISA.

  HL-93 Line Load............................. $HL93 + P_c = 85 \cdot kip$

  $HL93 - P_c = 72 \cdot kip$

- **Braking Force -** Unfactored beam reactions at the pier for BR loads

BRAKING FORCES AT PIER			
	BR Loads (kip)		
Beam	x	y	z
1	1.6	-0.5	-4.3
2	1.6	-0.5	-4.3
3	1.6	-0.5	-4.3
4	1.6	-0.5	-4.3
5	1.6	-0.5	-4.3
6	0.0	0.0	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0

Note*: The direction of braking was reversed in order to maximize the longitudinal braking moments, Mx caused by ž'łloads, to maximize the effects of WS and WL.*

- **Creep, Shrinkage and Temperature -** Unfactored beam reactions at the pier for CU, SH and TU loads

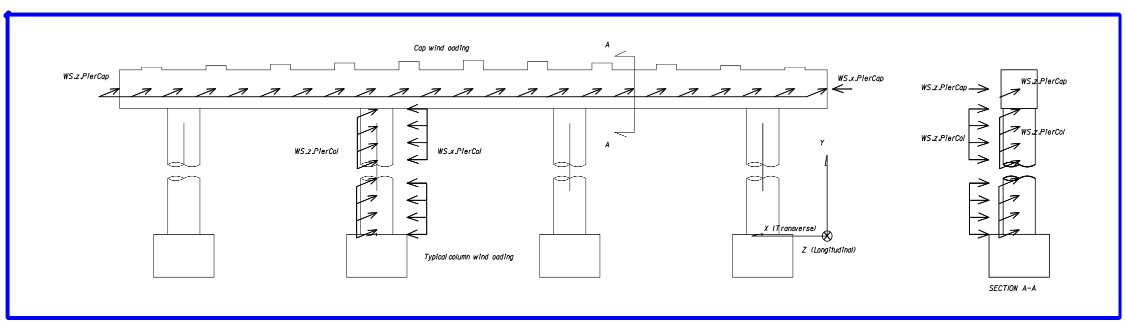
CREEP, SHRINKAGE, TEMPERATURE FORCES AT PIER	CR, SH, TU Loads (kip)		
Beam	x	y	z
1	0.0	0.0	0.0
2	0.0	0.0	0.0
3	0.0	0.0	0.0
4	0.0	0.0	0.0
5	0.0	0.0	0.0
6	0.0	0.0	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0

- **Wind on structure -** Unfactored beam reactions for WS loads

WIND ON STRUCTURE FORCES AT PIER Strength III, Service IV	WS Loads (kip)		
Beam	x	y	z
1	-2.9	0.0	-1.5
2	-2.9	0.0	-1.5
3	-2.9	0.0	-1.5
4	-2.9	0.0	-1.5
5	-2.9	0.0	-1.5
6	-2.9	0.0	-1.5
7	-2.9	0.0	-1.5
8	-2.9	0.0	-1.5
9	-2.9	0.0	-1.5

WIND ON STRUCTURE FORCES AT PIER Strength V, Service I	WS Loads (kip)		
Beam	x	y	z
1	-0.6	0.0	-0.3
2	-0.6	0.0	-0.3
3	-0.6	0.0	-0.3
4	-0.6	0.0	-0.3
5	-0.6	0.0	-0.3
6	-0.6	0.0	-0.3
7	-0.6	0.0	-0.3
8	-0.6	0.0	-0.3
9	-0.6	0.0	-0.3

***Note**: The direction of wind was reversed in order to maximize the -Mz moment about pier column 2*



$WS_{z.PierCap.StrIIIServIV} = 0.209 \cdot klf$

$WS_{x.PierCap.StrIIIServIV} = -2.16 \cdot kip$

$WS_{z.PierCol.StrIIIServIV} = 0.186 \cdot klf$

$WS_{x.PierCol.StrIIIServIV} = 0.216 \cdot klf$

$WS_{z.PierCap.StrVServI} = 0.046 \cdot klf$

$WS_{x.PierCap.StrVServI} = -0.47 \cdot kip$

$WS_{z.PierCol.StrVServI} = 0.041 \cdot klf$

$WS_{x.PierCol.StrVServI} = 0.047 \cdot klf$

- **Wind on load on vehicles -** Unfactored beam reactions for WL loads

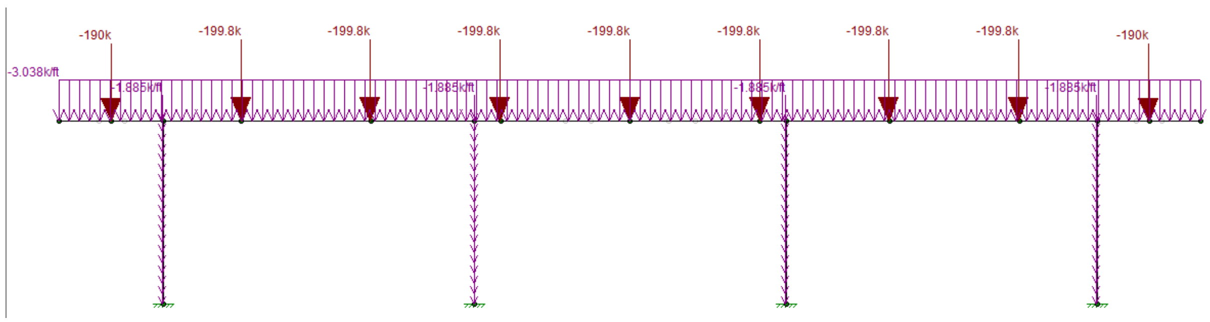
WIND ON LIVE LOAD FORCES AT PIER			
	WL Loads (kip)		
Beam	x	y	z
1	-1.1	9.4	-0.7
2	-1.1	-9.4	-0.7
3	-1.1	9.4	-0.7
4	-1.1	-9.4	-0.7
5	-1.1	9.4	-0.7
6	-1.1	-9.4	-0.7
7	-1.1	9.4	-0.7
8	-1.1	-9.4	-0.7
9	-1.1	0.0	-0.7

Note: *The direction of wind was reversed in order to maximize the -Mz moment about pier column 2*

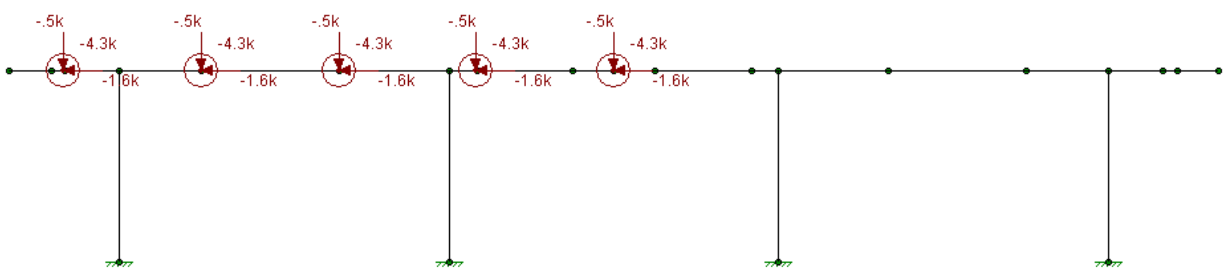
B. Design Limit States

The design loads for strength I, strength III, and strength V limit states are summarized in this section. For each limit state, three loading conditions are presented: maximum axial force and maximum moment in the y- and z-directions.

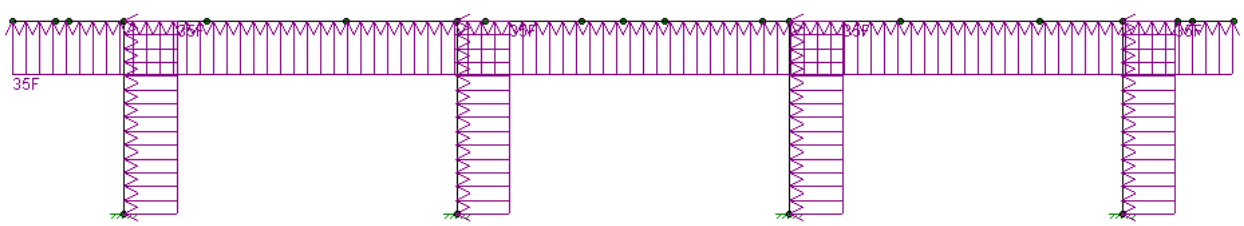
DC Loads:



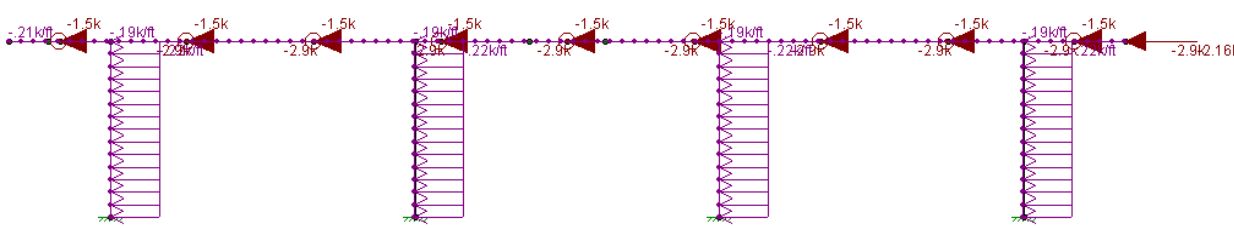
BR Loads:



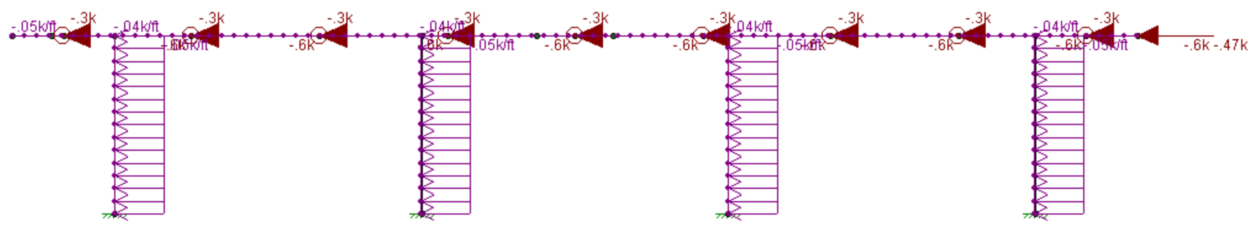
CR/SH/TU Loads:



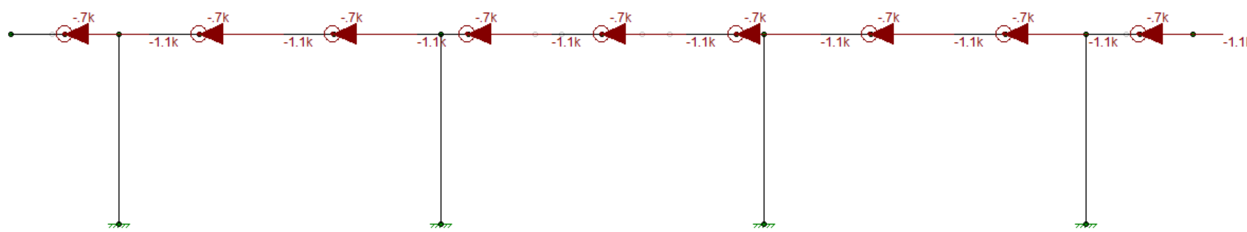
WS Loads: Strength III, Service IV



WS Loads: Strength V, Service I



WL Loads:

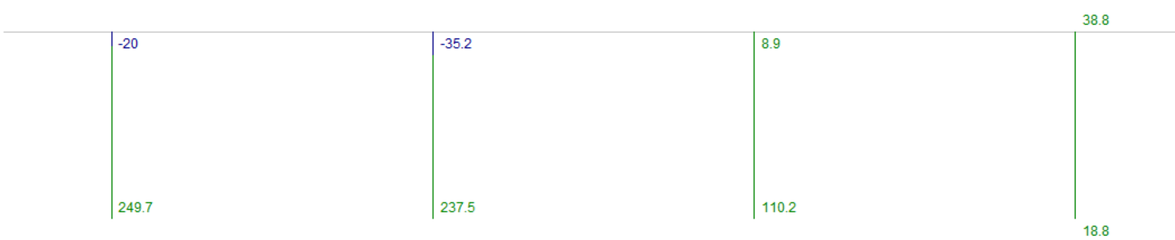


See Section 3.03 for Moving Load and Load Combination definitions.

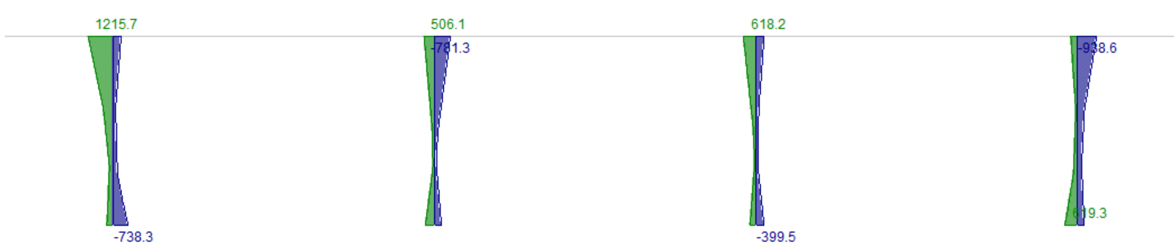
B1. Strength I Limit State

Strength1 **=** 1.25·DC + 1.5·DW + 1.75·LL + 1.75BR + 0.50·(TU + CR + SH)

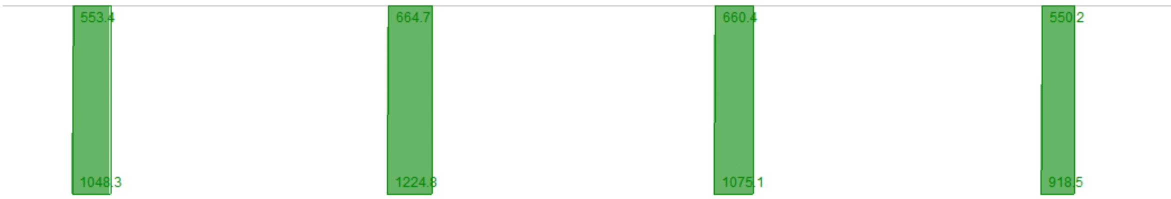
Column Y-Moment:



Column Z-Moment:



Column Axial:

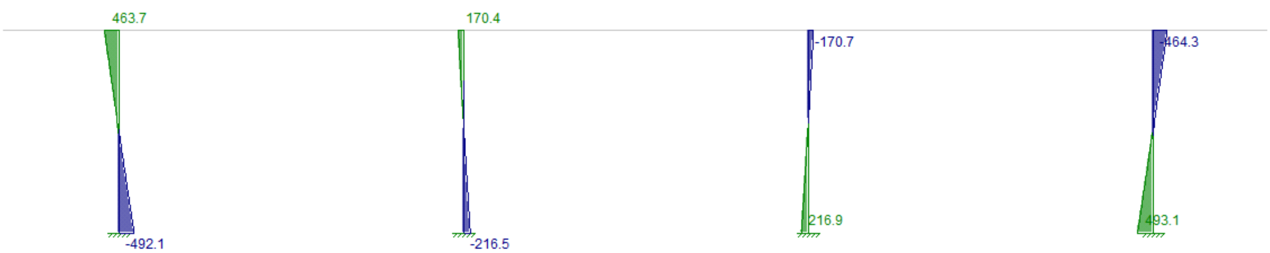


B2. Strength III Limit State

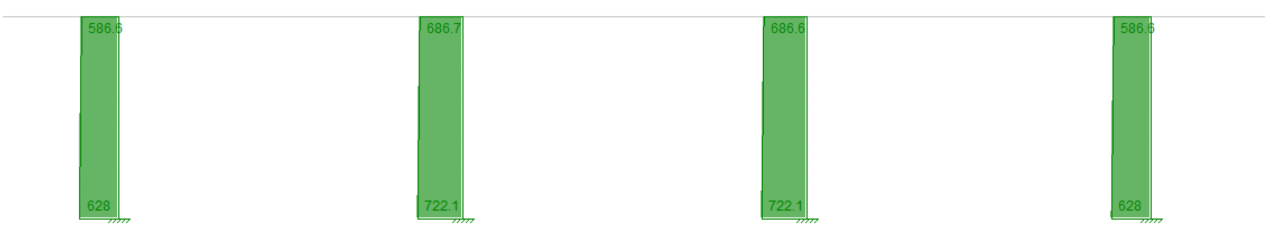
Strength3 **=** 1.25·DC + 1.5·DW + 1.4WS + 0.50·(TU + CR + SH)

Column Y-Moment:

Column Z-Moment:



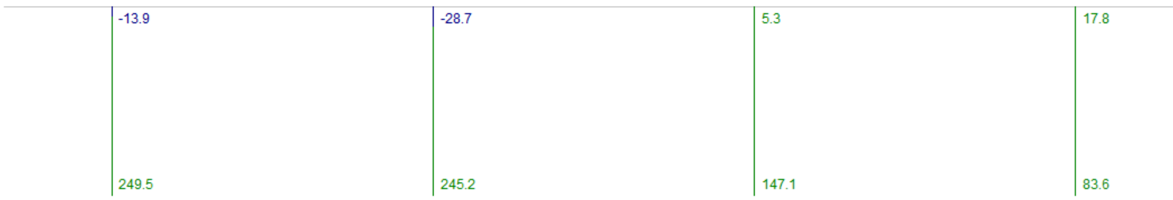
Column Axial:



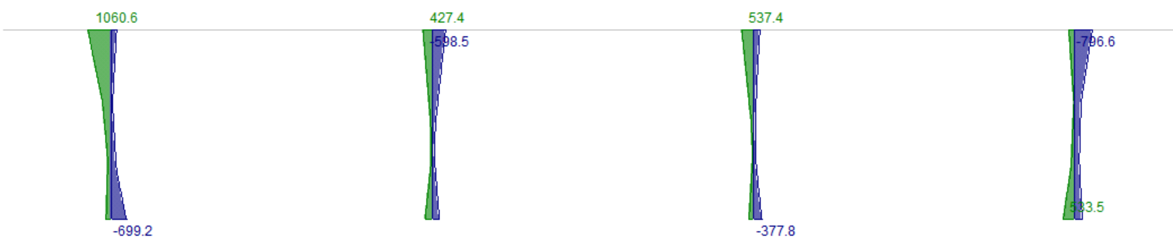
B3. Strength V Limit State

Strength5 = 1.25·DC + 1.50·DW + 1.35·LL + 1.35·BR + 1.3·WS + 1.0·WL + 0.50·(TU + CR + SH)

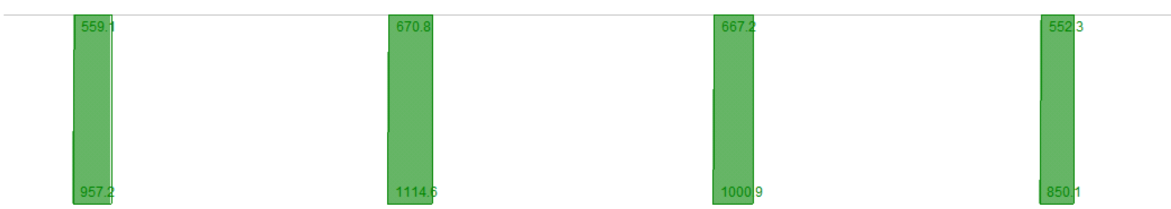
Column Y-Moment:



Column Z-Moment:



Column Axial:



B4. Summary of Results

RISA RESULTS

Result Case	My	Mz	Axial
Column 1	**kip-ft**	**kip-ft**	**kip**
Strength I - Top	20.0	1215.7	553.4
Strength I - Bottom	249.7	738.3	1048.3
Strength III - Top	4.7	463.7	586.6
Strength III - Bottom	174.8	492.1	628.0
Strength V - Top	13.9	1060.6	559.1
Strength V - Bottom	249.5	699.2	957.2
Column 2			
Strength I - Top	35.2	781.3	664.7
Strength I - Bottom	237.5	445.5	1224.8
Strength III - Top	4.7	170.4	686.7
Strength III - Bottom	190.7	216.5	722.1
Strength V - Top	28.7	598.5	670.8
Strength V - Bottom	245.2	333.7	1114.6

Pier Column Design

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\305PierColLoads.xmcd(R)

Description

This document provides the design check summary for columns 1 and 2. P−Δ or any secondary effects were not evaluated. (**Note**: Most higher-end analysis programs, such as Larsa 2000 have the capability to analyze for secondary effects on columns such that the resulting moments are already magnified by P−Δ. If not, programs like PCA Column have a "Slender" column option whereas some parameters for slenderness can be entered to include secondary effects.) The column analysis was done using the FDOT Biaxial Column Program V2.3.

Page Contents

A. General Criteria

A1. Pier Column Design Loads

Strength I, strength III, and strength V loads for columns 1 (exterior) and 2 (interior) were evaluated. The following table summarizes the results from RISA output for pier columns 1 and 2.

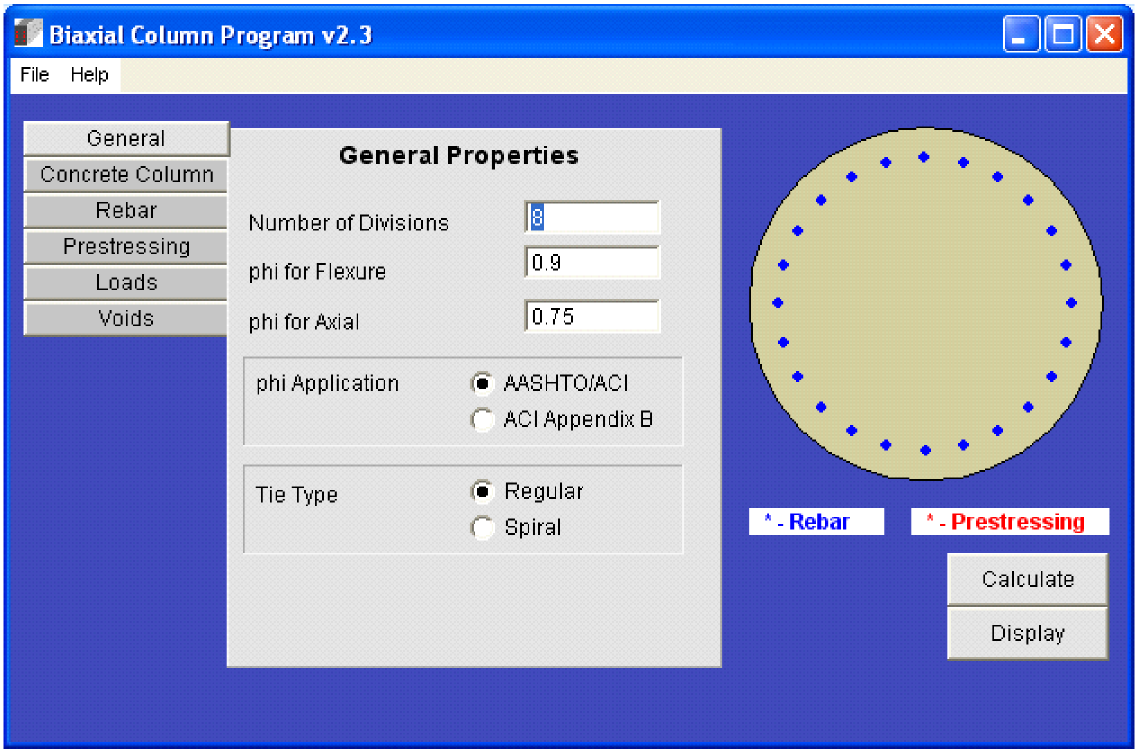
RISA COLUMN RESULTS

Result Case	Axial (kip)	My (kip-ft)	Mz (kip-ft)
Column 1			
Strength 1 - Top	553.4	20.0	1215.7
Strength 1 - Bottom	1048.3	249.7	738.3
Strength 3 - Top	586.6	4.7	463.7
Strength 3 - Bottom	628.0	174.8	492.1
Strength 5 - Top	559.1	13.9	1060.6
Strength 5 - Bottom	957.2	249.5	699.2
Column 2			
Strength 1 - Top	664.7	35.2	781.3
Strength 1 - Bottom	1224.8	237.5	445.5
Strength 3 - Top	686.7	4.7	170.4
Strength 3 - Bottom	722.1	190.7	216.5
Strength 5 - Top	670.8	28.7	598.5
Strength 5 - Bottom	1114.6	245.2	333.7

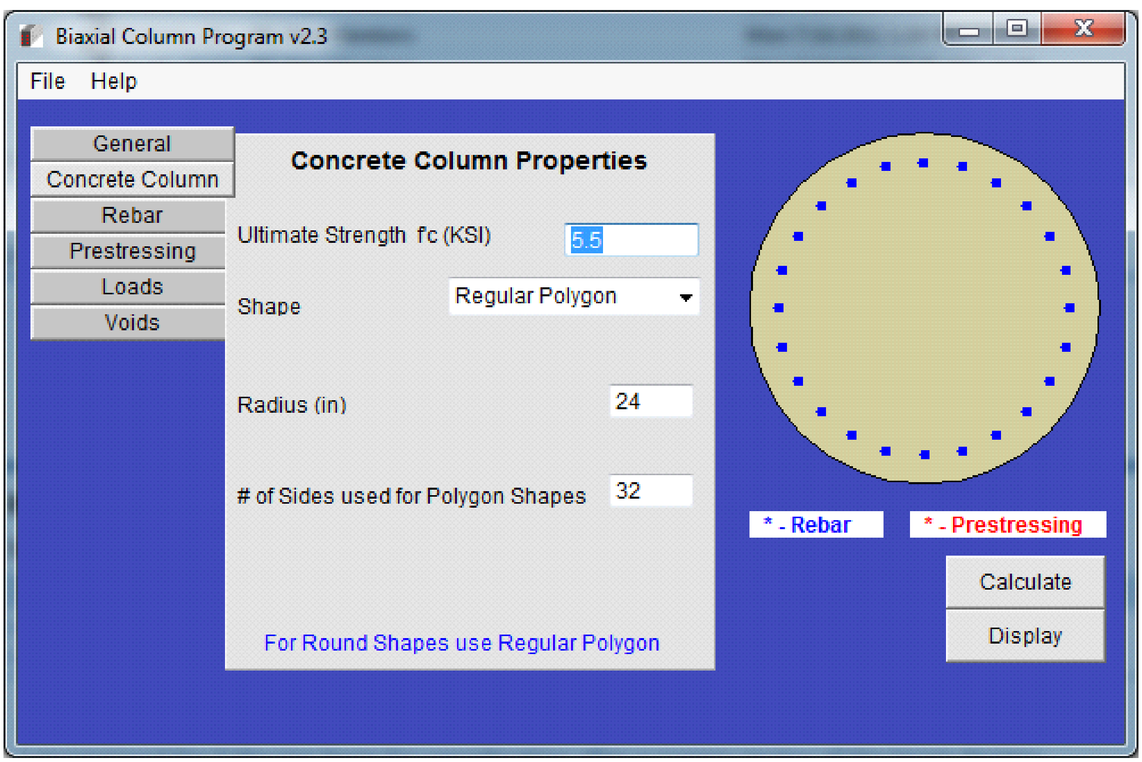
B. Biaxial Column Analysis

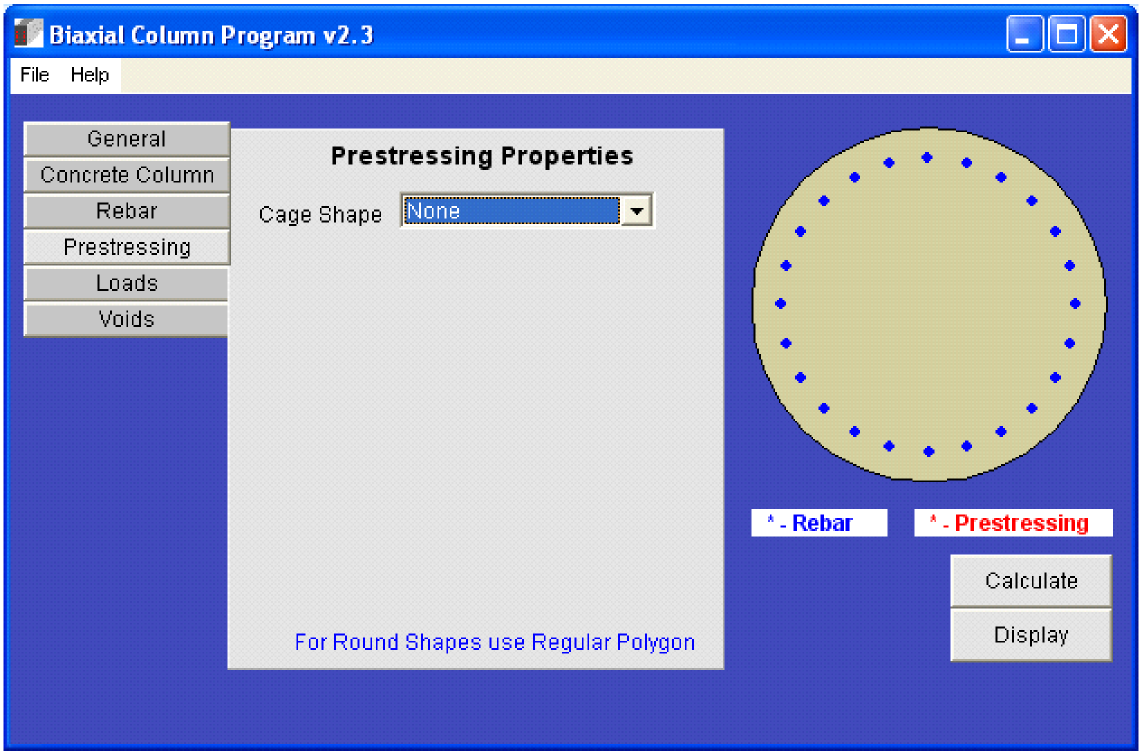
B1. Input Variables

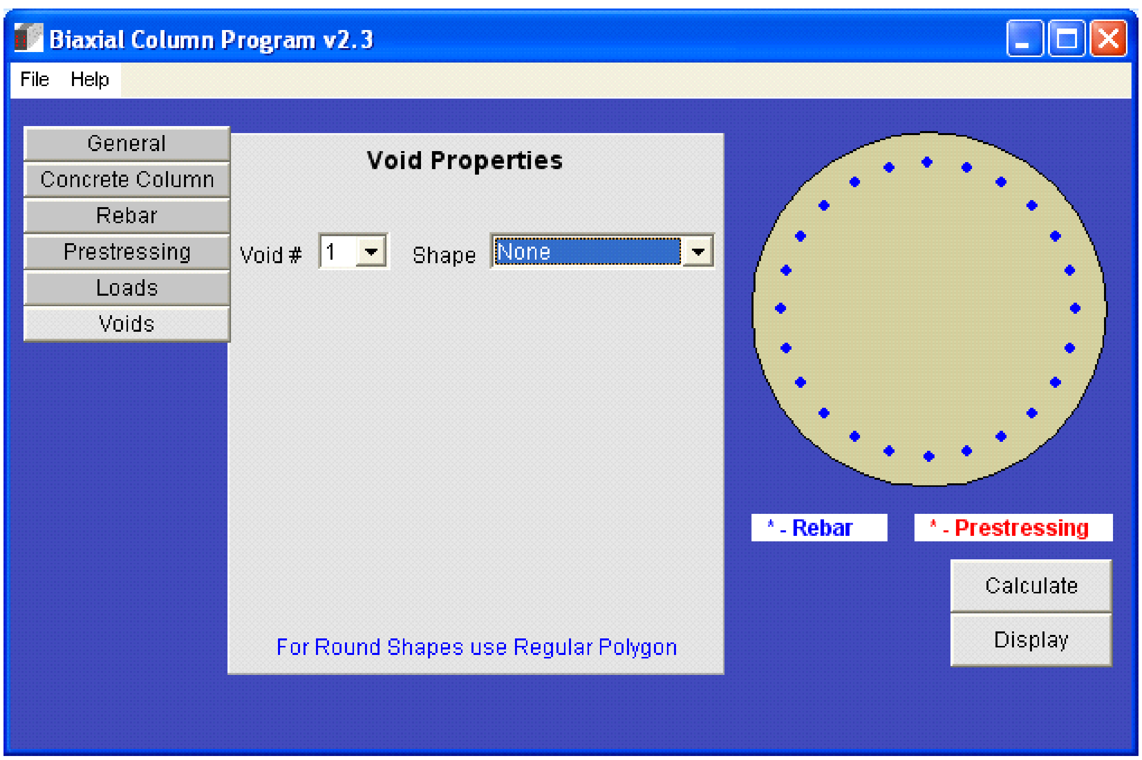
...Enter general information...



...Enter material properties...




Biaxial Column Program v2.3
File
Help
General
Concrete Column
Rebar
Prestressing
Loads
Voids
Prestressing Properties
Cage Shape
None
For Round Shapes use Regular Polygon
* - Rebar
* - Prestressing
Calculate
Display


Biaxial Column Program v2.3
File
Help
General
Concrete Column
Rebar
Prestressing
Loads
Voids
Void Properties
Void #
1
Shape
None
For Round Shapes use Regular Polygon
* - Rebar
* - Prestressing
Calculate
Display

Limits of Reinforcement [LRFD 5.7.4.2]

To account for the compressive strength of concrete, minimum reinforcement in flexural members is found to be proportional to $\left(\frac{f_c}{f_y}\right)$. Therefore, the longitudinal reinforcement in columns can be less than $0.01 \cdot A_g$ if allowed by the following equation:

Maximum area of reinforcement............. $\frac{A_s}{A_g} + \frac{A_{ps} \cdot f_{pu}}{A_g \cdot f_y} \le 0.08$

*(**Note**: 8% maximum is still applicable as per the LRFD).*

Minimum area of reinforcement.............. $\frac{A_s \cdot f_y}{A_g \cdot f'_c} + \frac{A_{ps} \cdot f_{pu}}{A_g \cdot f'_c} \ge 0.135$

For non-prestressed columns, the minimum percentage of reinforcement allowed is..... $A_{s\%} := 0.135 \cdot \frac{f_{c.sub}}{f_y} = 1.24 \cdot \%$

*(**Note**: This equation was written in the form of* $A_{s\%} = \frac{A_s}{A_g} \ge 0.135 \cdot \frac{f_{c.sub}}{f_y}$ *where* $A_{s\%}$ *is the percentage of reinforcement.*

In this situation, the minimum steel requirement was greater than 1% of the gross column area.

Gross concrete area............................... $A_g := \frac{\pi}{4} \cdot b_{Col}{}^2 = 1809.6 \cdot in^2$

Minimum steel area............................... $A_s := A_{s\%} \cdot A_g = 22.39 \cdot in^2$

Bar Size.. $bar := "9"$

Bar Area... $A_b := 1in^2$

Minimum number of bars....................... $n_b := \frac{A_s}{A_b} = 22.39$

A conservative 24 bars will be analyzed.

Spirals and Ties [LRFD 5.7.4.6]

Tie size [LRFD 5.10.6.3]....................... $bar_{tie} := "3"$

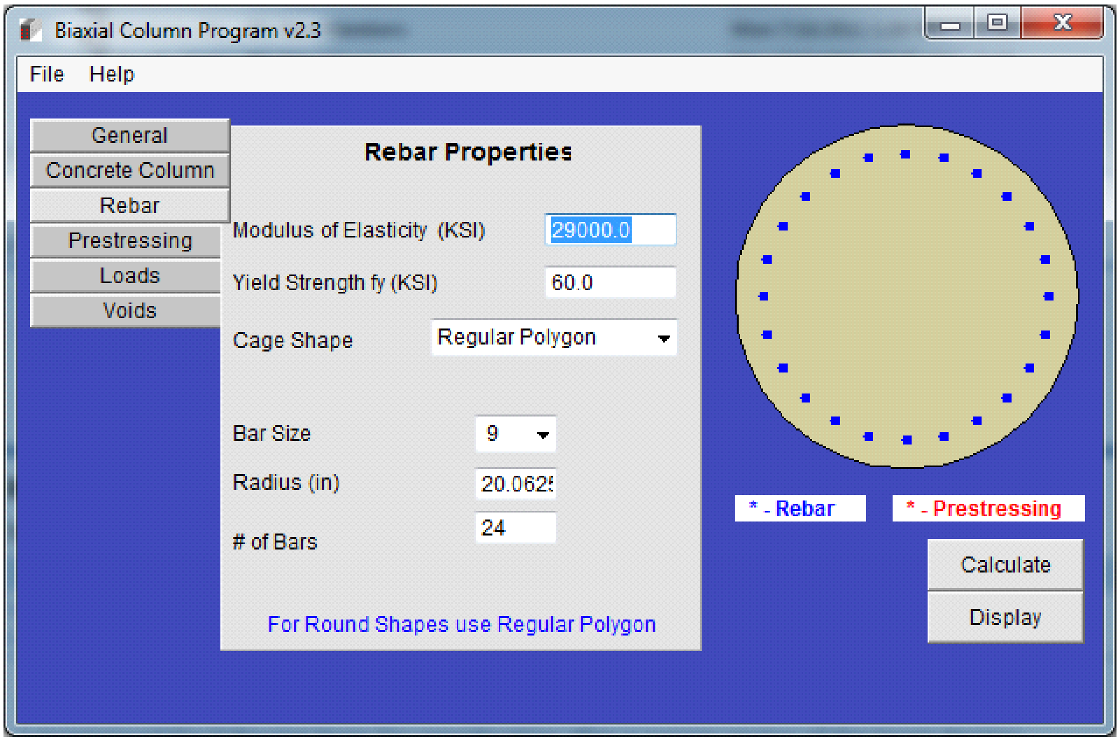
Tie area.. $A_{bar.tie} := 0.2in^2$

Area of core... $A_{core} := \frac{\pi}{4} \cdot \left(b_{Col} - 2 \cdot cover_{sub}\right)^2 = 1385.4 \cdot in^2$

Required tie area.................................... $\rho_s := 0.45 \cdot \left(\dfrac{A_g}{A_{core}} - 1 \right) \cdot \dfrac{f_{c.sub}}{f_y} \cdot \dfrac{in^2}{ft} = 0.0126 \cdot \dfrac{in^2}{ft}$

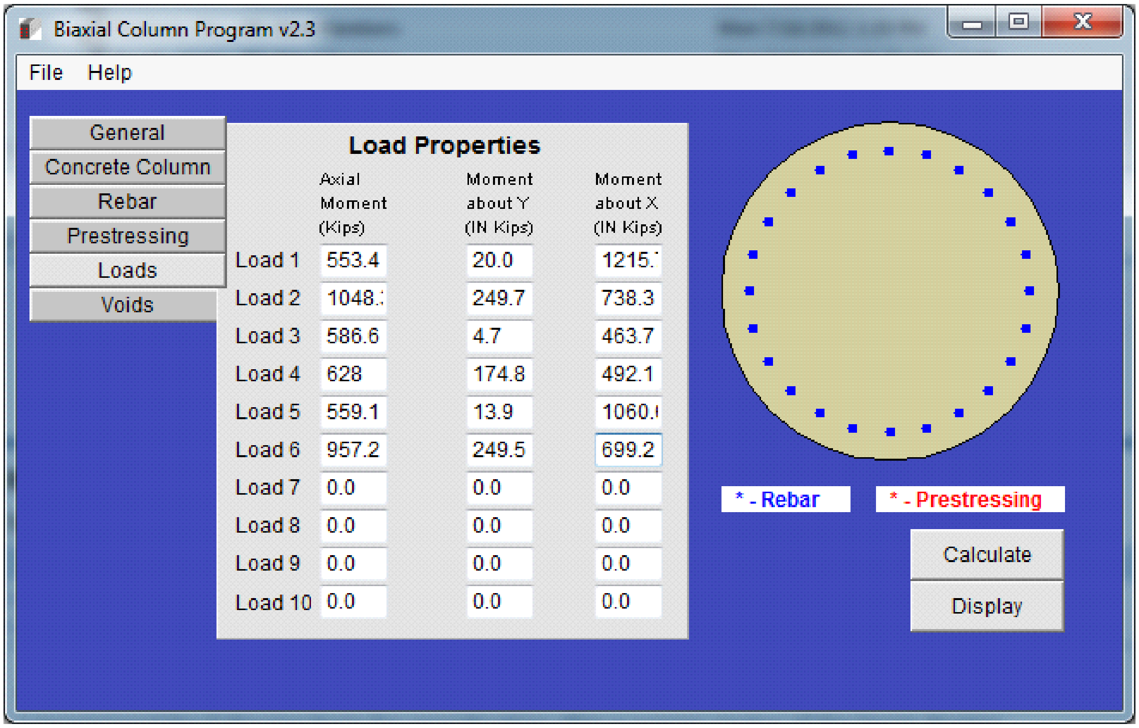
Required tie spacing................................ $s := \min\left(b_{Col}, 12in, \dfrac{A_{bar.tie}}{\rho_s} \right) = 12 \cdot in$

...Enter column design criteria...

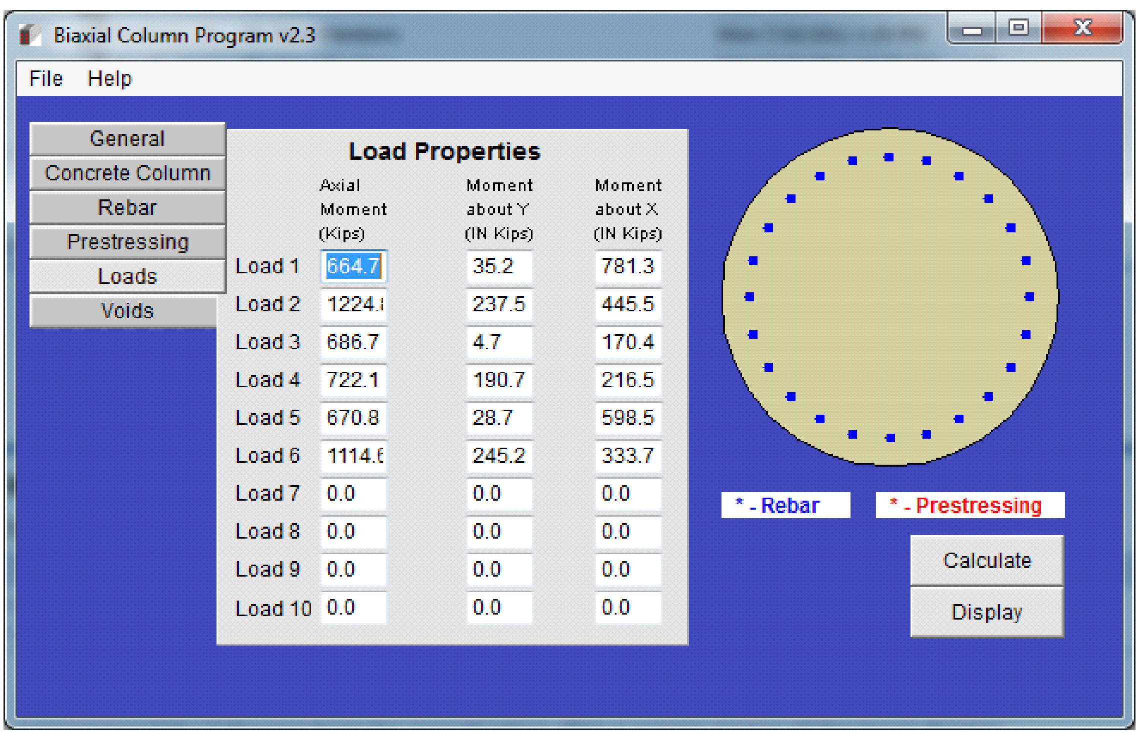


...Enter factored loads acting on column...

Column 1



Column 2

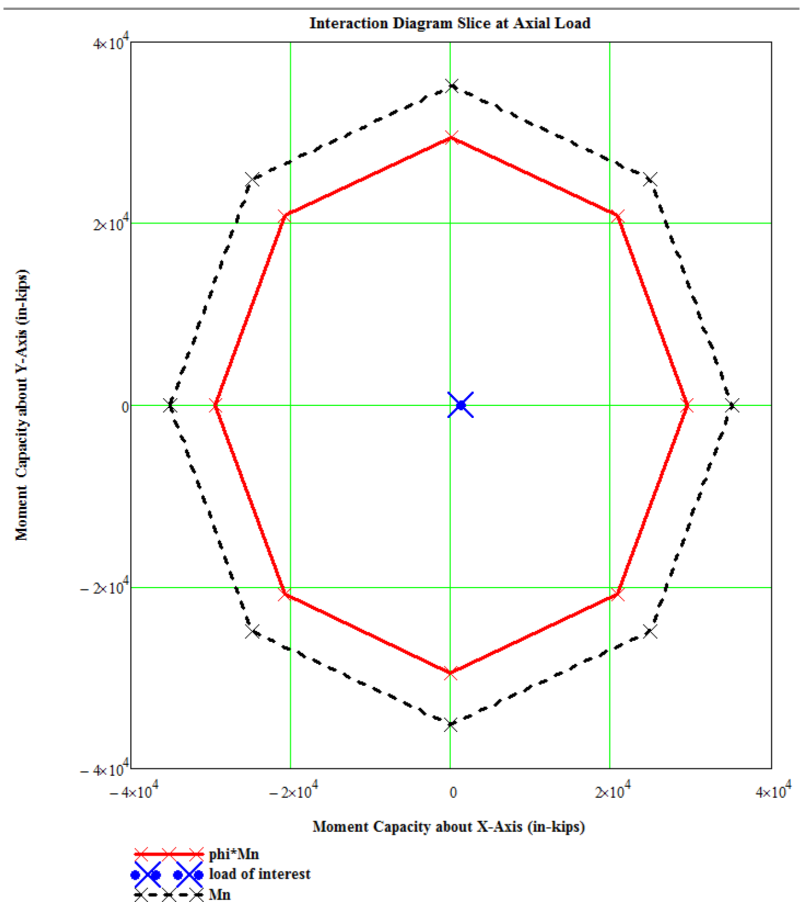


B2. Output

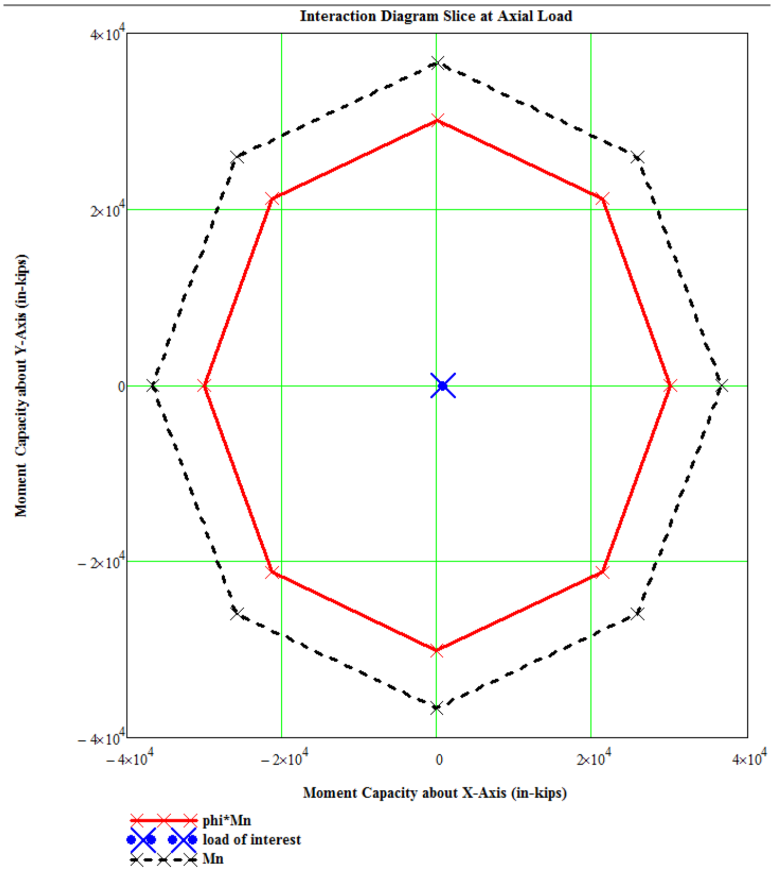
Based on the results, the columns have adequate capacity for the applied loads. The columns can be reduced in diameter, however, 4 foot diameter columns are typically found on intermediate piers over cross-streets. Another alternative to maximize the columns is to increase the column spacing, however, this will require greater reinforcing in the pier cap.

*(**Note**: For constructability, our experience has shown that if the bars are kept to a multiple of 4 then it improves placing the longitudinal steel around the column steel. In the plans, 24-#9 will be detailed.)*

<u>Column 1</u>



Column 2


Interaction Diagram Slice at Axial Load
4×10^4
2×10^4
0
− 2×10^4
− 4×10^4
Moment Capacity about Y-Axis (in-kips)
− 4×10^4
− 2×10^4
0
2×10^4
4×10^4
Moment Capacity about X-Axis (in-kips)
phi*Mn
load of interest
Mn

SUBSTRUCTURE DESIGN

Pier Foundation Design Loads

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\306PierColDesign.xmcd(R)

Description

This document provides the design parameters necessary for the substructure pile vertical load and footing design.

Page Contents

A. General Criteria

A1. Modification to Pier Column Live Loads for Foundation Design

The Dynamic Load Allowance (IM) is not required since the foundation components are entirely below ground level **[LRFD 3.6.2.1]**.

A2. Foundation Design Load Summary

For the foundation design, the impact on the truck will need to be removed from the load combinations since the footing is embedded in the ground. If the footing were a waterline footing, then impact should be included.

The RISA Analysis will be re-run with the impact factor removed from live load.

Revised HL-93 Line Load.................. $HL93_{No.IM} := \frac{R_{LLs}}{2} \cdot g_{v.Skew} = 65.86 \cdot kip$

$$HL93_{No.IM} + \frac{P_c}{IM} = 70.8 \cdot kip$$

$$HL93_{No.IM} - \frac{P_c}{IM} = 61 \cdot kip$$

For this design example, we will use the load combination that governed for the column design. In addition, the corresponding service limit state moments have been included and shown in the table below.

RISA COLUMN RESULTS

Result Case	Fy	Axial	Fz	My	Mz
Column 1					
Strength 1 - Top	126.9	556.2	19.3	20.0	1088.6
Strength 1 - Bottom	126.9	980.9	19.3	249.7	687.8
		980.9		**307.6**	**1068.5**
Service 1 - Top	63.1	435.5	9.1	10.1	963.7
Service 1 - Bottom	131.9	723.2	14.7	191.3	878.9
		723.2		**235.4**	**1274.6**

Note:
The values in **bold** have been translated from the bottom of the column to the top of the piles (= 3 ft).

Pier Piles Vertical Load Design

References

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\307PierFoundLoads.xmcd(R)

Description

This section provides the design of the piles for vertical loads (exclude lateral load design). For this design example, only the piles for column 1 footing will be evaluated.

Page Contents

FDOT Criteria

Minimum Sizes [SDG 3.5.1]

Use 18" square piling, except for extremely aggressive salt water environments.

Spacing, Clearances and Embedment and Size [SDG 3.5.4]

Minimum pile spacing center-to-center must be at least three times the pile diameter or 30 inches.

Resistance Factors [SDG 3.5.6]

The resistance factor for piles under compression with 10% dynamic testing, davisson capacity and EDC or PDA & CAPWAP analysis of test piles shall be $\phi_{pile} := 0.65$

Minimum Pile Tip [SDG 3.5.8]

The minimum pile tip elevation must be the deepest of the minimum elevations that satisfy uplift & lateral stability requirements for the three limit states. Since this bridge is not over water, scour and ship impact are not design issues. The design criteria for minimum tip elevation are based on vertical load requirements and lateral load analysis.

Pile Driving Resistance [SDG 3.5.12]

The Required Driving Resistance for an 18" square concrete pile must not exceed [SDG 3.5.12-1]..................... $R_{n.FDOT.max} := 300 \cdot Ton$

A. Input Variables

A1. Geometry

Depth of footing.................................... $h_{Ftg} = 4\,ft$

Width of footing.................................... $b_{Ftg} = 7.5\,ft$

Length of footing.................................... $L_{Ftg} = 7.5\,ft$

Pile Embedment Depth............................ $Pile_{embed} = 1\,ft$

A2. Forces on Top of Footing

Area of Footing...................................... $A_{ftg} := b_{Ftg} \cdot L_{Ftg} = 56.25\,ft^2$

Footing weight not included in RISA........ $wt_{Ftg} := \gamma_{conc} \cdot \left(A_{ftg} \cdot h_{Ftg}\right) = 33.75 \cdot kip$

Maximum service load............................ $P_y = 723.2 \cdot kip$

and corresponding moments............. $M_y = 235.4 \cdot ft \cdot kip$

$M_z = 1274.6 \cdot ft \cdot kip$

Maximum factored load.......................... $Pu_y = 980.9 \cdot kip$

and corresponding moments............. $Mu_y = 307.6 \cdot ft \cdot kip$

$Mu_z = 1068.5 \cdot ft \cdot kip$

B. Pile Loads

B1. 4- Pile Footing Investigation

So far, the design example has assumed that a 4-pile footing will be adequate.

Foundation Layout

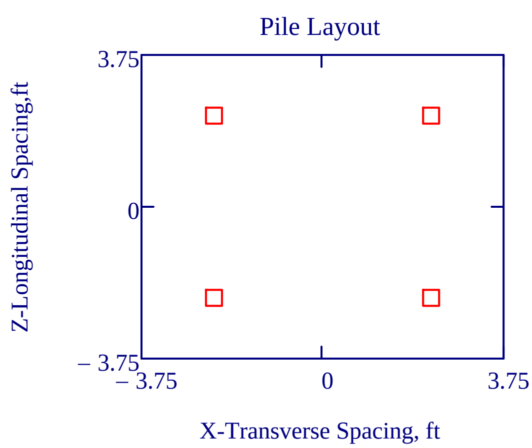
Size of the square concrete piles............. $Pile_{size} = 18 \cdot in$

Number of Piles.................................... $n_{pile} := 4$

Pile Coordinates....................................

$$Pile_{index} := \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \qquad X_{pile} := \begin{pmatrix} -2.25 \\ 2.25 \\ 2.25 \\ -2.25 \end{pmatrix} \cdot ft \qquad Z_{pile} := \begin{pmatrix} 2.25 \\ 2.25 \\ -2.25 \\ -2.25 \end{pmatrix} \cdot ft$$

$k := 0 .. n_{pile} - 1$



Overturning Forces due to Moments

General equation for axial load on any pile.. $Q_m = \frac{P_y}{n} + \frac{M_x \cdot z}{\sum_{z=0}^{n} (z^2)} + \frac{M_z \cdot x}{\sum_{x=0}^{n} (x^2)}$

Factored Axial Load on Pile

$$Q_{u_k} := \frac{\left|Pu_y + 1.25 \cdot wt_{Ftg}\right|}{n_{pile}} + \frac{\left|Mu_y\right| \cdot Z_{pile_k}}{\sum_{z=0}^{n_{pile}-1}\left[\left(Z_{pile_z}\right)^2\right]} + \frac{\left|Mu_z\right| \cdot X_{pile_k}}{\sum_{x=0}^{n_{pile}-1}\left[\left(X_{pile_x}\right)^2\right]}$$

$$Pile_{index} := \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} \qquad Q_u = \begin{pmatrix} 171.2 \\ 408.7 \\ 340.3 \\ 102.9 \end{pmatrix} \cdot kip$$

Maximum axial load on pile.................... $Q_{max} := \max\left(Q_u\right) = 204.34 \cdot Ton$

Required driving resistance (RDR)........... $RDR = R_n = \dfrac{\text{Factored Design Load} + \text{Net Scour} + \text{Downdrag}}{\phi}$

Using variables defined in this example...... $R_n := \dfrac{Q_{max}}{\phi_{pile}} = 314.36 \cdot Ton$

This value should not exceed the limit
specified by FDOT................................ $R_{n.FDOT.max} = 300 \cdot Ton$

A 4-pile footing is not acceptable. It is recommended not to design to the R_n limit since difficulties in pile driving can be encountered causing construction delays. Suggest consulting with the District geotechnical and structural engineers if within 5%-10%. We will investigate a 6-pile footing.

B2. 6- Pile Footing Investigation

The 4-pile footing design involves a limited amount of shear design, since the piles are outside the critical section for shear. To illustrate the shear design process, a 6-pile footing will be evaluated and designed.

New depth of footing............................ $h_{Ftg.new} := 4 \cdot ft$

New width of footing............................ $b_{Ftg.new} := 12 \cdot ft$

New length of footing............................ $L_{Ftg.new} := 7.5 \cdot ft$

New area of Footing.............................. $A_{ftg.new} := b_{Ftg.new} \cdot L_{Ftg.new} = 90\, ft^2$

Footing weight not included in RISA........ $wt_{Ftg} := \gamma_{conc} \cdot (A_{ftg.new} \cdot h_{Ftg.new}) = 54 \cdot kip$

Foundation Layout

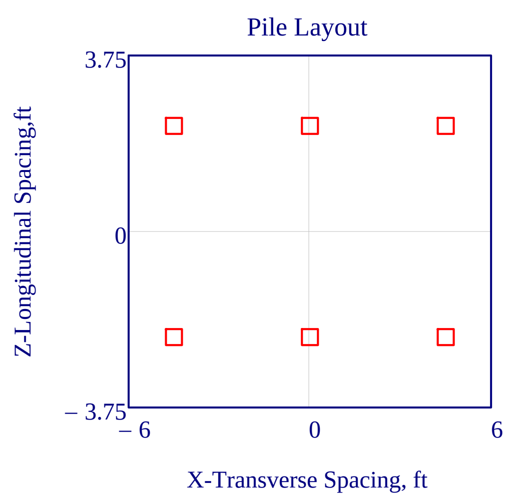
Size of the square concrete piles............. $Pile_{size} = 18 \cdot in$

Number of Piles.................................... $n_{pile} := 6$

Pile Coordinates....................................

$$Pile_{index} := \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \qquad X_{pile} := \begin{pmatrix} -4.5 \\ 0 \\ 4.5 \\ 4.5 \\ 0 \\ -4.5 \end{pmatrix} \cdot ft \qquad Z_{pile} := \begin{pmatrix} 2.25 \\ 2.25 \\ 2.25 \\ -2.25 \\ -2.25 \\ -2.25 \end{pmatrix} \cdot ft$$

$k := 0 .. n_{pile} - 1$



Note:
Pile numbering is from 0" to 5" and are numbered CLOCKWISE beginning with the upper top left side pile.

Service Axial Load on Pile

$$Q_k := \frac{\left|P_y + 1.0 \cdot wt_{Ftg}\right|}{n_{pile}} + \frac{\left|M_y\right| \cdot Z_{pile_k}}{\sum_{z=0}^{n_{pile}-1} \left[\left(Z_{pile_z}\right)^2\right]} + \frac{\left|M_z\right| \cdot X_{pile_k}}{\sum_{x=0}^{n_{pile}-1} \left[\left(X_{pile_x}\right)^2\right]}$$

$$Pile_{index} = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \qquad Q = \begin{pmatrix} 76.2 \\ 147.0 \\ 217.8 \\ 182.9 \\ 112.1 \\ 41.3 \end{pmatrix} \cdot kip$$

Factored Axial Load on Pile

$$Q_{u_k} := \frac{\left|Pu_y + 1.25 \cdot wt_{Ftg}\right|}{n_{pile}} + \frac{\left|Mu_y\right| \cdot Z_{pile_k}}{\sum_{z=0}^{n_{pile}-1} \left[\left(Z_{pile_z}\right)^2\right]} + \frac{\left|Mu_z\right| \cdot X_{pile_k}}{\sum_{x=0}^{n_{pile}-1} \left[\left(X_{pile_x}\right)^2\right]}$$

$$Pile_{index} = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \qquad Q_u = \begin{pmatrix} 138.2 \\ 197.5 \\ 256.9 \\ 211.3 \\ 151.9 \\ 92.6 \end{pmatrix} \cdot kip$$

Maximum axial load on pile..................... $Q_{max} := \max(Q_u) = 128.44 \cdot Ton$

Minimum axial load on pile (verify no uplift occurs)... $Q_{min} := \min(Q_u) = 46.29 \cdot Ton$

Required driving resistance (RDR)........... $RDR = R_n = \dfrac{\text{Factored Design Load} + \text{Net Scour} + \text{Downdrag}}{\phi}$

Using variables defined in this example...... $R_n := \frac{Q_{max}}{\phi_{pile}} = 197.6 \cdot Ton$

This value should not exceed the limit specified by FDOT................................ $R_{n.FDOT.max} = 300 \cdot Ton$

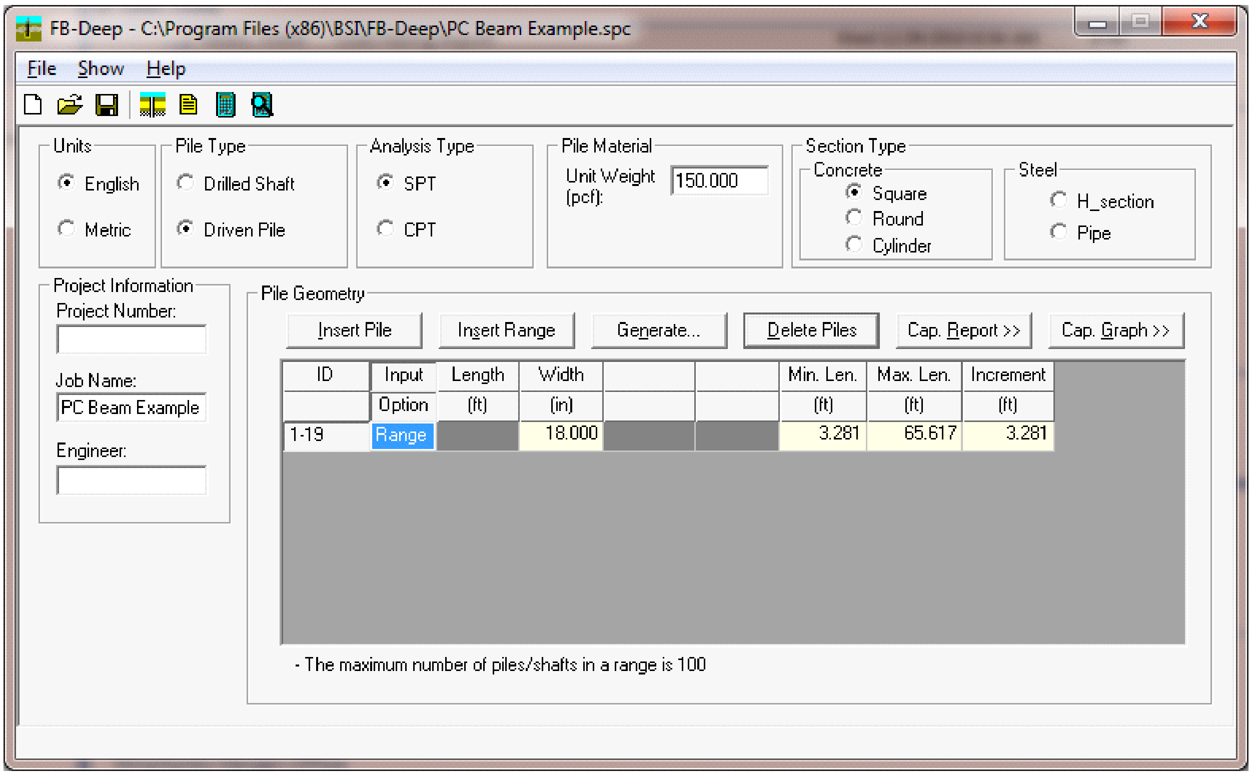
A 6-pile footing is acceptable.

C. Pile Tip Elevations for Vertical Load

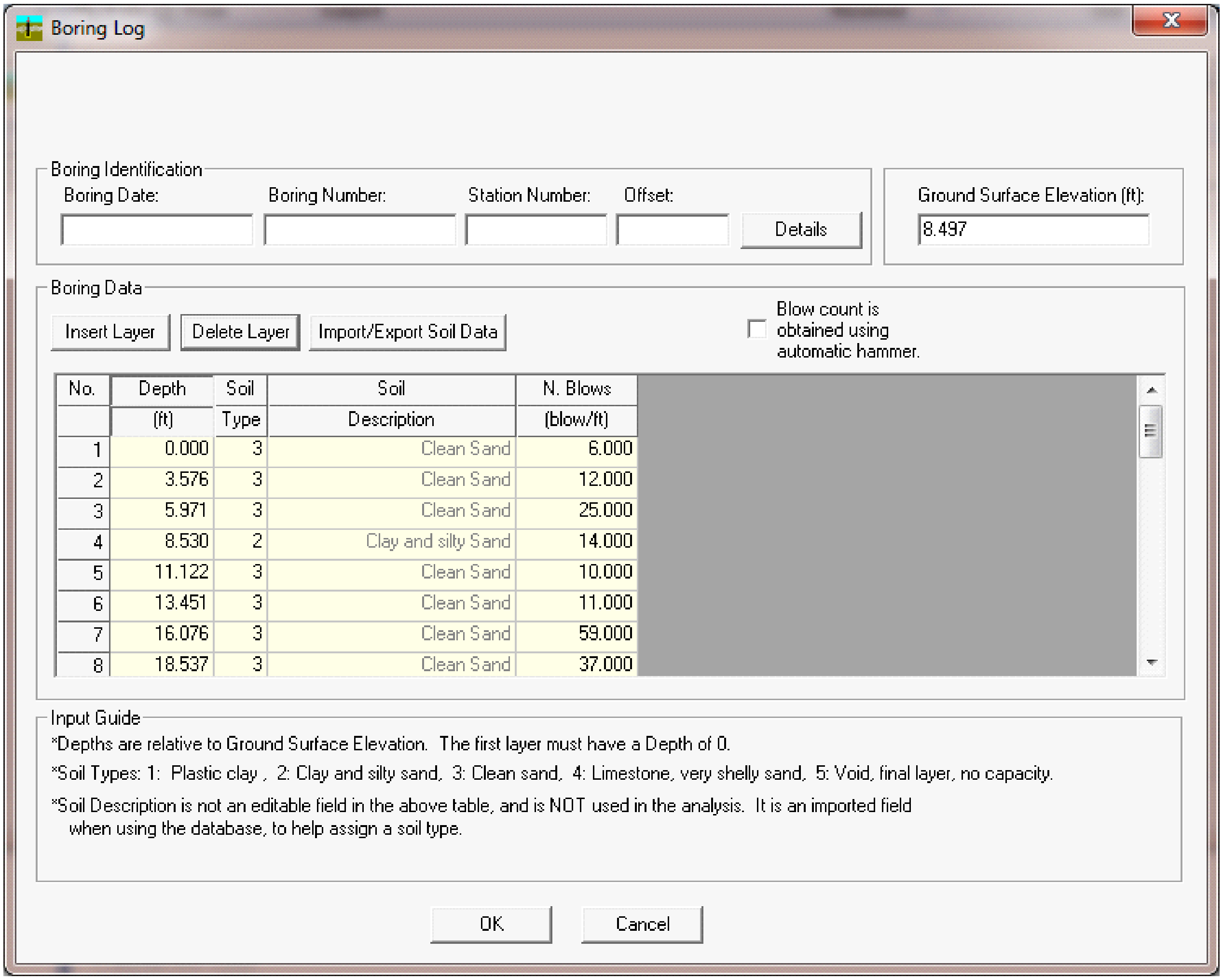
C1. Pile Capacities

The FB-Deep Program, Version 2.02, was utilized to determine the pile capacity. Using boring data, the program can analyze concrete piles, H-piles, pipe piles, and drilled shafts. It is available from BSI.

For this design example, the boring data is based on a similar project.



The following picture shows the boring log entries.



Recall that the factored bearing resistance, R_n, is given by..................................

$$R_n = \frac{\text{Factored Design Load} + \text{Net Scour} + \text{Downdrag}}{\phi}$$

In this design example, net scour and downdrag are zero, so the R_n is...........

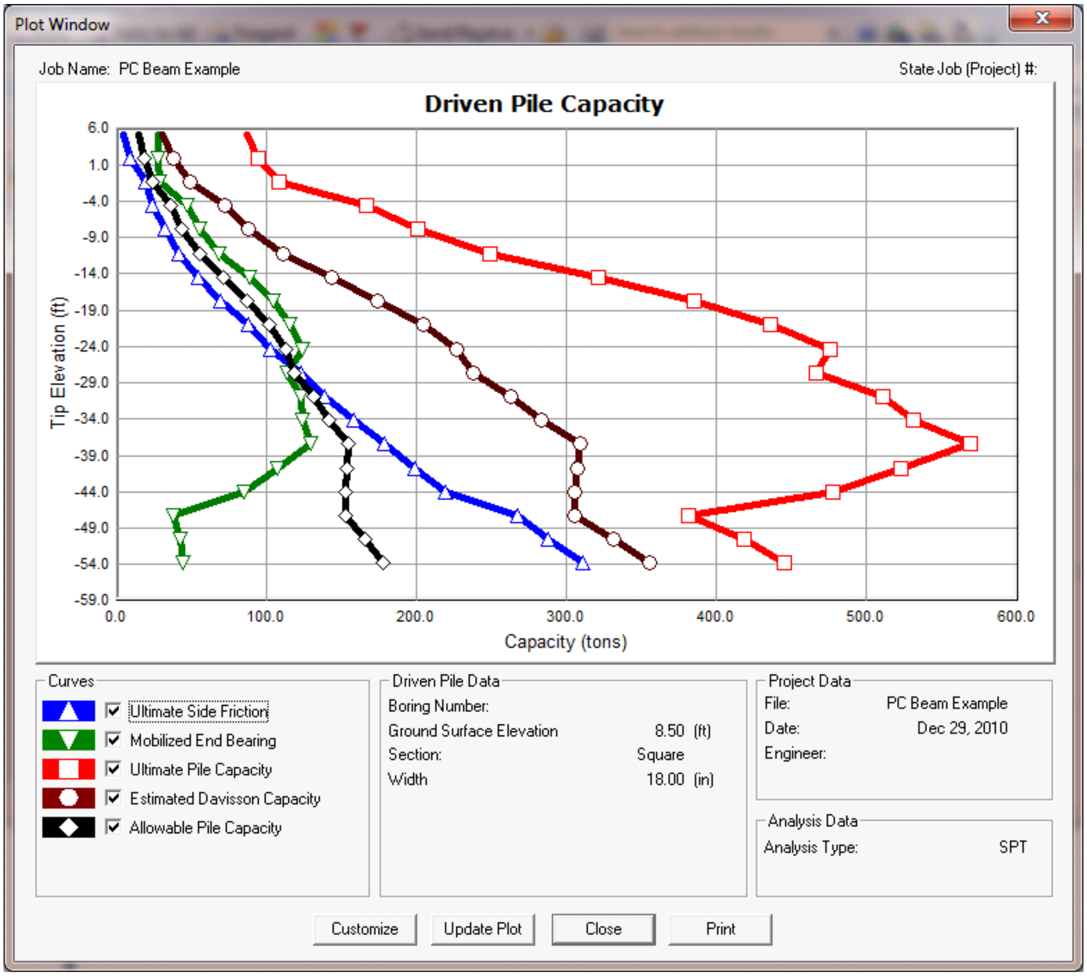
$$R_n = 197.6 \cdot \text{Ton}$$

The program was executed, and the output can be summarized as follows:

Shaft002.tmp - Notepad

File Edit Format View Help

```
Driven Pile Capacity:
=====================

   Test   Pile   Ultimate  Mobilized  Estimated  Allowable   Ultimate
   Pile  Width       Side        End   Davisson       Pile       Pile
 Length          Friction    Bearing   Capacity   Capacity   Capacity
   (ft)   (in)     (tons)     (tons)     (tons)     (tons)     (tons)
  ----- ------ ---------- ---------- ---------- ---------- ----------
   3.28   18.0       2.96      28.20      31.16      15.58      87.56
   6.56   18.0       9.91      28.34      38.25      19.13      94.94
   9.84   18.0      20.15      29.39      49.53      24.77     108.31
  13.12   18.0      24.67      47.44      72.12      36.06     167.01
  16.40   18.0      32.51      56.14      88.65      44.32     200.93
  19.69   18.0      42.44      68.80     111.24      55.62     248.83
  22.97   18.0      54.67      88.84     143.51      71.76     321.19
  26.25   18.0      69.43     105.26     174.70      87.35     385.22
  29.53   18.0      88.62     115.90     204.52     102.26     436.31
  32.81   18.0     102.91     124.27     227.19     113.59     475.73
  36.09   18.0     123.70     114.35     238.05     119.02     466.75
  39.37   18.0     139.07     123.86     262.93     131.47     510.66
  42.65   18.0     158.83     124.09     282.91     141.46     531.09
  45.93   18.0     179.23     129.82     309.05     154.53     568.70
  49.22   18.0     199.49     107.64     307.13     153.57     522.41
  52.50   18.0     219.74      85.93     305.67     152.83     477.52
  55.78   18.0     267.60      37.95     305.55     152.77     381.44
  59.06   18.0     287.86      43.39     331.25     165.62     418.03
  62.34   18.0     310.77      44.87     355.63     177.82     445.36

   NOTES
  -------
   1. MOBILIZED END BEARING IS 1/3 OF THE ORIGINAL RB-121 VALUES.

   2. DAVISSON PILE CAPACITY IS AN ESTIMATE BASED ON FAILURE CRITERIA,
      AND EQUALS ULTIMATE SIDE FRICTION PLUS MOBILIZED END BEARING.

   3. ALLOWABLE PILE CAPACITY IS 1/2 THE DAVISSON PILE CAPACITY.

   4. ULTIMATE PILE CAPACITY IS ULTIMATE SIDE FRICTION PLUS
      3 x THE MOBILIZED END BEARING.
      EXCEPTION: FOR H-PILES TIPPED IN SAND OR LIMESTONE, THE
      ULTIMATE PILE CAPACITY IS ULTIMATE SIDE FRICTION PLUS
      2 x THE MOBILIZED END BEARING.
```



A lateral load analysis may require the pile tip elevations to be driven deeper for stability purposes. This file only evaluates the vertical load requirements based on the boring capacity curves.

Calculate the pile length required.... $pile_{length} := \left(R_n - 174.7 \cdot Ton\right) \cdot \dfrac{29.5 \cdot ft - 26.2 \cdot ft}{204.52 \cdot Ton - 174.7 \cdot Ton} + 26.2 \cdot ft = 28.7\,ft$

Calculate the pile tip elevation required: $pile_{tip} := 8.497\,ft - pile_{length} = -20.2\,ft$

...based on the Estimated Davisson pile capacity curve given above, the pile lengths for vertical load will require a specified Tip Elevation = $pile_{tip} = -20.2\,ft$ and the pile in the ground length is $pile_{length} = 28.7\,ft$

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SUBSTRUCTURE DESIGN

Pier Footing Design

References

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\308PierPileDesign.xmcd(R)

Description

This document provides the criteria for the pier footing design. For this design example, only column 1 footing will be evaluated.

Page Contents

LRFD Criteria

STRENGTH I - Basic load combination relating to the normal vehicular use of the bridge without wind.

$WA = 0$ Water load and stream pressure are not applicable.

$FR = 0$ No friction forces.

TU Uniform temperature load effects on the pier will be generated by the substructure analysis model (RISA).

$$Strength1 = 1.25 \cdot DC + 1.50 \cdot DW + 1.75 \cdot LL + 1.75 \cdot BR + 0.50 \cdot (TU + CR + SH)$$

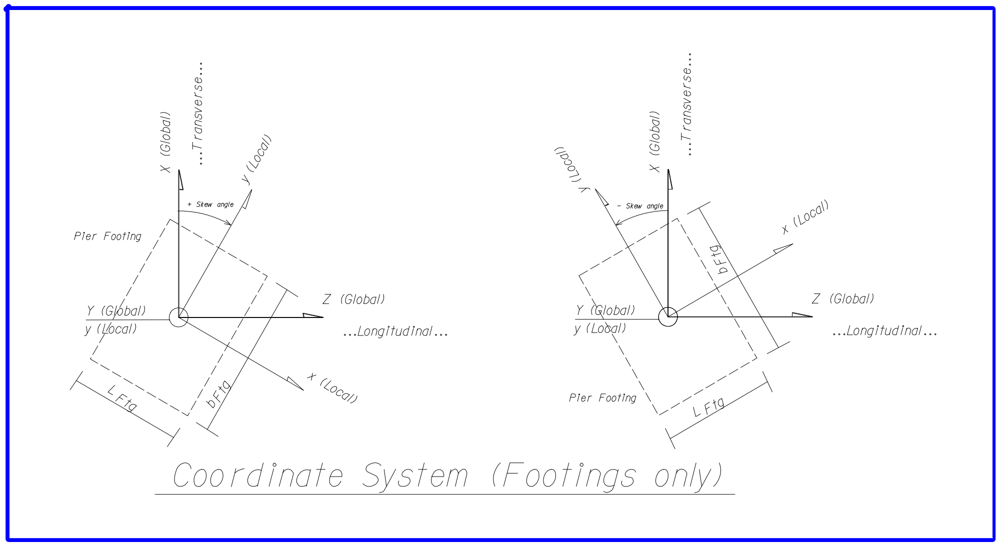
SERVICE I - Load combination relating to the normal operational use of the bridge with a 55 MPH wind and all loads taken at their nominal values.

$$Service1 = 1.0 \cdot DC + 1.0 \cdot DW + 1.0 \cdot LL + 1.0 \cdot BR + 1.0WS + 1.0 \cdot WL + 1.0 \cdot (TU + CR + SH)$$

"For the footing, utilized only to check for crack control"

A. Input Variables

A1. Design Parameters



Coordinate System (Footings only)

Transverse dimension of footing............. $b_{Ftg} = 12\,ft$

Longitudinal dimension of footing........... $L_{Ftg} = 7.5\,ft$

Depth of footing.................................... $h_{Ftg} = 4\,ft$

Area of footing...................................... $A_{Ftg} := b_{Ftg} \cdot L_{Ftg} = 90\,ft^2$

Embedment of pile in footing.................. $Pile_{embed} = 1\,ft$

Concrete cover above piles..................... $cover_{sub} = 3 \cdot in$

Height of surcharge (column height in ground).. $h_{Surcharge} = 2\,ft$

Diameter of column............................... $b_{Col} = 4\,ft$

Area of column.................................... $A_g = 12.6\,ft^2$

Equivalent square width for the circular column **[LRFD 5.13.3.4]**:...................... $b_{Col.eff} := \mathrm{round}\left(\sqrt{\dfrac{A_g}{ft^2}}, 1\right) \cdot ft = 3.5\,ft$

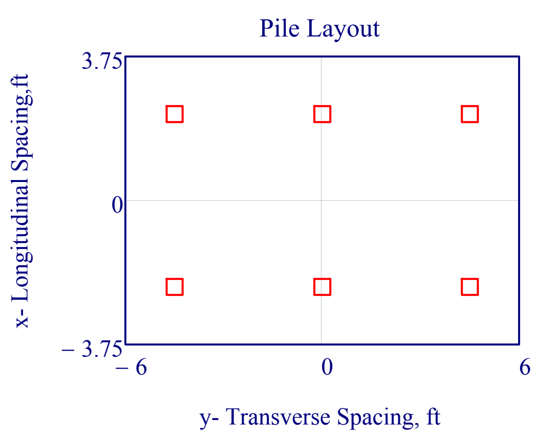
A2. Pile Layout

Pile size.. $Pile_{size} = 18 \cdot in$

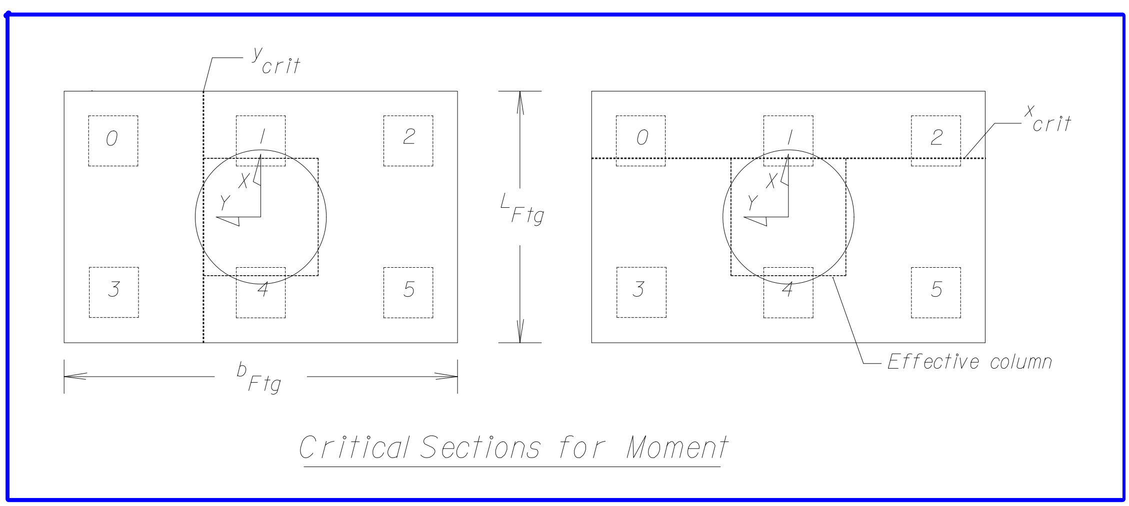
Number of piles.................................... $n_{pile} = 6$

Summary of pile loads

			Service I Limit State		Strength I Limit State	
Pile #	**x Coord.**	**y Coord.**	**Q, Tons**	**Q, kips**	**Qu, Tons**	**Qu, kips**
0	2.25	-4.5	38.1	76.2	69.1	138.2
1	2.25	0	73.5	147.0	98.8	197.5
2	2.25	4.5	108.9	217.8	128.4	256.9
3	-2.25	4.5	91.5	182.9	105.7	211.3
4	-2.25	0	56.0	112.1	76.0	151.9
5	-2.25	-4.5	20.6	41.3	46.3	92.6



A3. Flexural Design Parameters



Distance from centerline of piles to edge of footing...

$pile_{edge} := 1.5ft$

Distance from x-critical section (face of effective column) to edge of footing along the x-axis...

$$x_{edge} := \frac{L_{Ftg} - b_{Col.eff}}{2} = 2\ ft$$

Distance from x-critical section to centerline of piles along the x-axis...........

$x_{crit} := x_{edge} - pile_{edge} = 0.5\ ft$

Distance from y-critical section (face of effective column) to edge of footing along the y-axis...

$$y_{edge} := \frac{b_{Ftg} - b_{Col.eff}}{2} = 4.25\ ft$$

Distance from y-critical section to centerline of piles along the y-axis..........................

$y_{crit} := y_{edge} - pile_{edge} = 2.75\ ft$

A4. Moments - Y Critical Section

Unfactored pile loads contributing to transverse moment................................

$P := \max\left(Q_0 + Q_3, Q_2 + Q_5\right) = 259.1 \cdot kip$

Unfactored moments at critical section due to pile loads...

$Mx_{Pile} := P \cdot y_{crit} = 712.43 \cdot kip \cdot ft$

Unfactored moment at critical section due to footing weight.....................................

$$Mx_{Ftg} := \left(L_{Ftg} \cdot h_{Ftg} \cdot \gamma_{conc}\right) \cdot \frac{y_{edge}^{\ 2}}{2} = 40.64 \cdot kip \cdot ft$$

Unfactored moments at critical section due to surcharge ....

$$Mx_{Surcharge} := \left(L_{Ftg} \cdot h_{Surcharge} \cdot \gamma_{soil}\right) \cdot \frac{y_{edge}^{\ 2}}{2} = 15.58 \cdot kip \cdot ft$$

Factored pile loads contributing to transverse moment................................ $P_u := \max\left(Q_{u_0} + Q_{u_3}, Q_{u_2} + Q_{u_5}\right) = 349.5 \cdot kip$

Assure the critical section is within the footing dimensions................................ $P_u := \text{if}\left(y_{crit} \ge y_{edge}, 0 \cdot kip, P_u\right) = 349.5 \cdot kip$

Factored moments at critical section due to pile loads... $Mux_{Pile} := P_u \cdot y_{crit} = 961 \cdot kip \cdot ft$

A5. Moments - X Critical Section

Unfactored pile loads contributing to longitudinal moment.............................. $P := \max\left(Q_0 + Q_1 + Q_2, Q_3 + Q_4 + Q_5\right) = 440.9 \cdot kip$

Unfactored moments at critical section due to pile loads... $My_{Pile} := P \cdot x_{crit} = 220.5 \cdot kip \cdot ft$

Unfactored moment at critical section due to footing weight...................................... $My_{Ftg} := \left(b_{Ftg} \cdot h_{Ftg} \cdot \gamma_{conc}\right) \cdot \frac{x_{edge}^{\ 2}}{2} = 14.4 \cdot kip \cdot ft$

Unfactored moments at critical section due to surcharge.. $My_{Surcharge} := \left(b_{Ftg} \cdot h_{Surcharge} \cdot \gamma_{soil}\right) \cdot \frac{x_{edge}^{\ 2}}{2} = 5.5 \cdot kip \cdot ft$

Factored pile loads contributing to longitudinal moment............................... $P_u := \max\left(Q_{u_0} + Q_{u_1} + Q_{u_2}, Q_{u_3} + Q_{u_4} + Q_{u_5}\right) = 592.56 \cdot kip$

Assure the critical section is within the footing dimensions................................ $P_u := \text{if}\left(x_{crit} \ge x_{edge}, 0 \cdot kip, P_u\right) = 592.6 \cdot kip$

Factored moments at critical section due to pile loads... $Muy_{Pile} := P_u \cdot x_{crit} = 296.3 \cdot kip \cdot ft$

A6. Design Moments

Transverse Footing Design (Mx moments) - Strength I.............................. $Mx_{Strength1} := Mux_{Pile} - 0.9 \cdot Mx_{Ftg} - 0.9 \cdot Mx_{Surcharge} = 910.4 \cdot kip \cdot ft$

Transverse Footing Design (Mx moments) - Service I............................ $Mx_{Service1} := 1.0 \cdot Mx_{Pile} - 1.0 \cdot Mx_{Ftg} - 1.0 \cdot Mx_{Surcharge} = 656.2 \cdot kip \cdot$

Longitudinal Footing Design (My moments) - Strength I............................ $My_{Strength1} := Muy_{Pile} - 0.9 \cdot My_{Ftg} - 0.9 \cdot My_{Surcharge} = 278.3 \cdot kip \cdot ft$

Longitudinal Footing Design (My moments) - Service I.......................... $My_{Service1} := 1.0 \cdot My_{Pile} - 1.0 \cdot My_{Ftg} - 1.0 \cdot My_{Surcharge} = 200.5 \cdot kip \cdot ft$

B. Flexural Design

B1. Transverse Flexural Design [LRFD 5.7.3.2]

The design procedure consists of calculating the reinforcement required to satisfy the design moment, then checking this reinforcement against criteria for crack control, minimum reinforcement, maximum reinforcement, shrinkage and temperature reinforcement, and distribution of reinforcement. The procedure is the same for both the transverse and longitudinal moment designs.

Factored resistance................................ $M_r = \phi \cdot M_n$

Nominal flexural resistance.....................

$$M_n = A_{ps} \cdot f_{ps} \cdot \left(d_p - \frac{a}{2}\right) + A_s \cdot f_s \cdot \left(d_s - \frac{a}{2}\right) - A'_s \cdot f'_s \cdot \left(d'_s - \frac{a}{2}\right) + 0.85 \cdot f'_c \cdot (b - b_w) \cdot \beta_1 \cdot h_f \cdot \left(\frac{a}{2} - \frac{h_f}{2}\right)$$

For a rectangular, non-prestressed section, $M_n = A_s \cdot f_s \cdot \left(d_s - \frac{a}{2}\right)$

$$a = \frac{A_s \cdot f_s}{0.85 \cdot f'_c \cdot b}$$

Using variables defined in this example,

Factored resistance.......................... $M_u := Mx_{Strength1} = 910.4 \cdot kip \cdot ft$

Width of section $b := L_{Ftg} = 7.5\,ft$

Initial assumption for area of steel required

Number of bars.............................. $n_{ybar} := 9$

Size of bar..................................... $ybar := "9"$

*(**Note**: Bar size and spacing are governed by crack control criteria and not bending capacity).*

Note: if bar spacing is "-1", the spacing is less than 3", and a bigger bar size should be selected.

Bar area.. $A_{bar} = 1.000 \cdot in^2$

Bar diameter.................................. $ybar_{dia} = 1.128 \cdot in$

Equivalent bar spacing..................... $ybar_{spa} = 10.4 \cdot in$

Area of steel provided............................ $A_s := n_{ybar} \cdot A_{bar} = 9 \cdot in^2$

Assume $f_s := f_y$ [LRFD 5.7.2.1]

Distance from extreme compressive fiber to centroid of reinforcing steel................ $d_s := h_{Ftg} - cover_{sub} - Pile_{embed} - \frac{ybar_{dia}}{2} = 32.44 \cdot in$

Stress block factor................................ $\beta_1 := \min\left[\begin{bmatrix} \max\left[\begin{bmatrix} 0.85 - 0.05 \cdot \left(\frac{f_{c.sub} - 4000psi}{1000psi}\right) \\ 0.65 \end{bmatrix}\right] \\ 0.85 \end{bmatrix}\right] = 0.78$

Distance between the neutral axis and compressive face.................................. $c := \frac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot \beta_1 \cdot b} = 1.66 \cdot in$

Solve the quadratic equation for the area of steel required....................................... Given $M_r = \phi \cdot A_s \cdot f_s \cdot \left[d_s - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot b}\right)\right]$

Area of steel required............................. $A_{s.reqd} := Find(A_s) = 6.33 \cdot in^2$

Check assumption that $f_s = f_y$ .... $Check\_f_s := if\left(\frac{c}{d_s} < 0.6, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

The area of steel provided, $A_s = 9.00 \cdot in^2$, should be greater than the area of steel required, $A_{s.reqd} = 6.33 \cdot in^2$. If not, decrease the spacing of the reinforcement. Once A_s is greater than $A_{s.reqd}$, the proposed reinforcing is adequate for the applied moments.

Moment capacity provided..................... $M_{r.tran} := \phi \cdot A_s \cdot f_s \cdot \left[d_s - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot b}\right)\right] = 1287.7 \cdot kip \cdot ft$

B2. Transverse Limits for Reinforcement [LRFD 5.7.3.3]

Minimum Reinforcement

The minimum reinforcement requirements ensure the moment capacity provided is at least equal to the lesser of the cracking moment and 1.33 times the factored moment required by the applicable strength load combinations.

Modulus of rupture............................... $f_r := -0.24 \cdot \sqrt{f_{c.sub} \cdot ksi} = -562.8\ psi$ [SDG 1.4.1.B]

Section modulus of the footing above the piles.. $S := \frac{L_{Ftg} \cdot (h_{Ftg} - Pile_{embed})^2}{6} = 11.25 \cdot ft^3$

Flexural cracking variability factor........... $\gamma_1 := 1.6$

Ratio of specified minimum yield strength to ultimate tensile strength of reinforcement..................................... $\gamma_3 := 0.67$ *(for ASTM A615, Grade 60 reinforcing steel, as required by SDG 1.4.1.C)*

Cracking moment.................................. $M_{cr} := f_r \cdot S \cdot \gamma_3 \cdot \gamma_1 = -977.5 \cdot kip \cdot ft$

Required flexural resistance..................... $M_{r.reqd} := \min\left(\left|M_{cr}\right|, 133 \cdot \% \cdot M_r\right) = 977.5 \cdot kip \cdot ft$

Check that the capacity provided, $M_{r.tran} = 1287.7 \cdot ft \cdot kip$, exceeds minimum requirements, $M_{r.reqd} = 977.5 \cdot ft \cdot kip$.

$LRFD_{5.7.3.3.2} :=$ | "OK, minimum reinforcement for transverse moment is satisfied" if $M_{r.tran} \geq M_{r.reqd}$
| "NG, reinforcement for transverse moment is less than minimum" otherwise

$LRFD_{5.7.3.3.2} =$ "OK, minimum reinforcement for transverse moment is satisfied"

B3. Transverse Crack Control by Distribution Reinforcement [LRFD 5.7.3.4]

Concrete is subjected to cracking. Limiting the width of expected cracks under service conditions increases the longevity of the structure. Potential cracks can be minimized through proper placement of the reinforcement. The check for crack control requires that the actual stress in the reinforcement should not exceed the service limit state stress (LRFD 5.7.3.4). The stress equations emphasize bar spacing rather than crack widths.

The maximum spacing of the mild steel reinforcement for control of cracking at the service limit state shall satisfy.........

$$s \leq \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

where

$$\beta_s = 1 + \frac{d_c}{0.7\left(h - Pile_{embed} - d_c\right)}$$

Exposure factor for Class 1 exposure condition.. $\gamma_e := 1.00$ **[SDG 3.10]**

Distance from extreme tension fiber to center of closest bar.............................. $d_c := cover_{sub} + \frac{ybar_{dia}}{2} = 3.56 \cdot in$

$$\beta_s := 1 + \frac{d_c}{0.7\left(h_{Ftg} - Pile_{embed} - d_c\right)} = 1.16$$

The neutral axis of the section must be determined to determine the actual stress in the reinforcement. This process is iterative, so an initial assumption of the neutral axis must be made.

Guess value $x := 6.3 \cdot in$

Given $\frac{1}{2} \cdot b \cdot x^2 = \frac{E_s}{E_{c.sub}} \cdot A_s \cdot (d_s - x)$

$x_{na} := \text{Find}(x) = 6.28 \cdot in$

Tensile force in the reinforcing steel due to service limit state moment...................... $T_s := \frac{Mx_{Service1}}{d_s - \frac{x_{na}}{3}} = 259.52 \cdot kip$

Actual stress in the reinforcing steel due to service limit state moment...................... $f_{s.actual} := \frac{T_s}{A_s} = 28.84 \cdot ksi$

Required reinforcement spacing.............. $s_{required} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{s.actual}} - 2 \cdot d_c = 13.85 \cdot in$

Provided reinforcement spacing.............. $ybar_{spa} = 10.36 \cdot in$

The required spacing of mild steel reinforcement in the layer closest to the tension face shall not be less than the reinforcement spacing provided due to the service limit state moment.

$LRFD_{5.7.3.4} :=$ | "OK, crack control for Mx is satisfied" if $s_{required} \geq ybar_{spa}$
| "NG, crack control for Mx not satisfied, provide more reinforcement" otherwise

$LRFD_{5.7.3.4} =$ "OK, crack control for Mx is satisfied"

B4. Longitudinal Flexural Design [LRFD 5.7.3.2]

Factored resistance............................... $M_r = \phi \cdot M_n$

Using variables defined in this example,

Factored resistance......................... $M_r := My_{Strength1} = 278.3 \cdot kip \cdot ft$

Width of section............................ $b := b_{Ftg} = 12\,ft$

Initial assumption for area of steel required

Number of bars................................ $n_{xbar} := 12$

Size of bar...................................... $xbar := \text{"6"}$

Note: if bar spacing is "-1", the spacing is less than 3", and a bigger bar size should be selected.

Bar area... $A_{bar} = 0.440 \cdot in^2$

Bar diameter.................................... $xbar_{dia} = 0.750 \cdot in$

Equivalent bar spacing...................... $xbar_{spa} = 12.4 \cdot in$

Area of steel provided............................ $A_s := n_{xbar} \cdot A_{bar} = 5.28 \cdot in^2$

Distance from extreme compressive fiber to centroid of reinforcing steel................

$$d_s := h_{Ftg} - cover_{sub} - Pile_{embed} - ybar_{dia} - \frac{xbar_{dia}}{2} = 31.5 \cdot in$$

Assume $f_s := f_y$ [LRFD 5.7.2.1]

Stress block factor................................

$$\beta_1 := \min\left[\left[\begin{array}{c} 0.85 \\ \max\left[\left[\begin{array}{c} 0.85 - 0.05 \cdot \left(\dfrac{f_{c.sub} - 4000psi}{1000psi}\right) \\ 0.65 \end{array}\right]\right] \end{array}\right]\right] = 0.78$$

Distance between the neutral axis and compressive face..................................

$$c := \frac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot \beta_1 \cdot b} = 0.61 \cdot in$$

Solve the quadratic equation for the area of steel required..

$$\text{Given} \quad M_r = \phi \cdot A_s \cdot f_s \cdot \left[d_s - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot b}\right)\right]$$

Area of steel required............................. $A_{s.reqd} := \text{Find}(A_s) = 1.97 \cdot in^2$

Check assumption that $f_s = f_y$ ....

$$Check\_f_s := \text{if}\left(\frac{c}{d_s} < 0.6, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$$

The area of steel provided, $A_s = 5.28 \cdot in^2$, should be greater than the area of steel required, $A_{s.reqd} = 1.97 \cdot in^2$. If not, decrease the spacing of the reinforcement. Once A_s is greater than $A_{s.reqd}$, the proposed reinforcing is adequate for the applied moments.

Moment capacity provided......................

$$M_{r.long} := \phi \cdot A_s \cdot f_s \cdot \left[d_s - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot b}\right)\right]$$

$M_{r.long} = 742.8 \cdot ft \cdot kip$

B5. Longitudinal Limits for Reinforcement [LRFD 5.7.3.3]

Minimum Reinforcement

The minimum reinforcement requirements ensure the moment capacity provided is at least equal to the lesser of the cracking moment and 1.33 times the factored moment required by the applicable strength load combinations.

Modulus of rupture................................ $f_r = -562.8 \cdot psi$

Section modulus of the footing above piles $S := \dfrac{b_{Ftg} \cdot \left(h_{Ftg} - Pile_{embed}\right)^2}{6} = 31104 \cdot in^3$

Flexural cracking variability factor........... $\gamma_1 := 1.6$

Ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement....................................... $\gamma_3 := 0.67$ *(for ASTM A615, Grade 60 reinforcing steel, as required by SDG 1.4.1.C)*

Cracking moment.................................. $M_{cr} := f_r \cdot S \cdot \gamma_1 \cdot \gamma_3 = -1563.9 \cdot kip \cdot ft$

Required flexural resistance..................... $M_{r.reqd} := \min\left(\left|M_{cr}\right|, 133 \cdot \% \cdot M_r\right) = 370.2 \cdot kip \cdot ft$

Check that the capacity provided, $M_{r.long} = 742.8 \cdot ft \cdot kip$, exceeds minimum requirements, $M_{r.reqd} = 370.2 \cdot ft \cdot kip$.

$$LRFD_{5.7.3.3.2} := \begin{cases} \text{"OK, minimum reinforcement for longitudinal moment is satisfied"} & \text{if } M_{r.long} \geq M_{r.reqd} \\ \text{"NG, reinforcement for longitudinal moment is less than minimum"} & \text{otherwise} \end{cases}$$

$LRFD_{5.7.3.3.2} =$ "OK, minimum reinforcement for longitudinal moment is satisfied"

B6. Longitudinal Crack Control by Distribution Reinforcement [LRFD 5.7.3.4]

The maximum spacing of the mild steel reinforcement for control of cracking at the service limit state shall satisfy............ $s \leq \dfrac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$

where $\beta_s = 1 + \dfrac{d_c}{0.7\left(h - Pile_{embed} - d_c\right)}$

Exposure factor for Class 1 exposure condition... $\gamma_e := 1.00$ **[SDG 3.10]**

Distance from extreme tension fiber to center of closest bar..............................

$$d_c := cover_{sub} + ybar_{dia} + \frac{xbar_{dia}}{2} = 4.5 \cdot in$$

$$\beta_s := 1 + \frac{d_c}{0.7\left(h_{Ftg} - Pile_{embed} - d_c\right)} = 1.2$$

The neutral axis of the section must be determined to determine the actual stress in the reinforcement. This process is iterative, so an initial assumption of the neutral axis must be made.

Guess value $x := 6 \cdot in$

Given $$\frac{1}{2} \cdot b \cdot x^2 = \frac{E_s}{E_{c.sub}} \cdot A_s \cdot \left(d_s - x\right)$$

$$x_{na} := Find(x) = 3.91 \cdot in$$

Tensile force in the reinforcing steel due to service limit state moment......................

$$T_s := \frac{My_{Service1}}{d_s - \frac{x_{na}}{3}} = 79.7 \cdot kip$$

Actual stress in the reinforcing steel due to service limit state moment......................

$$f_{s.actual} := \frac{T_s}{A_s} = 15.09 \cdot ksi$$

Required reinforcement spacing..............

$$s_{required} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{s.actual}} - 2 \cdot d_c = 29.5 \cdot in$$

Provided reinforcement spacing..............

$$xbar_{spa} = 12.41 \cdot in$$

The required spacing of mild steel reinforcement in the layer closest to the tension face shall not be less than the reinforcement spacing provided due to the service limit state moment.

$$LRFD_{5.7.3.4} := \begin{cases} \text{"OK, crack control for My is satisfied"} & \text{if } s_{required} \geq xbar_{spa} \\ \text{"NG, crack control for My not satisfied, provide more reinforcement"} & \text{otherwise} \end{cases}$$

$LRFD_{5.7.3.4}$ = "OK, crack control for My is satisfied"

B7. Shrinkage and Temperature Reinforcement [LRFD 5.10.8]

Initial assumption for area of steel required

Size of bar...................................... $bar_{st} := "6"$

Spacing of bar................................ $bar_{spa.st} := 12 \cdot in$

Bar area... $A_{bar} = 0.44 \cdot in^2$

Bar diameter.................................. $dia = 0.750 \cdot in$

Minimum area of shrinkage and temperature reinforcement...........

$$A_{shrink.temp} := \min\left(\begin{bmatrix} 0.6 \dfrac{in^2}{ft} \\ \max\left(\begin{bmatrix} 0.11 \dfrac{in^2}{ft} \\ \dfrac{1.3 \cdot b_{Ftg} \cdot h_{Ftg} \cdot \dfrac{kip}{in \cdot ft}}{2 \cdot (b_{Ftg} + h_{Ftg}) \cdot f_y} \end{bmatrix}\right) \end{bmatrix}\right) = 0.39 \cdot \frac{in^2}{ft}$$

Maximum spacing of shrinkage and temperature reinforcement

$$spacing_{shrink.temp} := \begin{cases} \min\left(\dfrac{A_{bar}}{A_{shrink.temp}}, 12in\right) & \text{if } \min(b_{Ftg}, L_{Ftg}, h_{Ftg}) > 18in \\ \min\left(\dfrac{A_{bar}}{A_{shrink.temp}}, 18in, 3 \cdot \min(b_{Ftg}, L_{Ftg}, h_{Ftg})\right) & \text{otherwise} \end{cases} = 12 \cdot in$$

The bar spacing should be less than the maximum spacing for shrinkage and temperature reinforcement

$$LRFD_{5.10.8} := \begin{cases} \text{"OK, minimum shrinkage and temperature requirements"} & \text{if } bar_{spa.st} \le spacing_{shrink.temp} \\ \text{"NG, minimum shrinkage and temperature requirements"} & \text{otherwise} \end{cases}$$

$LRFD_{5.10.8} = $ "OK, minimum shrinkage and temperature requirements"

B8. Mass Concrete Provisions

Volume to surface area ratio for footing....

$$Ratio_{VS} := \frac{b_{Ftg} \cdot h_{Ftg} \cdot L_{Ftg}}{2 \cdot b_{Ftg} \cdot L_{Ftg} + (2b_{Ftg} + 2L_{Ftg}) \cdot h_{Ftg}} = 1.07\,ft$$

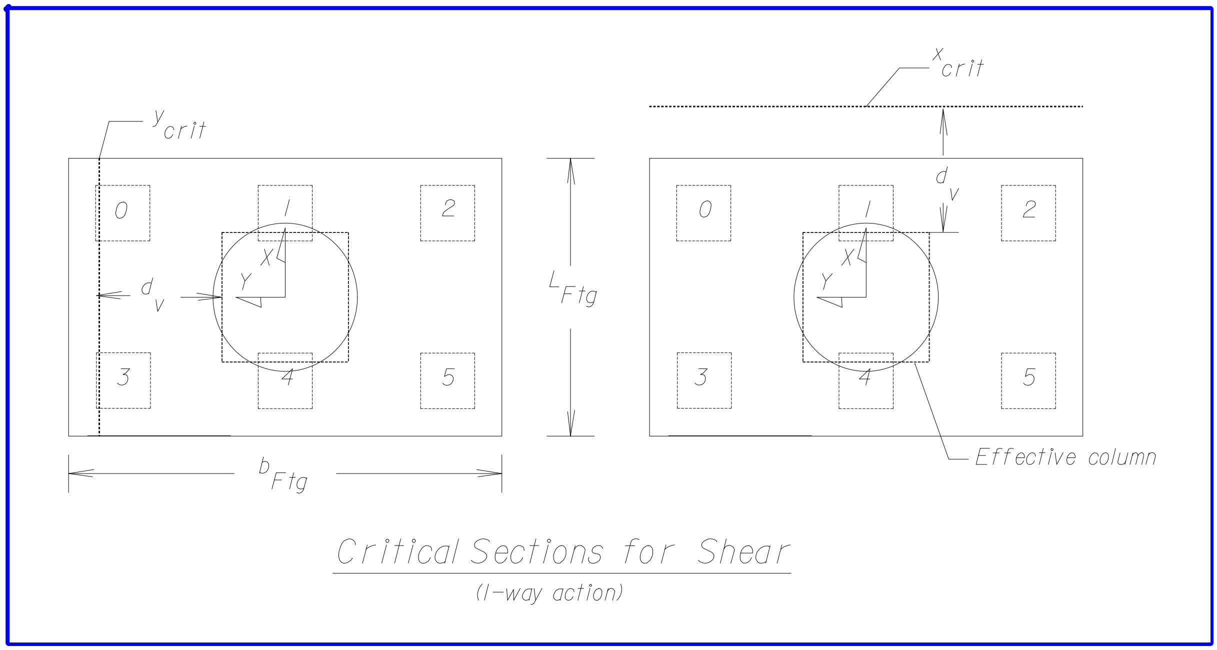
Mass concrete provisions apply if the volume to surface area ratio exceeds 1 ft and the minimum dimension exceeds 3 feet.

$$SDG_{3.9} := \begin{cases} \text{"Use mass concrete provisions"} & \text{if } Ratio_{VS} > 1.0 \cdot ft \wedge (b_{Ftg} > 3ft \wedge h_{Ftg} > 3ft \wedge L_{Ftg} > 3ft) \\ \text{"Use regular concrete provisions"} & \text{otherwise} \end{cases}$$

$SDG_{3.9} = $ "Use mass concrete provisions"

C. Shear Design Parameters [LRFD 5.13.3.6]

C1. Shear Design Parameters - One Way Shear



Distance from extreme compression fiber to centroid of tension steel (use the top of the main transverse steel or bottom of the longitudinal steel)................................. $d_e := h_{Ftg} - Pile_{embed} - cover_{sub} - ybar_{dia} = 2.66\,ft$

Effective shear depth **[LRFD 5.8.2.9]**...... $d_v = \max\left(\begin{pmatrix} 0.9 \cdot d_e \\ 0.72 \cdot h \end{pmatrix}\right)$

Using variables defined in this example, $d_v := \max\left[\begin{bmatrix} 0.9 \cdot d_e \\ 0.72 \cdot (h_{Ftg} - Pile_{embed}) \end{bmatrix}\right] = 2.39\,ft$

C2. Shear Resistance - Simplified Procedure [LRFD 5.8.3.4.1]

Values of B = 2 and θ = 45 deg may be assumed for concrete footings in which the distance from point of zero shear to the face of the column, pier or wall is less than $3d_v$, with or without transverse reinforcement.

$Check_{5.8.3.4.1} := if(y_{crit} < 3 \cdot d_v, \text{"OK, Simplified procedure may be used."}, \text{"NG, Use general procedure."})$

$Check_{5.8.3.4.1} = \text{"OK, Simplified procedure may be used."}$

Factor indicating ability of diagonally cracked concrete to transmit tension.. $\beta := 2.0$

Angle of inclination for diagonal compressive stresses....................... $\theta := 45deg$

C3. One Way Shear - Y Critical Section

Factored pile loads contributing to transverse shear ....

$Vu_T := \max\left(Q_{u_0} + Q_{u_3}, Q_{u_2} + Q_{u_5}\right) = 349.5 \cdot kip$

Distance between face of equivalent square column and face of pile.................

$dy_{face} := \frac{\left(b_{Ftg} - b_{Col.eff}\right)}{2} - \left(\frac{Pile_{size}}{2} + pile_{edge}\right) = 2\,ft$

The location of the piles relative to the critical shear plane determines the amount of shear design. According to LRFD 5.13.3.6.1, if a portion of the pile lies inside the critical section, the pile load shall be uniformly distributed over the pile width, and the portion of the load outside the critical section shall be included in shear calculations for the critical section.

$$Transverse_{shear} := \begin{cases} \text{"Full shear, piles are outside of the y-critical shear plane"} & \text{if } dy_{face} \geq d_v \\ \text{"No shear, Y-critical shear plane is outside pile dimension"} & \text{if } d_v \geq \left(dy_{face} + Pile_{size}\right) \\ \text{"Partial shear, piles intersect y-critical shear plane"} & \text{if } dy_{face} < d_v \wedge d_v < \left(dy_{face} + Pile_{size}\right) \end{cases}$$

$Transverse_{shear} = \text{"Partial shear, piles intersect y-critical shear plane"}$

If the piles partially intersect the shear plane, the shear for the critical section can be linearly reduced by the following factor.

$$\psi_y := \begin{cases} 1 & \text{if } dy_{face} \geq d_v \\ 0 & \text{if } d_v \geq \left(dy_{face} + Pile_{size}\right) \\ \left| 1 - \frac{d_v - dy_{face}}{Pile_{size}} \right| & \text{if } dy_{face} < d_v \wedge d_v < \left(dy_{face} + Pile_{size}\right) \end{cases} = 0.74$$

Factored shear along transverse y-critical section..

$Vu_T := \psi_y \cdot Vu_T = 258.5 \cdot kip$

The nominal shear resistance for footings with no prestressing or transverse reinforcing is the minimum of the following equations...............................

$V_n = 0.0316 \cdot \beta \cdot \sqrt{f'_c} \cdot b_v \cdot d_v$ or $V_n = 0.25 \cdot f'_c \cdot b_v \cdot d_v$

Using variables defined in this example, $b_v := L_{Ftg} = 7.5\,ft$

and the corresponding shear values..... $V_{c1} := 0.0316 \cdot \beta \cdot \sqrt{f_{c.sub} \cdot ksi} \cdot b_v \cdot d_v = 382.6 \cdot kip$

$V_{c2} := 0.25 \cdot f_{c.sub} \cdot b_v \cdot d_v = 3549.7 \cdot kip$

Nominal shear resistance........................ $V_n := \min(V_{c1}, V_{c2}) = 382.6 \cdot kip$

Check the section has adequate shear capacity

$$LRFD_{5.8.3.3} := \begin{cases} \text{"OK, footing depth for Y-critical section is adequate for 1-way shear"} & \text{if } V_n \geq \dfrac{Vu_T}{\phi_v} \\ \text{"NG, footing depth for Y-critical section is not adequate for 1-way shear"} & \text{otherwise} \end{cases}$$

$LRFD_{5.8.3.3} =$ "OK, footing depth for Y-critical section is adequate for 1-way shear"

C4. One Way Shear - X Critical Section

Factored pile loads contributing to longitudinal shear.................................

$$Vu_L := \max\left(Q_{u_0} + Q_{u_1} + Q_{u_2}, Q_{u_3} + Q_{u_4} + Q_{u_5}\right) = 592.6 \cdot kip$$

Distance between face of equivalent square column and face of pile..........................

$$dx_{face} := \frac{\left(L_{Ftg} - b_{Col.eff}\right)}{2} - \left(\frac{Pile_{size}}{2} + pile_{edge}\right) = -0.25\ ft$$

The location of the piles relative to the critical shear plane determines the amount of shear design.

$$Longitudinal_{shear} := \begin{cases} \text{"Full shear, piles are outside of the x-critical shear plane"} & \text{if } dx_{face} \ge d_v \\ \text{"No shear, X-critical shear plane is outside pile dimension"} & \text{if } d_v \ge \left(dx_{face} + Pile_{size}\right) \\ \text{"Partial shear, piles intersect x-critical shear plane"} & \text{if } dx_{face} < d_v \wedge d_v < \left(dx_{face} + Pile_{size}\right) \end{cases}$$

$$Longitudinal_{shear} = \text{"No shear, X-critical shear plane is outside pile dimension"}$$

If the piles partially intersect the shear plane, the shear affecting the critical section can be linearly reduced by the following factor

$$\psi_x := \begin{cases} 1 & \text{if } dx_{face} \ge d_v \\ 0 & \text{if } d_v \ge \left(dx_{face} + Pile_{size}\right) \\ \left|1 - \dfrac{d_v - dx_{face}}{Pile_{size}}\right| & \text{if } dx_{face} < d_v \wedge d_v < \left(dx_{face} + Pile_{size}\right) \end{cases} = 0$$

Factored shear along longitudinal x-critical section..

$$Vu_L := \psi_x \cdot Vu_L = 0 \cdot kip$$

The nominal shear resistance for footings with no prestressing or transverse reinforcing is the minimum of the following equations..

$$V_n = 0.0316 \cdot \beta \cdot \sqrt{f'_c} \cdot b_v \cdot d_v \qquad \text{OR} \qquad V_n = 0.25 \cdot f'_c \cdot b_v \cdot d_v$$

Using variables defined in this example, $b_v := b_{Ftg} = 12\ ft$

and the corresponding shear values......

$$V_{c1} := 0.0316 \cdot \beta \cdot \sqrt{f_{c.sub} \cdot ksi} \cdot b_v \cdot d_v = 612.2 \cdot kip$$

$$V_{c2} := 0.25 \cdot f_{c.sub} \cdot b_v \cdot d_v = 5679.6 \cdot kip$$

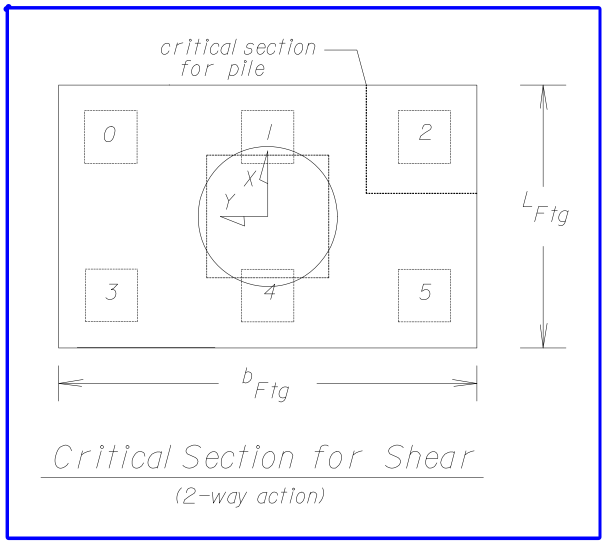
Nominal shear resistance......................... $V_n := \min(V_{c1}, V_{c2}) = 612.23 \cdot kip$

Check the section has adequate shear capacity

$$LRFD_{5.8.3.3} := \begin{cases} \text{"OK, X-critical section footing depth is adequate for 1-way shear"} & \text{if } V_n \ge \dfrac{Vu_L}{\phi_v} \\ \text{"NG, X-critical section footing depth is NO GOOD for 1-way shear"} & \text{otherwise} \end{cases}$$

$LRFD_{5.8.3.3}$ = "OK, X-critical section footing depth is adequate for 1-way shear"

C5. Two Way Shear Design (Punching Shear)



Critical section for 2-way shear **[LRFD 5.13.3.6]**.. $d_{v2} := 0.5 \cdot d_v = 1.2\,ft$

Maximum shear force for pile 2.............. $Vu_{pile} := Q_{max} = 256.9 \cdot kip$

Nominal shear resistance for 2-way action in sections without shear reinforcement.... $V_n = \left(0.063 + \frac{0.126}{\beta_c}\right) \cdot \left(\sqrt{f_c} \cdot b_o \cdot d_v\right) \le 0.126 \sqrt{f_c} \cdot b_o \cdot d_v$

Perimeter of critical section.................... $b_o := \min\left[\begin{bmatrix} 4 \cdot \left(Pile_{size} + 2 \cdot d_{v2}\right) \\ Pile_{size} + 2 \cdot \left(d_{v2} + pile_{edge}\right) \end{bmatrix}\right] = 6.89\,ft$

Ratio of long side to short side of the rectangle, which the concentrated load or reaction is transmitted

$\beta_c := 1.0$

Nominal shear resistance........................

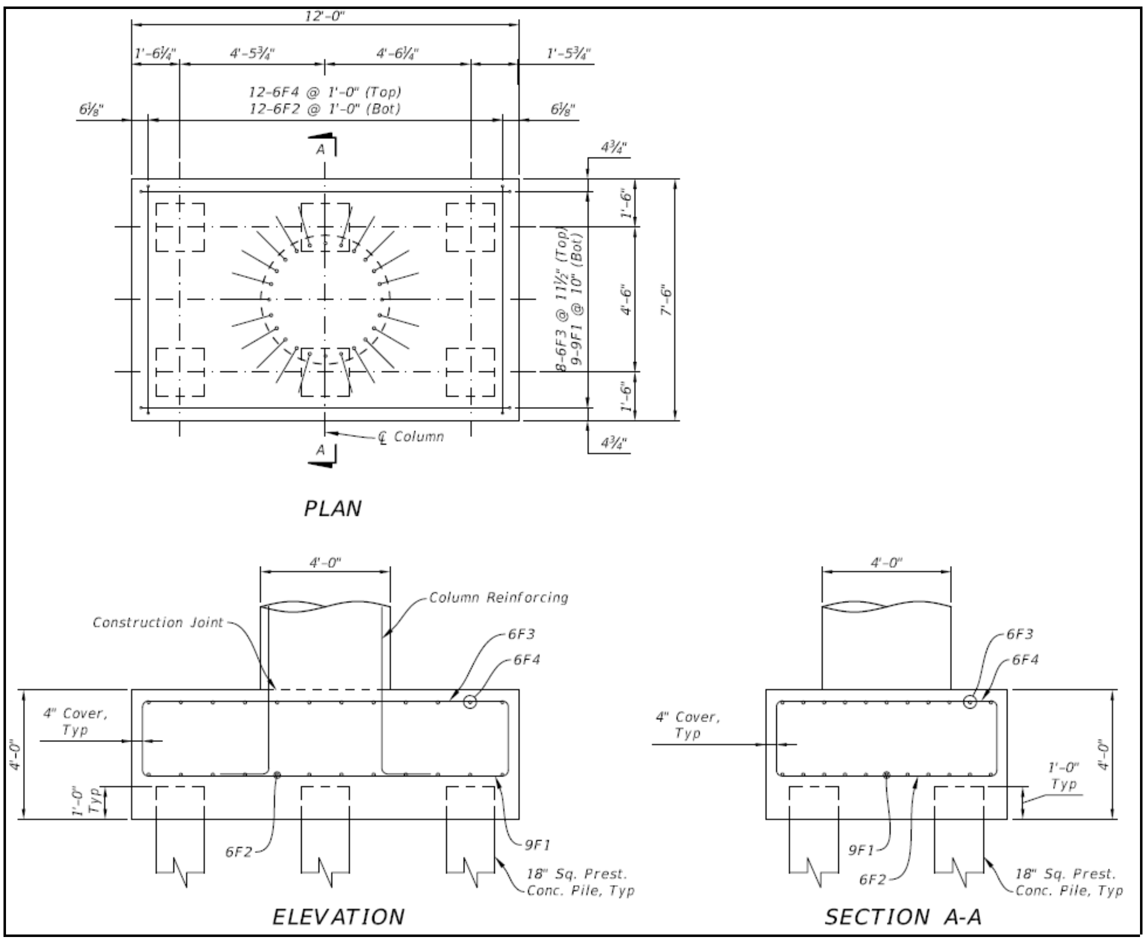
$$V_n := \min\left[\begin{bmatrix} \left(0.063 + \frac{0.126}{\beta_c}\right) \cdot \left(\sqrt{f_{c.sub} \cdot ksi} \cdot b_o \cdot d_v\right) \\ 0.126 \sqrt{f_{c.sub} \cdot ksi} \cdot b_o \cdot d_v \end{bmatrix}\right] = 700.86 \cdot kip$$

$$LRFD_{5.13.3.6.3} := \begin{cases} \text{"OK, Footing depth for 2-way pile punching shear"} & \text{if } V_n > \frac{Vu_{pile}}{\phi_v} \\ \text{"NG, Footing depth for 2-way pile punching shear"} & \text{otherwise} \end{cases}$$

$LRFD_{5.13.3.6.3} =$ "OK, Footing depth for 2-way pile punching shear"

D. Design Summary

Footing properties	Transverse dimension of footing........	$b_{Ftg} = 12 \cdot ft$	
	Longitudinal dimension of footing......	$L_{Ftg} = 7.5 \cdot ft$	
	Depth of footing..............................	$h_{Ftg} = 4 \cdot ft$	
Bottom reinforcement (transverse)	Number of bars..............................	$n_{ybar} = 9$	
	Selected bar size.............................	$ybar = "9"$	
	Approximate spacing.......................	$ybar_{spa} = 10.4 \cdot in$	**use 10" +/-spacing**
Bottom reinforcement (longitudinal)	Number of bars..............................	$n_{xbar} = 12$	
	Selected bar size.............................	$xbar = "6"$	
	Approximate spacing.......................	$xbar_{spa} = 12.4 \cdot in$	**use 12" +/-spacing**
Temp and shrinkage (top and side)	Selected bar size.............................	$bar_{st} = "6"$	
	Spacing...	$bar_{spa.st} = 12 \cdot in$	



Redefine Variables

SUBSTRUCTURE DESIGN

End Bent Live Load Analysis

References

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\309PierFootingDesign.xmcd(R)

Description

This document provides the criteria for the end bent live load design. Since the piles are placed directly under the beams at the end bent, no positive or negative moment due to live load is introduced in the end bent cap, therefore, the maximum live load placement will try to maximize a beam reaction or pile load.

Page Contents

A. Input Variables

A1. Shear: Skewed Modification Factor [LRFD 4.6.2.2.3c]

Skew modification factor for shear **shall** be applied to the exterior beam at the obtuse corner (θ>90 deg) and to all beams in a multibeam bridge, whereas $g_{v.Skew} = 1.06$.

A2. Maximum Live Load Reaction at End Bent - One HL-93 Vehicle

Since each beam is directly over each pile, live load will not contribute to any moments or shears in the bent cap. For the pile design, the live load will not include dynamic amplification since the piles are considered to be in the ground.

Reaction induced by HL-93 truck load...... $V_{truck}(Support) = 64.34 \cdot kip$

Reaction induced by lane load.................. $V_{lane}(Support) = 28.1 \cdot kip$

Impact factor....................................... $IM = 1.33$

The truck reaction (including impact and skew modification factors) is applied on the deck as two wheel-line loads.............

$$wheel_{line} := \left(\frac{V_{truck}(Support)}{2}\right) \cdot g_{v.Skew} \cdot IM = 45.5 \cdot kip$$

The lane load reaction (including skew modification factor) is applied on the deck as two wheel-line loads...........................

$$lane_{load} := \left(\frac{V_{lane}(Support)}{2}\right) \cdot g_{v.Skew} = 14.9 \cdot kip$$

HL-93 Line Load.................................... $HL93 := wheel_{line} + lane_{load} = 60.4 \cdot kip$

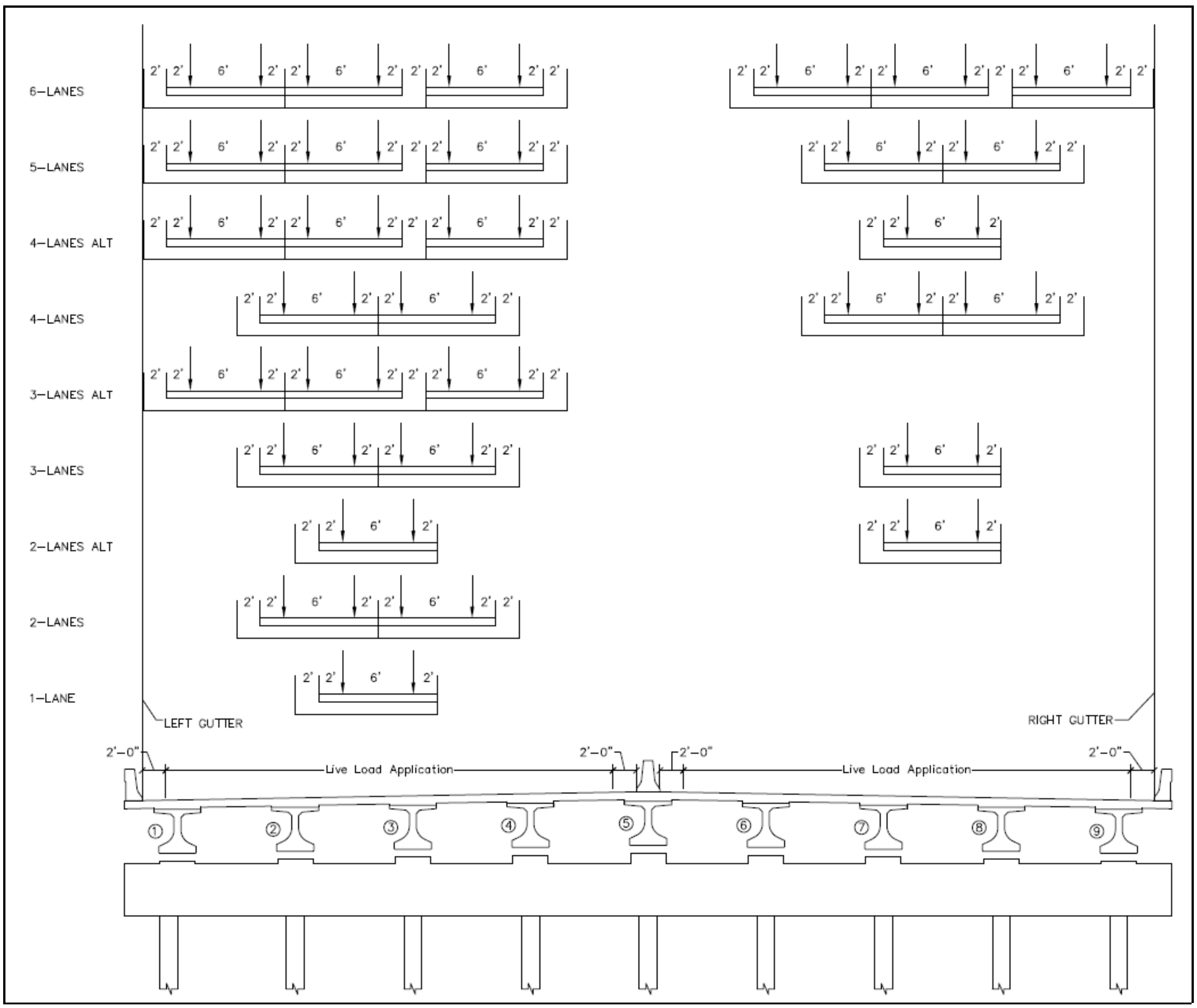
The HL-93 line load can be placed within 2'-0" of the overhang and median barriers.

A3. HL-93 Vehicle Placement

HL-93 vehicles, comprising of wheel line loads and lane loads, should be placed on the deck to maximize the axial force in the end bent piles and moments in the pier cap. Note that for maximum effects, live load may be placed on both sides of the roadway. Utilizing our engineering judgement, it is possible to have up to six lanes of HL-93 vehicles at a single time. However, note that for the calculation of braking forces, vehicles in only one roadway were applied since the braking forces would be counterproductive, in opposite directions.

Depending on the number of design lanes, a multiple presence factor (LRFD Table 3.6.1.1.2-1) is applied to the HL-93 wheel line loads and lane load.

$$\text{Lanes} := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix} \qquad \text{MPF} := \begin{pmatrix} 1.2 \\ 1.0 \\ 0.85 \\ 0.65 \\ 0.65 \\ 0.65 \end{pmatrix} \qquad \text{HL93\_Line\_Load} := \text{HL93} \cdot \text{MPF} \qquad \text{HL93\_Line\_Load} = \begin{pmatrix} 72.5 \\ 60.4 \\ 51.4 \\ 39.3 \\ 39.3 \\ 39.3 \end{pmatrix} \cdot \text{kip}$$



SUBSTRUCTURE DESIGN

End Bent Design Loads

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\310EndBentLiveLoads.xmcd(R)

Description

This section provides the design parameters necessary for the substructure end bent design. The loads calculated in this file are only from the superstructure. Substructure self-weight, wind on substructure and uniform temperature on substructure can be generated by the substructure analysis model/program chosen by the user.

For this design example, RISA was chosen as the analysis model/program.

Page Contents

A. General Criteria

A1. Bearing Design Movement/Strain

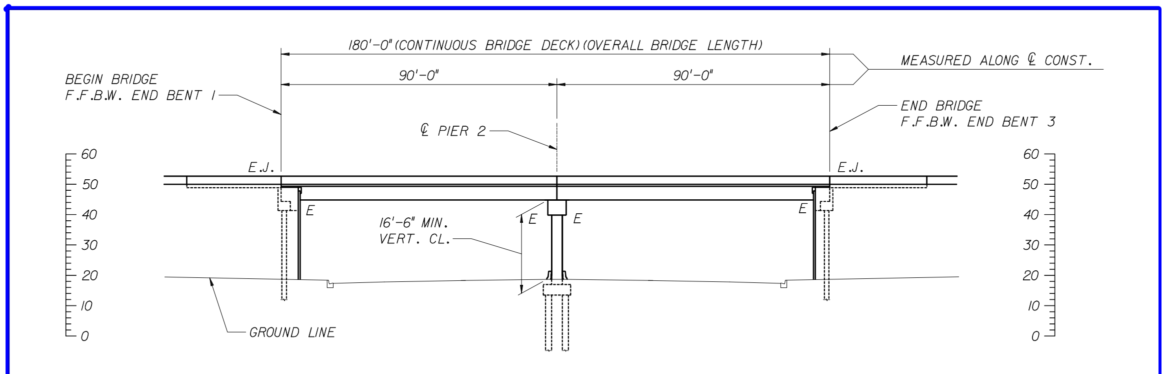
Strain due to temperature, creep and shrinkage.. $\varepsilon_{CST} = 0.00029$ *(**Note**: See Sect. 2.09.B4 - Bearing Design Movement/Strain)*

A2. End Bent Dead Load Summary

Unfactored beam reactions at the end bent for DC and DW loads

UNFACTORED BEAM REACTIONS AT END BENTS						
	DC Loads (kip)			DW Loads (kip)		
Beam	**x**	**y**	**z**	**x**	**y**	**z**
1	0.0	-95.0	0.0	0.0	0.0	0.0
2	0.0	-99.9	0.0	0.0	0.0	0.0
3	0.0	-99.9	0.0	0.0	0.0	0.0
4	0.0	-99.9	0.0	0.0	0.0	0.0
5	0.0	-99.9	0.0	0.0	0.0	0.0
6	0.0	-99.9	0.0	0.0	0.0	0.0
7	0.0	-99.9	0.0	0.0	0.0	0.0
8	0.0	-99.9	0.0	0.0	0.0	0.0
9	0.0	-95.0	0.0	0.0	0.0	0.0

A3. Center of Movement



ELEVATION

By inspection, the center of movement will be the intermediate pier.

$L_0 := L_{span}$

$L_0 = 90.0\,\text{ft}$

B. Lateral Load Analysis

B1. Centrifugal Force: CE [LRFD 3.6.3]

Since the design speed is not specified, it will be conservatively taken as the maximum specified in the AASHTO publication, A Policy on Geometric Design of Highways and Streets.

Design Speed....................................... $V_{design} := 70mph$

Factor per LRFD 3.6.3........................... $f := \frac{4}{3}$

Horizontal radius.................................... $R := 3800ft$

Centrifugal factor.................................. $C := \frac{f \cdot V_{design}^2}{g \cdot R} = 0.11$

Centrifugal force.................................... $P_c := C \cdot wheel_{line} = 5.23 \cdot kip$

$HL93 + P_c = 65.68 \cdot kip$

$HL93 - P_c = 55.22 \cdot kip$

B2. Braking Force: BR [LRFD 3.6.4]

The braking force should be taken as the greater of:

25% of axle weight for design truck / tandem

5% of design truck / tandem and lane

The number of lanes for braking force calculations depends on future expectations of the bridge. For this example, the bridge is not expected to become one-directional in the future, and future widening is expected to occur to the outside. From this information, the number of lanes is

$N_{lanes} = 3$

The multiple presence factor (LRFD Table 3.6.1.1.2-1) should be taken into account..

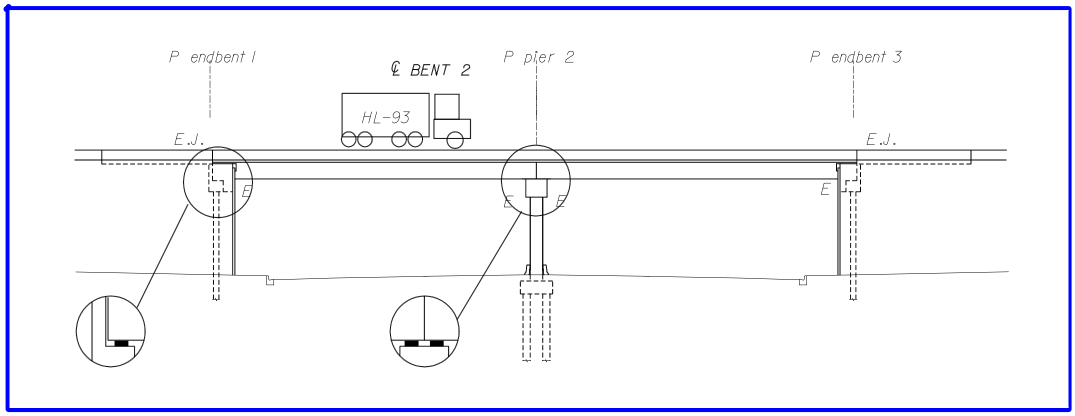
$$MPF := \begin{vmatrix} 1.2 \text{ if } N_{lanes} = 1 \\ 1.0 \text{ if } N_{lanes} = 2 \\ 0.85 \text{ if } N_{lanes} = 3 \\ 0.65 \text{ otherwise} \end{vmatrix} = 0.85$$

Braking force as 25% of axle weight for design truck / tandem............................. $BR_{Force.1} := 25\% \cdot (72 \cdot kip) \cdot N_{lanes} \cdot MPF = 45.9 \cdot kip$

Braking force as 5% of axle weight for design truck / tandem and lane................. $BR_{Force.2} := 5\% \cdot (72 \cdot kip + w_L \cdot L_{span}) \cdot N_{lanes} \cdot MPF = 16.52 \cdot kip$

Governing braking force......................... $BR_{Force} := \max(BR_{Force.1}, BR_{Force.2}) = 45.9 \cdot kip$

Distribution of Braking Forces to End Bent



The same bearing pads are provided at the pier and end bent to distribute the braking forces. The braking force transferred to the pier or end bents is a function of the bearing pad and pier column stiffnesses. For this example, (1) the pier column stiffnesses are ignored, (2) the deck is continuous over pier 2 and expansion joints are provided only at the end bents.

Braking force at End Bent....................... $BR_{Endbent} = BR_{Force} \cdot (K_{Endbent})$

where.. $K_{Endbent} = \dfrac{N_{pads.endbent} \cdot K_{pad}}{\sum (N_{pads.pier} + N_{pads.endbent}) \cdot K_{pad}}$

Simplifying and using variables defined in this example,

end bent stiffness can be calculated as $K_{Endbent} := \dfrac{N_{beams}}{(1 + 2 + 1) \cdot N_{beams}} = 0.25$

corresponding braking force.............. $BR_{Endbent} := BR_{Force} \cdot (K_{Endbent}) = 11.47 \cdot kip$

Since the bridge superstructure is very stiff in the longitudinal direction, the braking forces are assumed to be equally distributed to the beams under the respective roadway.

$beams := 5$

Braking force at end bent per beam.......... $BR_{Endbent} := \dfrac{BR_{Endbent}}{beams} = 2.3 \cdot kip$

Adjustments for Skew

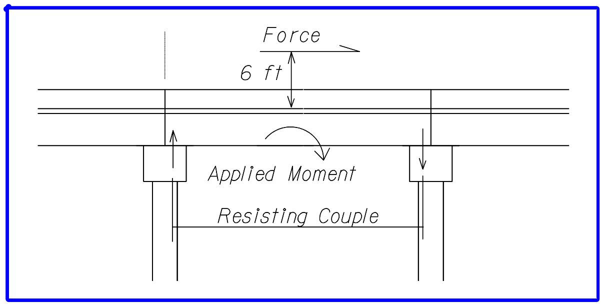
The braking force is transferred to the pier by the bearing pads. The braking forces need to be resolved along the axis of the bearing pads for design of the pier substructure.

Braking force perpendicular (z-direction) to the skew.. $BR_{z.Endbent} := BR_{Endbent} \cdot \cos(Skew) = 2.16 \cdot kip$

Braking force parallel (x-direction) to the skew... $BR_{x.Endbent} := BR_{Endbent} \cdot \sin(Skew) = -0.78 \cdot kip$

Adjustments for Braking Force Loads Applied 6' above Deck

The longitudinal moment induced by braking forces over a pier is resisted by the moment arm. Conservatively, assume the braking occurs over one span only, then the result is an uplift reaction on the downstation end bent or pier and a downward reaction at the upstation end bent or pier. In this example, the braking is assumed to occur in span 1 and the eccentricity of the downward load with the bearing and centerline of pier eccentricities is ignored.



Moment arm from top of bearing pad to location of applied load........................... $M_{arm} := 6ft + h = 9.75\,ft$

Braking force in end bent (y-direction), vertical.. $BR_{y.Endbent} := \dfrac{-BR_{Endbent} \cdot M_{arm}}{L_{span}} = -0.25 \cdot kip$

Only the downward component of this force is considered. Typically, the vertical forces (uplift) are small and can be ignored.

BRAKING FORCES AT END BENT			
	BR Loads (kip)		
Beam	x	y	z
1	-0.8	-0.2	2.2
2	-0.8	-0.2	2.2
3	-0.8	-0.2	2.2
4	-0.8	-0.2	2.2
5	-0.8	-0.2	2.2
6	0.0	0.0	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0

B3. Creep, Shrinkage, and Temperature Forces

The forces transferred from the superstructure to the substructure due to temperature, creep, and shrinkage are influenced by the shear displacements in the bearing pad. In this example, only temperature and shrinkage effects are considered. Creep is ignored, since this example assumes the beams will creep towards their center and the composite deck will offer some restraint.

$$\varepsilon_{CST} = 0.00029$$

Displacements at top of end bent due to temperature, creep, and shrinkage...........

$$\Delta_{Endbent1} := \left(L_0 - x_{dist_0}\right)\cdot\varepsilon_{CST} = 0.31\cdot in$$

Shear force transferred through each bearing pad due to creep, shrinkage, and temperature..

$$CST_{Endbent} := \frac{G_{bp}\cdot L_{pad}\cdot W_{pad}\cdot\Delta_{Endbent1}}{h_{pad}} = 3.87\cdot kip$$

This force needs to be resolved along the direction of the skew

Shear force perpendicular (z-direction) to the end bent per beam............................

$$CST_{z.Endbent} := CST_{Endbent}\cdot\cos(Skew) = 3.64\cdot kip$$

Shear force parallel (x-direction) to the end bent per beam......................................

$$CST_{x.Endbent} := CST_{Endbent}\cdot\sin(Skew) = -1.32\cdot kip$$

Summary of beam reactions at the end bent due to creep, shrinkage, and temperature

CREEP, SHRINKAGE, TEMPERATURE FORCES AT END BENT			
	CR, SH, TU Loads (kip)		
Beam	x	y	z
1	-1.3	0.0	3.6
2	-1.3	0.0	3.6
3	-1.3	0.0	3.6
4	-1.3	0.0	3.6
5	-1.3	0.0	3.6
6	-1.3	0.0	3.6
7	-1.3	0.0	3.6
8	-1.3	0.0	3.6
9	-1.3	0.0	3.6

B4. Wind Pressure on Structure: WS

The wind loads are applied to the superstructure and substructure.

Loads from Superstructure [SDG 2.4], Strength III and Service IV Limit States

The wind pressure on the superstructure consists of lateral (x-direction) and longitudinal (z-direction) components.

Height above ground that the wind pressure is applied.. $z_{sup} = 20.5\,ft$

Design wind pressure .... $P_{z.super} := P_{z.sup.StrIII.ServIV} \cdot ksf = 48.83 \cdot psf$

For prestressed beam bridges, the following wind pressures factors are given to account for the angle of attack [SDG Table 2.4.1-3]

$$Wind_{skew} := \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{Factor} := \begin{array}{cc} \underline{x} & \underline{z} \\ \multicolumn{2}{c}{\begin{pmatrix} 1.0 & 0.0 \\ 0.88 & 0.12 \\ 0.82 & 0.24 \\ 0.66 & 0.32 \\ 0.34 & 0.38 \end{pmatrix}} \end{array} \quad [\text{Global}]$$

Wind pressures based on angle of attack are as follows......... .... $Wind_{super} := P_{z.super} \cdot Wind_{Factor}$

$$Wind_{skew} := \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{super} = \begin{array}{cc} \underline{x} & \underline{z} \\ \multicolumn{2}{c}{\begin{pmatrix} 48.83 & 0 \\ 42.97 & 5.86 \\ 40.04 & 11.72 \\ 32.23 & 15.63 \\ 16.6 & 18.56 \end{pmatrix}} \end{array} \cdot psf \quad [\text{Global}]$$

The exposed superstructure area influences the wind forces that are transferred to the supporting substructure. Tributary areas are used to determine the exposed superstructure area.

Exposed superstructure area at end bent... $A_{Super} := \dfrac{L_{span}}{2} \cdot (h + 2.667\,ft) = 288.76\,ft^2$

Forces due to wind applied to the superstructure....................................... $WS_{Super.Endbent} := Wind_{super} \cdot A_{Super}$

$$WS_{Super.Endbent} = \begin{array}{cc} \underline{x} & \underline{z} \\ \multicolumn{2}{c}{\begin{pmatrix} 14.1 & 0.0 \\ 12.4 & 1.7 \\ 11.6 & 3.4 \\ 9.3 & 4.5 \\ 4.8 & 5.4 \end{pmatrix}} \end{array} \cdot kip \quad [\text{Global}]$$

Case 1 - Skew Angle of Wind = 0 degrees

Maximum transverse force..................... $F_{WS.x.case1} := WS_{Super.Endbent_{0,0}} = 14.1 \cdot kip$

Maximum longitudinal force................... $F_{WS.z.case1} := WS_{Super.Endbent_{0,1}} = 0 \cdot kip$

The forces due to wind need to be resolved along the direction of the skew.

Force perpendicular (z-direction) to the end bent...

$$WS_{z.Endbent.case1} := \left|F_{WS.z.case1} \cdot \cos(Skew)\right| + \left|F_{WS.x.case1} \cdot \sin(Skew)\right| = 4.82 \cdot kip$$

Force parallel (x-direction) to the end bent

$$WS_{x.Endbent.case1} := \left|F_{WS.z.case1} \cdot \sin(Skew)\right| + \left|F_{WS.x.case1} \cdot \cos(Skew)\right| = 13.25 \cdot kip$$

Case 2 - Skew Angle of Wind = 60 degrees

Maximum transverse force..................... $F_{WS.x.case2} := WS_{Super.Endbent_{4,0}} = 4.79 \cdot kip$

Maximum longitudinal force................... $F_{WS.z.case2} := WS_{Super.Endbent_{4,1}} = 5.36 \cdot kip$

The forces due to wind need to be resolved along the direction of the skew.

Force perpendicular (z-direction) to the pier..

$$WS_{z.Endbent.case2} := \left|F_{WS.z.case2} \cdot \cos(Skew)\right| + \left|F_{WS.x.case2} \cdot \sin(Skew)\right| = 6.67 \cdot kip$$

Force parallel (x-direction) to the pier.......

$$WS_{x.Endbent.case2} := \left|F_{WS.z.case2} \cdot \sin(Skew)\right| + \left|F_{WS.x.case2} \cdot \cos(Skew)\right| = 6.34 \cdot kip$$

A conservative approach is taken to minimize the analysis required. The maximum transverse and longitudinal forces are used in the following calculations.

Force perpendicular (z-direction) to the end bent

$$WS_{z.Endbent.StrIII.ServIV} := \max\left(WS_{z.Endbent.case1}, WS_{z.Endbent.case2}\right) = 6.67 \cdot kip$$

Force parallel (x-direction) to the end bent

$$WS_{x.Endbent.StrIII.ServIV} := \max\left(WS_{x.Endbent.case1}, WS_{x.Endbent.case2}\right) = 13.25 \cdot kip$$

Loads from Superstructure, Strength V and Service I Limit States:

Design wind pressure .... $P_{z.super} := P_{z.sup.StrV.ServI} \cdot ksf = 10.63 \cdot psf$

Wind pressures based on angle of attack are as follows....................................... $Wind_{super} := P_{z.super} \cdot Wind_{Factor}$

$$Wind_{skew} = \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{super} = \begin{array}{cc} \underline{x} & \underline{z} \end{array} \; [\text{Global}] \qquad Wind_{super} = \begin{pmatrix} 10.63 & 0 \\ 9.36 & 1.28 \\ 8.72 & 2.55 \\ 7.02 & 3.4 \\ 3.62 & 4.04 \end{pmatrix} \cdot psf$$

The exposed superstructure area influences the wind forces that are transferred to the supporting substructure. Tributary areas are used to determine the exposed superstructure area.

Exposed superstructure area at end bent... $A_{Super} := \dfrac{L_{span}}{2} \cdot (h + 2.667\,ft) = 288.8\,ft^2$

Forces due to wind applied to the superstructure....................................... $WS_{Super.Endbent} := Wind_{super} \cdot A_{Super}$

$$\begin{array}{cc} \underline{x} & \underline{z} \end{array} \; [\text{Global}]$$

$$WS_{Super.Endbent} = \begin{pmatrix} 3.1 & 0.0 \\ 2.7 & 0.4 \\ 2.5 & 0.7 \\ 2.0 & 1.0 \\ 1.0 & 1.2 \end{pmatrix} \cdot kip$$

Case 1 - Skew Angle of Wind = 0 degrees

Maximum transverse force...................... $F_{WS.x.case1} := WS_{Super.Endbent_{0,0}} = 3.07 \cdot kip$

Maximum longitudinal force.................... $F_{WS.z.case1} := WS_{Super.Endbent_{0,1}} = 0 \cdot kip$

The forces due to wind need to be resolved along the direction of the skew.

Force perpendicular (z-direction) to the end bent...

$$WS_{z.Endbent.case1} := \left|F_{WS.z.case1} \cdot \cos(Skew)\right| + \left|F_{WS.x.case1} \cdot \sin(Skew)\right| = 1.05 \cdot kip$$

Force parallel (x-direction) to the end bent

$$WS_{x.Endbent.case1} := \left|F_{WS.z.case1} \cdot \sin(Skew)\right| + \left|F_{WS.x.case1} \cdot \cos(Skew)\right| = 2.89 \cdot kip$$

Case 2 - Skew Angle of Wind = 60 degrees

Maximum transverse force...................... $F_{WS.x.case2} := WS_{Super.Endbent_{4,0}} = 1.04 \cdot kip$

Maximum longitudinal force.................... $F_{WS.z.case2} := WS_{Super.Endbent_{4,1}} = 1.17 \cdot kip$

The forces due to wind need to be resolved along the direction of the skew.

Force perpendicular (z-direction) to the end bent...

$$WS_{z.Endbent.case2} := \left|F_{WS.z.case2} \cdot \cos(Skew)\right| + \left|F_{WS.x.case2} \cdot \sin(Skew)\right| = 1.45 \cdot kip$$

Force parallel (x-direction) to the end bent

$$WS_{x.Endbent.case2} := \left|F_{WS.z.case2} \cdot \sin(Skew)\right| + \left|F_{WS.x.case2} \cdot \cos(Skew)\right| = 1.38 \cdot kip$$

A conservative approach is taken to minimize the analysis required. The maximum transverse and longitudinal forces are used in the following calculations.

Force perpendicular (z-direction) to the end bent

$$WS_{z.Endbent.StrV.ServI} := \max\left(WS_{z.Endbent.case1}, WS_{z.Endbent.case2}\right) = 1.45 \cdot kip$$

Force parallel (x-direction) to the end bent

$$WS_{x.Endbent.StrV.ServI} := \max\left(WS_{x.Endbent.case1}, WS_{x.Endbent.case2}\right) = 2.89 \cdot kip$$

The force due to wind acts on the full superstructure. This force needs to be resolved into the reactions in each beam. The following table(s) summarize the beam reactions due to wind.

Strength V & Service I

WIND ON STRUCTURE FORCES AT END BENT			
	WS Loads (kip)		
Beam	x	y	z
1	0.3	0.0	0.2
2	0.3	0.0	0.2
3	0.3	0.0	0.2
4	0.3	0.0	0.2
5	0.3	0.0	0.2
6	0.3	0.0	0.2
7	0.3	0.0	0.2
8	0.3	0.0	0.2
9	0.3	0.0	0.2

Strength III & Service IV

WIND ON STRUCTURE FORCES AT END BENT			
	WS Loads (kip)		
Beam	x	y	z
1	1.5	0.0	0.7
2	1.5	0.0	0.7
3	1.5	0.0	0.7
4	1.5	0.0	0.7
5	1.5	0.0	0.7
6	1.5	0.0	0.7
7	1.5	0.0	0.7
8	1.5	0.0	0.7
9	1.5	0.0	0.7

Loads on Substructure [SDG 2.4]

The end bents are usually shielded from wind by a MSE wall or an embankment fill, so wind on the end bent substructure is **ignored**.

B5. Wind Pressure on Vehicles [LRFD 3.8.1.3]

The LRFD specifies that wind load should be applied to vehicles on the bridge..........

$$Skew_{wind} := \begin{pmatrix} 0 \\ 15 \\ 30 \\ 45 \\ 60 \end{pmatrix} \qquad Wind_{LRFD} := \begin{array}{c} \begin{array}{cc} x & z \end{array} \\ \begin{pmatrix} .100 & 0 \\ .088 & .012 \\ .082 & .024 \\ .066 & .032 \\ .034 & .038 \end{pmatrix} \end{array} \cdot \frac{kip}{ft}$$

The wind forces on vehicles are transmitted to the end bent using tributary lengths.......

$$L_{Endbent} := \frac{L_{span}}{2} = 45\,ft$$

Forces due to wind on vehicles applied to the superstructure.................................

$$WL_{Super.Endbent} := Wind_{LRFD} \cdot L_{Endbent}$$

$$WL_{Super.Endbent} = \begin{array}{c} \begin{array}{cc} x & z \end{array} \\ \begin{pmatrix} 4.5 & 0.0 \\ 4.0 & 0.5 \\ 3.7 & 1.1 \\ 3.0 & 1.4 \\ 1.5 & 1.7 \end{pmatrix} \end{array} \cdot kip$$

A conservative approach is taken to minimize the analysis required. The maximum transverse and longitudinal forces are used in the following calculations.

Maximum transverse force.....................
$F_{WL.x} := WL_{Super.Endbent_{0,0}} = 4.5 \cdot kip$

Maximum longitudinal force...................
$F_{WL.z} := WL_{Super.Endbent_{4,1}} = 1.71 \cdot kip$

The forces due to wind need to be resolved along the direction of the skew.

Force perpendicular (z-direction) to the end bent..
$WL_{z.Endbent} := \left|F_{WL.z} \cdot \cos(Skew)\right| + \left|F_{WL.x} \cdot \sin(Skew)\right| = 3.15 \cdot kip$

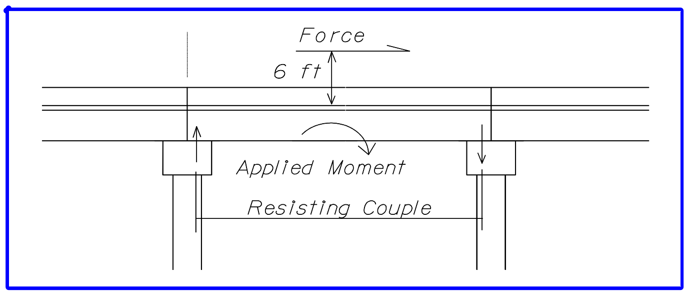
Force perpendicular (z-direction) to the end bent per beam.................................
$$WL_{z.Beam} := \frac{WL_{z.Endbent}}{N_{beams}} = 0.35 \cdot kip$$

Force parallel (x-direction) to the cap.......
$WL_{x.Endbent} := \left|F_{WL.z} \cdot \sin(Skew)\right| + \left|F_{WL.x} \cdot \cos(Skew)\right| = 4.81 \cdot kip$

Force parallel (x-direction) to the cap per beam...
$$WL_{x.Beam} := \frac{WL_{x.Endbent}}{N_{beams}} = 0.53 \cdot kip$$

Longitudinal Adjustments for Wind on Vehicles

The longitudinal moment is resisted by the moment arm.



Moment arm from top of bearing pad to location of applied load...........................
$M_{arm} = 9.750\ ft$ *($M_{arm} = h + 6\ ft$)*

Vertical force in end bent due to wind pressure on vehicle................................
$$WL_{y.Endbent} := \frac{-WL_{z.Beam} \cdot M_{arm}}{L_{span}} = -0.04 \cdot kip$$

For this design example, this component of the load is **ignored**.

Transverse Adjustments for Wind on Vehicles

Using the principles of the lever rule for transverse distribution of live load on beams, the wind on live load can be distributed similarly. It assumes that the wind acting on the live load will cause the vehicle to tilt over. Using the lever rule, the tilting effect of the vehicle is resisted by up and down reactions on the beams assuming the deck to act as a simple span between beams. Conservatively, assume all beams that can see live load can develop this load since the placement of the vehicle(s) and number of vehicles within the deck is constantly changing.

Moment arm from top of bearing pad to location of applied load............................ $M_{arm} = 9.750\ \text{ft}$

Vertical reaction on pier from transverse wind pressure on vehicles....................... $WL_{y.Endbent} := \frac{-WL_{x.Endbent} \cdot M_{arm}}{BeamSpacing} = -4.69 \cdot \text{kip}$

Since this load can occur at any beam location, conservatively apply this load to all beams

WIND ON LIVE LOAD FORCES AT END BENT			
	WL Loads (kip)		
Beam	x	y	z
1	0.5	-4.7	0.3
2	0.5	4.7	0.3
3	0.5	-4.7	0.3
4	0.5	4.7	0.3
5	0.5	-4.7	0.3
6	0.5	4.7	0.3
7	0.5	-4.7	0.3
8	0.5	4.7	0.3
9	0.5	0.0	0.0

C. Design Limit States

The design loads for strength I, strength III, strength V, and service I limit states are summarized in this section.

These reactions are from the superstructure **only**, acting on the substructure. In the RISA analysis model, include the following loads:

- DC: self-weight of the substructure, include end bent cap and backwall.

- TU: a temperature increase and fall on the pier substructure utilizing the following parameters:

  coefficient of expansion $\alpha_t = 6 \times 10^{-6} \cdot \frac{1}{°F}$

  temperature change $temperature_{increase} = temperature_{fall} = 35 \cdot °F$

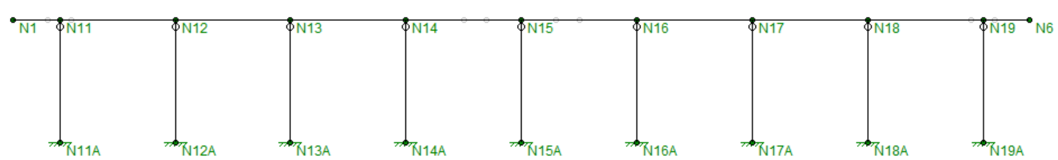
  Two load cases would be required for temperature with a positive and negative strain being inputed.

  For the end bent, assuming the cap to be supported on pin supports at every pile location is an acceptable modeling decision. A depth of fixity of 7 pile diameters, or 10.5 feet, is assumed.

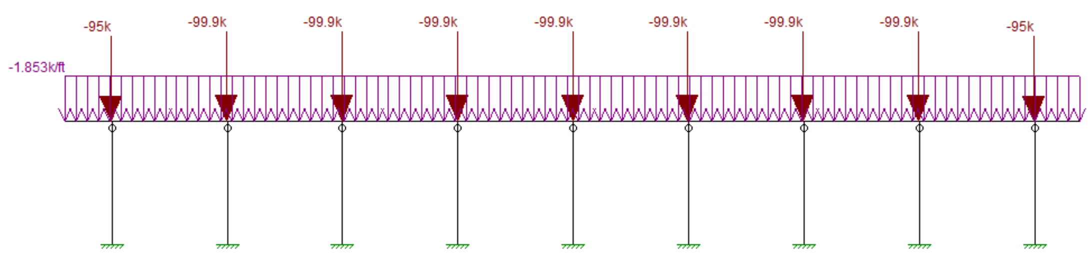
- WS: Wind on the substructure should be applied directly to the analysis model. For the end bent surrounded by MSE wall, the wind loads on substructure are non-existant since the substructure can be considered shielded by the wall.

- All applied loads in the substructure analysis model should be multiplied by the appropriate load factor values and combined with the limit state loads calculated in this file for the final results.

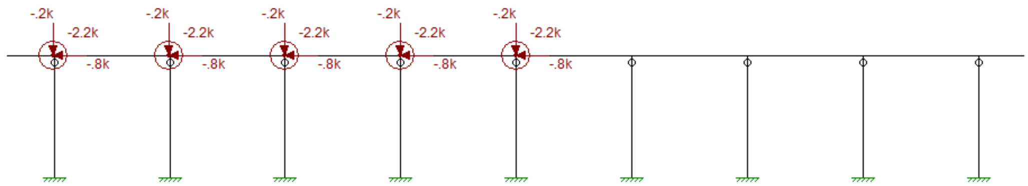
Substructure Model



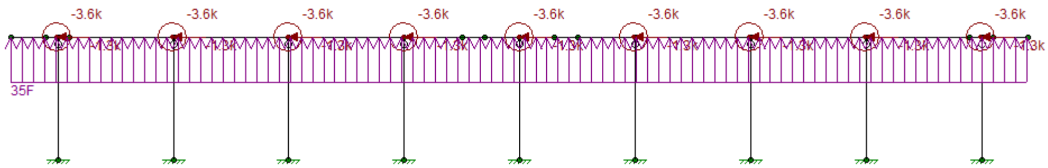
DC Loads:



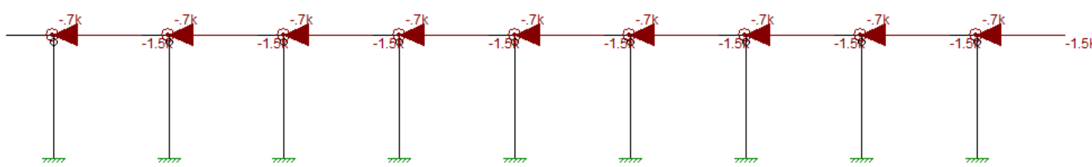
BR Loads:



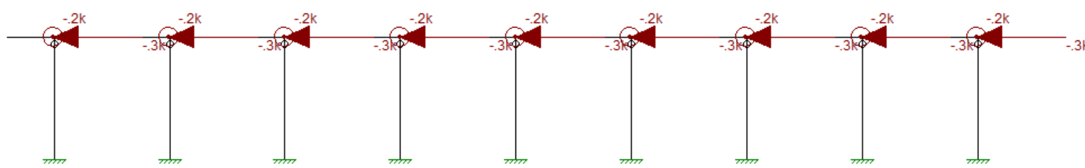
CR/SH/TU Loads:



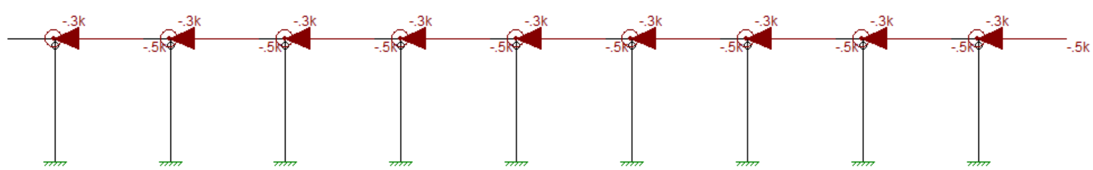
WS Strength III/Service IV Loads:



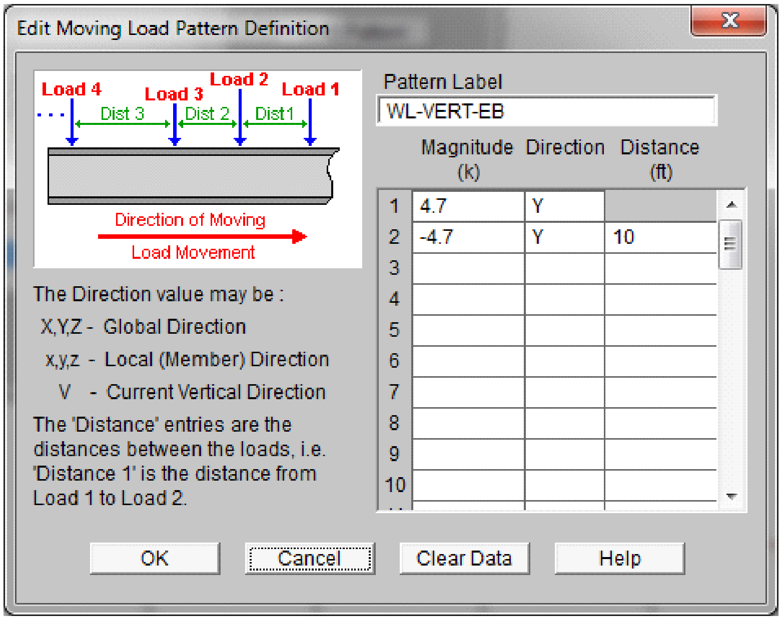
WS Strength V/Service I Loads:



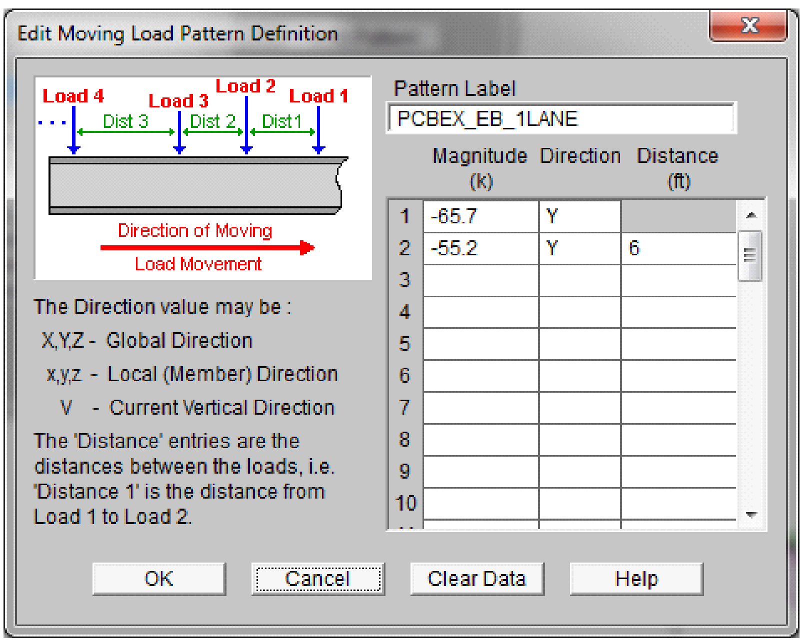
WL Loads:



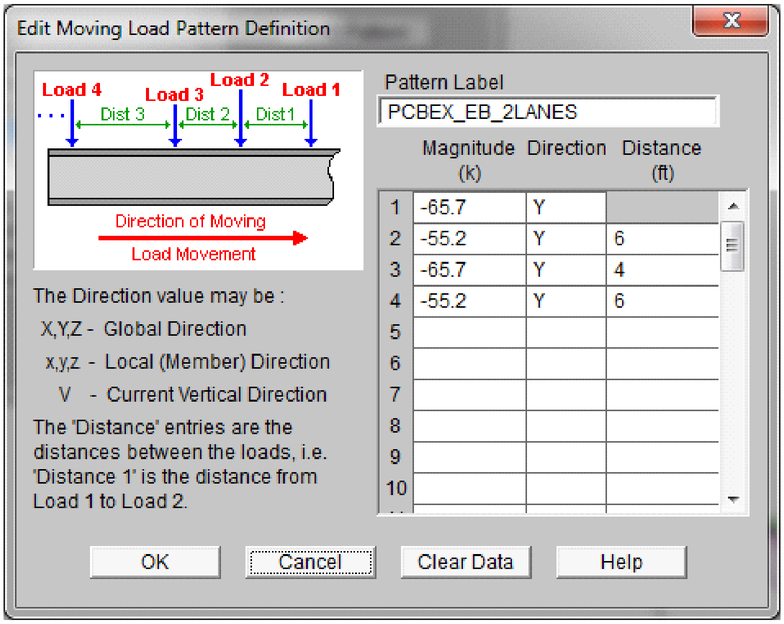
Moving Load Pattern: WL Vertical



1 Lane Moving Load Pattern:

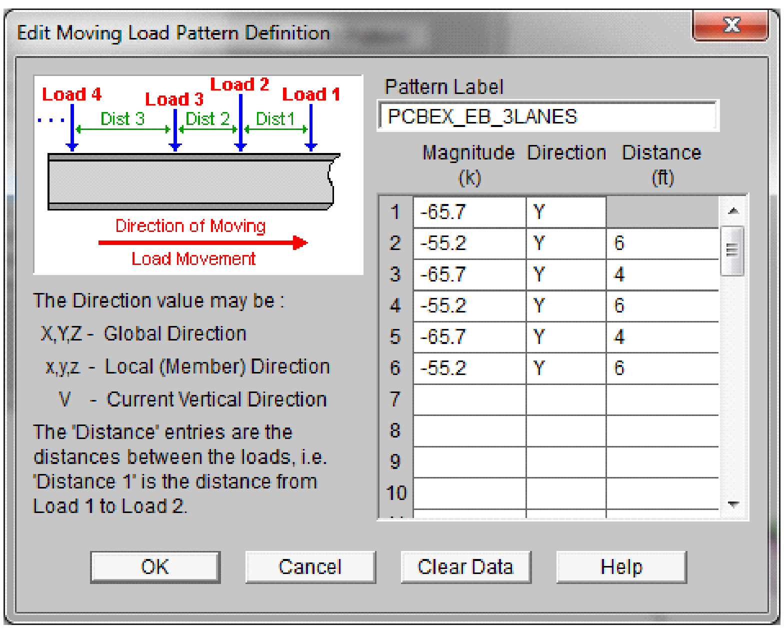


2 Lane Moving Load Pattern:



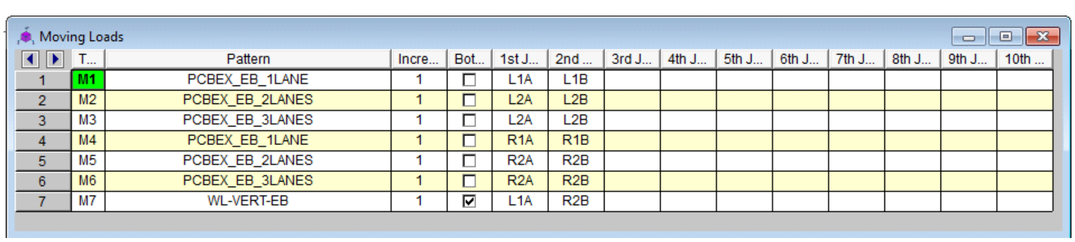
	Magnitude (k)	Direction	Distance (ft)
1	-65.7	Y	
2	-55.2	Y	6
3	-65.7	Y	4
4	-55.2	Y	6
5			
6			
7			
8			
9			
10			

3 Lane Moving Load Pattern:



	Magnitude (k)	Direction	Distance (ft)
1	-65.7	Y	
2	-55.2	Y	6
3	-65.7	Y	4
4	-55.2	Y	6
5	-65.7	Y	4
6	-55.2	Y	6
7			
8			
9			
10			

Moving Load Cases:



	T...	Pattern	Incre...	Bot...	1st J...	2nd ...	3rd J...	4th J...	5th J...	6th J...	7th J...	8th J...	9th J...	10th ...
1	M1	PCBEX_EB_1LANE	1	☐	L1A	L1B								
2	M2	PCBEX_EB_2LANES	1	☐	L2A	L2B								
3	M3	PCBEX_EB_3LANES	1	☐	L2A	L2B								
4	M4	PCBEX_EB_1LANE	1	☐	R1A	R1B								
5	M5	PCBEX_EB_2LANES	1	☐	R2A	R2B								
6	M6	PCBEX_EB_3LANES	1	☐	R2A	R2B								
7	M7	WL-VERT-EB	1	☑	L1A	R2B								

Basic Load Cases:

Basic Load Cases

	BLC Description	Category	X Gravity	Y Gravity	Z Gravity	Joint	Point	Distrib...	Area (...	Surfac...
1	DC Load	None				9		1		
2	DW Load	None								
3	BR Load	None				15				
4	CR/SH/TU Loads	None						1		
5	WS StrIII/ServIV	None				18				
6	WS StrV/ServI	None				18				
7	WL	None				18				

Load Combinations:

Load Combinations

Combinations | Design

	Description	Sol...	PD...	SR...	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor
1	StrI-1Lane +TU	☑			1	1.25	3	2.1	4	.5	M1	2.1								
2	StrI-1Lane -TU	☑			1	1.25	3	2.1	4	-.5	M1	2.1								
3	StrI-2Lanes(2) +TU	☑			1	1.25	3	1.75	4	.5	M2	1.75								
4	StrI-2Lanes(2) -TU	☑			1	1.25	3	1.75	4	-.5	M2	1.75								
5	StrI-2Lanes(1+1) +TU	☑			1	1.25	3	1.75	4	.5	M1	1.75	M4	1.75						
6	StrI-2Lanes(1+1) -TU	☑			1	1.25	3	1.75	4	-.5	M1	1.75	M4	1.75						
7	StrI-3Lanes(3) +TU	☑			1	1.25	3	1.488	4	.5	M3	1.488								
8	StrI-3Lanes(3) -TU	☑			1	1.25	3	1.488	4	-.5	M3	1.488								
9	StrI-3Lanes(2+1) +TU	☑			1	1.25	3	1.488	4	.5	M2	1.488	M4	1.488						
10	StrI-3Lanes(2+1) -TU	☑			1	1.25	3	1.488	4	-.5	M2	1.488	M4	1.488						
11	StrI-4Lanes(3+1) +TU	☑			1	1.25	3	1.138	4	.5	M3	1.138	M4	1.138						
12	StrI-4Lanes(3+1) -TU	☑			1	1.25	3	1.138	4	-.5	M3	1.138	M4	1.138						
13	StrI-4Lanes(2+2) +TU	☑			1	1.25	3	1.138	4	.5	M2	1.138	M5	1.138						
14	StrI-4Lanes(2+2) -TU	☑			1	1.25	3	1.138	4	-.5	M2	1.138	M5	1.138						
15	StrI-5Lanes +TU	☑			1	1.25	3	1.138	4	.5	M3	1.138	M5	1.138						
16	StrI-5Lanes -TU	☑			1	1.25	3	1.138	4	-.5	M3	1.138	M5	1.138						
17	StrI-6Lanes +TU	☑			1	1.25	3	1.138	4	.5	M3	1.138	M6	1.138						
18	StrI-6Lanes -TU	☑			1	1.25	3	1.138	4	-.5	M3	1.138	M6	1.138						
19	StrIII-1Lane +TU	☑			1	1.25	4	.5	5	1.4										
20	StrIII-1Lane -TU	☑			1	1.25	4	-.5	5	1.4										
21	StrIII-2Lanes(2) +TU	☑			1	1.25	4	.5	5	1.4										
22	StrIII-2Lanes(2) -TU	☑			1	1.25	4	-.5	5	1.4										
23	StrIII-2Lanes(1+1) +TU	☑			1	1.25	4	.5	5	1.4										
24	StrIII-2Lanes(1+1) -TU	☑			1	1.25	4	-.5	5	1.4										
25	StrIII-3Lanes(3) +TU	☑			1	1.25	4	.5	5	1.4										
26	StrIII-3Lanes(3) -TU	☑			1	1.25	4	-.5	5	1.4										
27	StrIII-3Lanes(2+1) +TU	☑			1	1.25	4	.5	5	1.4										
28	StrIII-3Lanes(2+1) -TU	☑			1	1.25	4	-.5	5	1.4										
29	StrIII-4Lanes(3+1) +TU	☑			1	1.25	4	.5	5	1.4										
30	StrIII-4Lanes(3+1) -TU	☑			1	1.25	4	-.5	5	1.4										
31	StrIII-4Lanes(2+2) +TU	☑			1	1.25	4	.5	5	1.4										
32	StrIII-4Lanes(2+2) -TU	☑			1	1.25	4	-.5	5	1.4										
33	StrIII-5Lanes +TU	☑			1	1.25	4	.5	5	1.4										
34	StrIII-5Lanes -TU	☑			1	1.25	4	-.5	5	1.4										
35	StrIII-6Lanes +TU	☑			1	1.25	4	.5	5	1.4										
36	StrIII-6Lanes -TU	☑			1	1.25	4	-.5	5	1.4										

Load Combinations

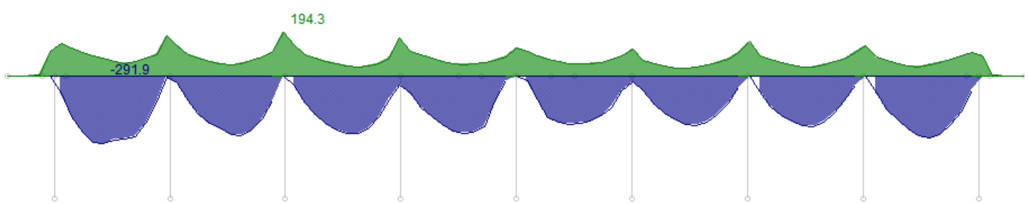
Combinations | Design

	Description	Sol...	PD...	SR...	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor	BLC	Factor
37	StrV-1Lane +TU	☑			1	1.25	3	1.62	4	.5	6	1.3	7	1	M7	1	M1	1.62		
38	StrV-1Lane -TU	☑			1	1.25	3	1.62	4	-.5	6	1.3	7	1	M7	1	M1	1.62		
39	StrV-2Lanes(2) +TU	☑			1	1.25	3	1.35	4	.5	6	1.3	7	1	M7	1	M2	1.35		
40	StrV-2Lanes(2) -TU	☑			1	1.25	3	1.35	4	-.5	6	1.3	7	1	M7	1	M2	1.35		
41	StrV-2lanes(1+1) +TU	☑			1	1.25	3	1.35	4	.5	6	1.3	7	1	M7	1	M1	1.35	M4	1.35
42	StrV-2lanes(1+1) -TU	☑			1	1.25	3	1.35	4	-.5	6	1.3	7	1	M7	1	M1	1.35	M4	1.35
43	StrV-3Lanes(3) +TU	☑			1	1.25	3	1.15	4	.5	6	1.3	7	1	M7	1	M3	1.15		
44	StrV-3Lanes(3) -TU	☑			1	1.25	3	1.15	4	-.5	6	1.3	7	1	M7	1	M3	1.15		
45	StrV-3Lanes(2+1) +TU	☑			1	1.25	3	1.15	4	.5	6	1.3	7	1	M7	1	M2	1.15	M4	1.15
46	StrV-3Lanes(2+1) -TU	☑			1	1.25	3	1.15	4	-.5	6	1.3	7	1	M7	1	M2	1.15	M4	1.15
47	StrV-4Lanes(3+1) +TU	☑			1	1.25	3	.88	4	.5	6	1.3	7	1	M7	1	M3	.88	M4	.88
48	StrV-4Lanes(3+1) -TU	☑			1	1.25	3	.88	4	-.5	6	1.3	7	1	M7	1	M3	.88	M4	.88
49	StrV-4Lanes(2+2) +TU	☑			1	1.25	3	.88	4	.5	6	1.3	7	1	M7	1	M2	.88	M5	.88
50	StrV-4Lanes(2+2) -TU	☑			1	1.25	3	.88	4	-.5	6	1.3	7	1	M7	1	M2	.88	M5	.88
51	StrV-5Lanes +TU	☑			1	1.25	3	.88	4	.5	6	1.3	7	1	M7	1	M3	.88	M5	.88
52	StrV-5Lanes -TU	☑			1	1.25	3	.88	4	-.5	6	1.3	7	1	M7	1	M3	.88	M5	.88
53	StrV-6Lanes +TU	☑			1	1.25	3	.88	4	.5	6	1.3	7	1	M7	1	M3	.88	M6	.88
54	StrV-6Lanes -TU	☑			1	1.25	3	.88	4	-.5	6	1.3	7	1	M7	1	M3	.88	M6	.88
55	ServI-1Lane +TU	☑			1	1	3	1.2	4	1	6	1	7	1	M7	1	M1	1.2		
56	ServI-1Lane -TU	☑			1	1	3	1.2	4	-1	6	1	7	1	M7	1	M1	1.2		
57	ServI-2Lanes(2) +TU	☑			1	1	3	1	4	1	6	1	7	1	M7	1	M2	1		
58	ServI-2Lanes(2) -TU	☑			1	1	3	1	4	-1	6	1	7	1	M7	1	M2	1		
59	ServI-2Lanes(1+1) +TU	☑			1	1	3	1	4	1	6	1	7	1	M7	1	M1	1	M4	1
60	ServI-2Lanes(1+1) -TU	☑			1	1	3	1	4	-1	6	1	7	1	M7	1	M1	1	M4	1
61	ServI-3Lanes(3) +TU	☑			1	1	3	.85	4	1	6	1	7	1	M7	1	M3	.85		
62	ServI-3Lanes(3) -TU	☑			1	1	3	.85	4	-1	6	1	7	1	M7	1	M3	.85		
63	ServI-3Lanes(2+1) +TU	☑			1	1	3	.85	4	1	6	1	7	1	M7	1	M2	.85	M4	.85
64	ServI-3Lanes(2+1) -TU	☑			1	1	3	.85	4	-1	6	1	7	1	M7	1	M2	.85	M4	.85
65	ServI-4Lanes(3+1) +TU	☑			1	1	3	.65	4	1	6	1	7	1	M7	1	M3	.65	M4	.65
66	ServI-4Lanes(3+1) -TU	☑			1	1	3	.65	4	-1	6	1	7	1	M7	1	M3	.65	M4	.65
67	ServI-4Lanes(2+2) +TU	☑			1	1	3	.65	4	1	6	1	7	1	M7	1	M2	.65	M5	.65
68	ServI-4Lanes(2+2) -TU	☑			1	1	3	.65	4	-1	6	1	7	1	M7	1	M2	.65	M5	.65
69	ServI-5Lanes +TU	☑			1	1	3	.65	4	1	6	1	7	1	M7	1	M3	.65	M5	.65
70	ServI-5Lanes -TU	☑			1	1	3	.65	4	-1	6	1	7	1	M7	1	M3	.65	M5	.65
71	ServI-6Lanes +TU	☑			1	1	3	.65	4	1	6	1	7	1	M7	1	M3	.65	M6	.65
72	ServI-6Lanes -TU	☑			1	1	3	.65	4	-1	6	1	7	1	M7	1	M3	.65	M6	.65

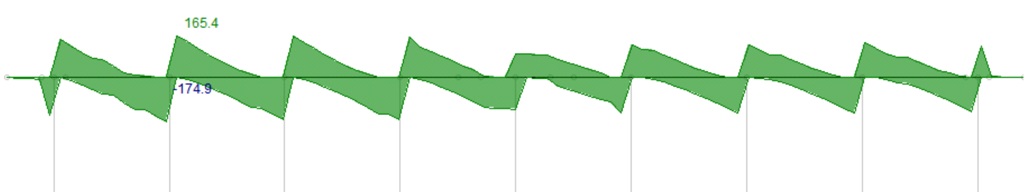
C1. Strength I Limit State Loads

$$Strength1 = 1.25 \cdot DC + 1.5 \cdot DW + 1.75 \cdot LL + 1.75BR + 0.5 \cdot (TU + CR + SH)$$

Beam Moment:



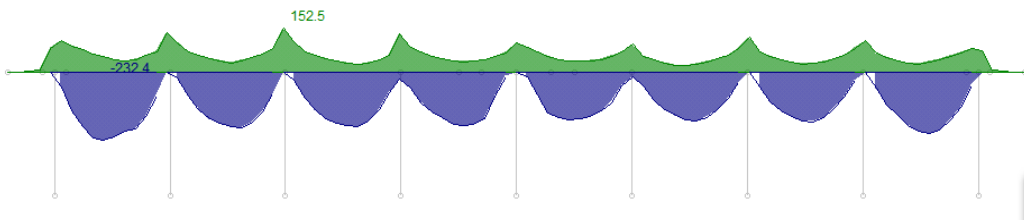
Beam Shear:



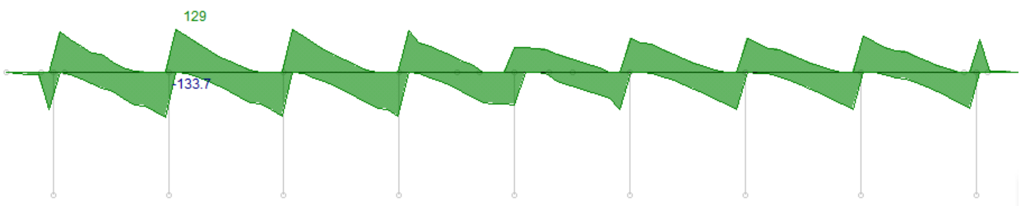
C2. Strength V Limit State Loads

Strength5 = 1.25·DC + 1.50·DW + 1.35·LL + 1.35·BR + 1.30·WS + 1.0·WL + 0.50·(TU + CR + SH)

Beam Moment:



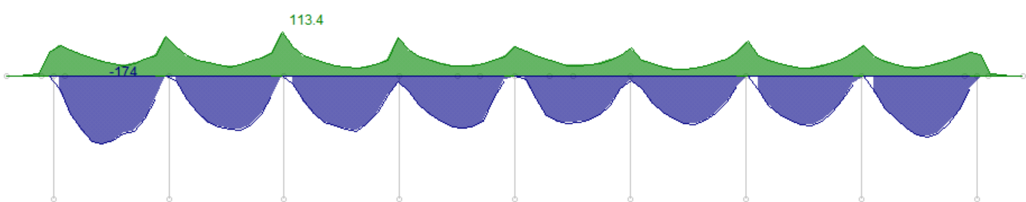
Beam Shear:



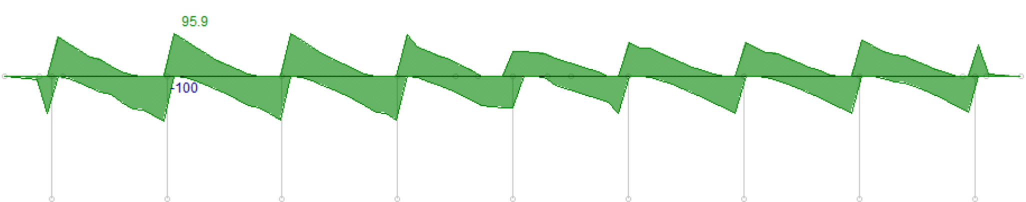
C3. Service 1 Limit State Loads

Service1 = 1.0·DC + 1.0·DW + 1.0·LL + 1.0·BR + 1.0WS + 1.0·WL + 1.0·(TU + CR + SH)

Beam Moment:



Beam Shear:



C4. Summary of Results

From the results of the analysis, the governing moments for the design of the end bent cap and the corresponding service moments were as follows:

$M_{Strength1.Negative} := -194.3 \cdot ft \cdot kip$

$M_{Service1.Negative} := -113.4 \cdot ft \cdot kip$

$M_{Strength1.Positive} := 291.9 \cdot ft \cdot kip$

$M_{Service1.Positive} := 174.0 \cdot ft \cdot kip$

For purposes of this design example, these values are given for references purposes. The method of obtaining the design values has been shown and the user will then utilize design equations and methodologies similar to Section 3.04 Pier Cap Design to design the end bent cap. For the piles, the approach is similar to Section 3.08 Pier Pile Vertical Load design. There are no moments transferred from the end bent cap to the piles since for a 1 foot embedment of the pile into the cap, the connection is considered to be a pin connection.

Defined Units

End Bent Cap Design

Reference

Description

The actual design of the end bent cap for the governing moments and shears has not been performed in this design example. For a similar design approach, refer to Section 3.04 Pier Cap Design.

End Bent Foundation Design Loads

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\311EndBentLoads.xmcd(R)

Description

This document provides the design parameters necessary for the substructure pile vertical load and footing design.

Page Contents

A. General Criteria

A1. Modification to End Bent Live Loads for Foundation Design

The Dynamic Load Allowance (IM) is not required since the foundation components are entirely below ground level **[LRFD 3.6.2.1]**.

For the foundation design, the impact on the truck will need to be removed from the load combinations since the piles are embedded in the ground.

The RISA Analysis will be re-run with the impact factor removed from live load.

Revised HL-93 Line Load................. $HL93_{No.IM} := \frac{wheel_{line}}{IM} + lane_{load} = 49.15 \cdot kip$

$$HL93_{No.IM} + \frac{P_c}{IM} = 53.09 \cdot kip$$

$$HL93_{No.IM} - \frac{P_c}{IM} = 45.22 \cdot kip$$

B. Foundation Vertical Design Load Summary

For this design example, we will use the load combinations that likely govern pile design. For example purposes, only piles 1-3 are presented.

RISA PILE RESULTS

Result Case	**Fx**	**Fy**	**Fz**
Pile 1			
Strength I	**3.9**	291.9	**6.9**
Strength III	**5.1**	139.7	**2.8**
Service I	**7.3**	196.7	**7.0**
Pile 2			
Strength I	3.4	**336.1**	6.5
Strength III	4.5	147.7	2.8
Service I	6.2	**227.1**	6.8
Pile 3			
Strength I	2.8	328.3	6.2
Strength III	3.9	**148.3**	2.8
Service I	5.0	222.4	6.6

C. Lateral Design Load

C1. Design Parameters

Depth of end bent cap............................ $h_{EB} = 2.5\,ft$

Width of end bent cap............................ $b_{EB} = 3.5\,ft$

Length of end bent cap.......................... $L_{EB} = 88\,ft$

Height of back wall................................ $h_{BW} = 3.6\,ft$

Backwall design width........................... $L_{BW} = 1\,ft$

Thickness of back wall.......................... $t_{BW} = 1\,ft$

Approach slab thickness........................ $t_{ApprSlab} = 13.75 \cdot in$

Approach slab length............................. $L_{ApprSlab} = 32\,ft$

Concrete cover..................................... $cover_{sub} = 3 \cdot in$

Load factor for EH and ES (**LRFD 3.4.1**). $\gamma_{p.max} := 1.5$ $\gamma_{p.min} := 0.90$

Load factor for dead load....................... $\gamma_{DC} := 1.25$

Load factor for live load surcharge (LS)... $\gamma_{LS} := 1.75$ *(for Str. I, 1.0 for Serv. I)*

Number of piles.................................... $N_{piles} := 9$

C2. Soil Parameters

Values for the active lateral earth pressure, k_a, [**LRFD 3.11.5.3, 3.11.5.6]** may be taken as:

$$k_a = \frac{\sin^2(\theta + \phi'_f)}{\Gamma \cdot (\sin^2\theta \cdot \sin(\theta - \delta))}$$

where

$$\Gamma = \left(1 + \sqrt{\frac{\sin(\phi'_f + \delta) \cdot \sin(\phi'_f - \beta)}{\sin(\theta - \delta)\sin(\theta + \beta)}}\right)^2$$

From LRFD Table 3.11.5.3-1:

Formed or precast concrete or concrete sheet piling against the following soils:		
• Clean gravel, gravel-sand mixture, well-graded rock fill with spalls	22 to 26	0.40 to 0.49
• Clean sand, silty sand-gravel mixture, single-size hard rock fill	17 to 22	0.31 to 0.40
• Silty sand, gravel or sand mixed with silt or clay	17	0.31
• Fine sandy silt, nonplastic silt	14	0.25

defining the following:

$\gamma_{soil} = 115 \cdot pcf$ — Unit weight of soil

$\theta := 90 \cdot deg$ — angle of the end bent back face of the wall to the horizontal

$\phi'_f := 29 \cdot deg$ — effective angle of internal friction, assumed

$\delta := 20 \cdot deg$ — friction angle between fill and wall given by **LRFD Table 3.11.5.3-1** *(**Note**: based on concrete on clean fine to medium sand)*

$\beta := 0 \cdot deg$ — angle of fill to the horizontal

therefore

$$\Gamma := \left(1 + \sqrt{\frac{\sin(\phi'_f + \delta) \cdot \sin(\phi'_f - \beta)}{\sin(\theta - \delta)\,\sin(\theta + \beta)}}\right)^2 = 2.64$$

and

$$k_a := \frac{\left(\sin(\theta + \phi'_f)\right)^2}{\Gamma \cdot \left(\sin(\theta)^2 \cdot \sin(\theta - \delta)\right)} = 0.31$$

The horizontal earth pressure due to live load, Δ_p , [**LRFD 3.11.6.4]** may be approximated as follows:

$\Delta_p = k \cdot \gamma_{soil} \cdot h_{eq}$ where

$\gamma_{soil} = 115 \cdot pcf$ — Unit weight of soil

$k := |k_a|$ — Coefficient of lateral earth pressure

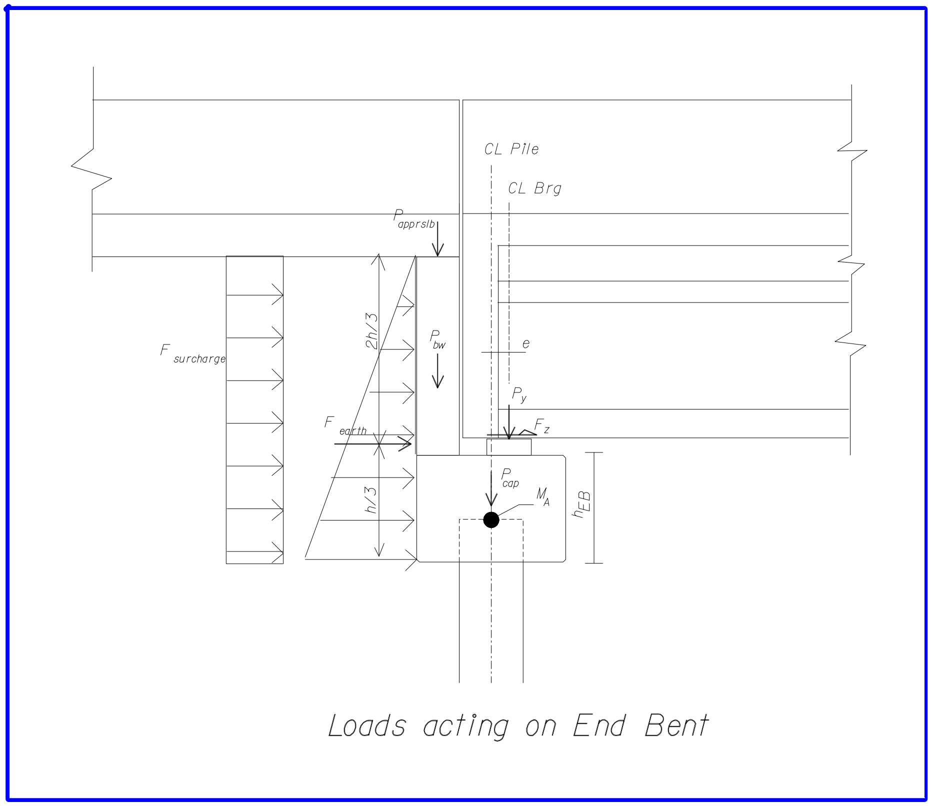
$h_{eq} := 4.0 \cdot ft$ — equivalent height of soil for vehicular loading, **LRFD Table 3.11.6.4-1 or 3.11.6.4-2**

therefore

$$\Delta_p := k \cdot \gamma_{soil} \cdot h_{eq} = 0.14 \cdot ksf$$

C3. Applied Loads

The following is a free body diagram of the loads acting on the end bent.



The loads due to the end bent cap and back wall height were included in the RISA analysis of the end bent, and therefore are not included here.

Calculate moment at top of pile due to earth pressure per foot of backwall

Lateral force.................................... $F_{earth} := \dfrac{|k_a| \cdot \gamma_{soil} \cdot (h_{BW} + h_{EB})^2}{2} = 0.66 \cdot \dfrac{kip}{ft}$

Lateral force moment arm................. $y_{earth} := \dfrac{h_{BW} + h_{EB}}{3} - Pile_{embed} = 1.03\,ft$

Moment at top of pile....................... $M_{earth} := F_{earth} \cdot y_{earth} = 0.68 \cdot \dfrac{kip \cdot ft}{ft}$

Calculate moment at top of pile due to live load surcharge per foot of backwall

Lateral force.................................... $F_{surcharge} := \left[\Delta_p \cdot \left(h_{BW} + h_{EB}\right)\right] = 0.87 \cdot \frac{kip}{ft}$

Lateral force moment arm................. $y_{surcharge} := \frac{h_{BW} + h_{EB}}{2} - Pile_{embed} = 2.05\,ft$

Moment at top of pile....................... $M_{surcharge} := F_{surcharge} \cdot y_{surcharge} = 1.78 \cdot \frac{kip \cdot ft}{ft}$

Calculate moment at top of pile due to approach slab per foot of backwall

Vertical force.................................. $P_{AS} := \left(\gamma_{conc} \cdot \frac{t_{ApprSlab} \cdot L_{ApprSlab}}{3}\right) = 1.83 \cdot \frac{kip}{ft}$

*(**Note**: assume 1/3 of weight is seen at back wall)*

Vertical force moment arm.............. $e_{AS} := \frac{t_{BW} - b_{EB}}{2} = -1.25\,ft$

Moment at top of pile....................... $M_{AS} := P_{AS} \cdot e_{AS} = -2.29 \cdot \frac{kip \cdot ft}{ft}$

D. Design Load Summary

Strength I

Calculate the Strength I limit state pile reaction.

Vertical force from RISA analysis....... $P_{Str1} := F_{y.StrI} \cdot kip = 336.1 \cdot kip$

Vertical force moment arm................ $e_{Py} := t_{BW} + K2 - \frac{b_{EB}}{2} = 0.58\,ft$

Moment at top of pile due to vertical force... $M_{P.Str1} := P_{Str1} \cdot e_{Py} = 196.06 \cdot kip \cdot ft$

Lateral force perpendicular to bent, excluding soil forces, from RISA analysis.. $F_{Str1} := F_{z.StrI} \cdot kip = 6.9 \cdot kip$

Lateral force moment arm................ $e_{Fy} := h_{EB} - Pile_{embed} + 4 \cdot in = 1.83\,ft$ *(**Note**: Use 4" pedestal height).*

Moment at top of pile due to lateral force... $M_{F.Str1} := F_{Str1} \cdot e_{Fy} = 12.65 \cdot kip \cdot ft$

Distribution of loads to piles............. $L_D := \frac{L_{EB}}{N_{piles}} = 9.78\,ft$

Total Moment.................................

$$M_{strength1} := \max\left(\left|\begin{array}{l} M_{P.Str1} + M_{F.Str1} + \gamma_{DC} \cdot L_D \cdot M_{AS} \cdots \\ + \gamma_{p.min} \cdot L_D \cdot M_{earth} + \gamma_{LS} \cdot L_D \cdot M_{surcharge} \end{array}\right|, \begin{array}{l} M_{P.Str1} + M_{F.Str1} + \gamma_{p.max} \cdot L_D \cdot M_{earth} \cdots \\ + M_{surcharge} \cdot \gamma_{LS} \cdot L_D \end{array}\right)$$

$$M_{strength1} = 249.1 \cdot kip \cdot ft$$

Total lateral force perpendicular to bent.. $F_{z.strength1} := F_{Str1} + \gamma_{p.max} \cdot F_{earth} \cdot L_D + \gamma_{LS} \cdot F_{surcharge} \cdot L_D = 31.41 \cdot kip$

Total lateral force parallel to bent........ $F_{x.strength1} := F_{x.StrI} \cdot kip = 3.9 \cdot kip$

Total vertical force........................... $F_{y.strength1} := P_{Str1} + \gamma_{DC} \cdot P_{AS} \cdot L_D = 358.51 \cdot kip$

Strength III

Calculate the Strength III limit state pile reaction.

Vertical force from RISA analysis....... $P_{Str3} := F_{y.StrIII} \cdot kip = 148.3 \cdot kip$

Vertical force moment arm............... $e_{Py} = 0.58\,ft$

Moment at top of pile due to vertical force.. $M_{P.Str3} := P_{Str3} \cdot e_{Py} = 86.51 \cdot kip \cdot ft$

Lateral force perpendicular to bent, excluding soil forces, from RISA analysis.. $F_{Str3} := F_{z.StrIII} \cdot kip = 2.8 \cdot kip$

Lateral force moment arm............... $e_{Fy} = 1.83\,ft$

Moment at top of pile due to lateral force.. $M_{F.Str3} := F_{Str3} \cdot e_{Fy} = 5.13 \cdot kip \cdot ft$

Distribution of loads to piles............. $L_D = 9.78\,ft$

Total Moment.................................

$$M_{strength3} := \max\left(\left(\begin{array}{c} \left|M_{P.Str3} + M_{F.Str3} + \gamma_{DC} \cdot L_D \cdot M_{AS} + \gamma_{p.min} \cdot L_D \cdot M_{earth}\right| \\ M_{P.Str3} + M_{F.Str3} + \gamma_{p.max} \cdot L_D \cdot M_{earth} \end{array}\right)\right) = 101.65\,ft \cdot kip$$

Total lateral force perpendicular to bent.. $F_{z.strength3} := F_{Str3} + \gamma_{p.max} \cdot F_{earth} \cdot L_D = 12.49 \cdot kip$

Total lateral force parallel to bent........ $F_{x.strength3} := F_{x.StrIII} \cdot kip = 5.1 \cdot kip$

Total vertical force.......................... $F_{y.strength3} := P_{Str3} + \gamma_{DC} \cdot P_{AS} \cdot L_D = 170.71 \cdot kip$

Service I

Calculate the Service I limit state pile reaction.

Vertical force from RISA analysis...... $P_{Srv1} := F_{y.ServI} \cdot kip = 227.1 \cdot kip$

Vertical force moment arm............... $e_{Py} = 0.58\,ft$

Moment at top of pile due to vertical force.. $M_{P.Srv1} := P_{Srv1} \cdot e_{Py} = 132.47 \cdot kip \cdot ft$

Lateral force perpendicular to bent, excluding soil forces from RISA analysis... $F_{Srv1} := F_{z.ServI} \cdot kip = 7 \cdot kip$

Lateral force moment arm................ $e_{Fy} = 1.83\,ft$

Moment at top of pile due to lateral force... $M_{F.Srv1} := F_{Srv1} \cdot e_{Fy} = 12.83 \cdot kip \cdot ft$

Distribution of loads to piles............. $L_D = 9.78\,ft$

Total Moment.................................

$$M_{service1} := \max\left(\left(\begin{array}{c} \left|M_{P.Srv1} + M_{F.Srv1} + L_D \cdot 1.00 \cdot M_{AS} + 1.00 \cdot L_D \cdot M_{earth} + 1.0 \cdot L_D \cdot M_{surcharge}\right| \\ M_{P.Srv1} + M_{F.Srv1} + 1.0 \cdot L_D \cdot M_{earth} + 1.0 \cdot L_D \cdot M_{surcharge} \end{array}\right)\right) = 169.3\,ft \cdot k$$

Total lateral force perpendicular to bent... $F_{z.service1} := F_{Srv1} + 1.00 \cdot \left(F_{earth} + F_{surcharge}\right) \cdot L_D = 21.93 \cdot kip$

Total lateral force parallel to bent........ $F_{x.service1} := F_{x.ServI} \cdot kip = 7.3 \cdot kip$

Total vertical force......................... $F_{y.service1} := P_{Srv1} + 1.0 \cdot P_{AS} \cdot L_D = 245.03 \cdot kip$

Redefine Variables

End Bent Pile Vertical Load Design

Reference

Description

The actual design of the end bent piles for the vertical loads has not been performed in this design example. For a similar design approach, refer to **Section 3.08** Pier Pile Vertical Load Design.

SUBSTRUCTURE DESIGN

End Bent Backwall Design

Reference

Reference:C:\Users\st986ch\AAAdata\LRFD PS Beam Design Example\313EndBentFoundLoads.xmcd(R)

Description

This section provides the design for the end bent backwall.

Page Contents

A. General Criteria

Resistance Factor for flexure and tension.. $\phi = 0.9$

Resistance Factor for shear and torsion.... $\phi_v = 0.9$

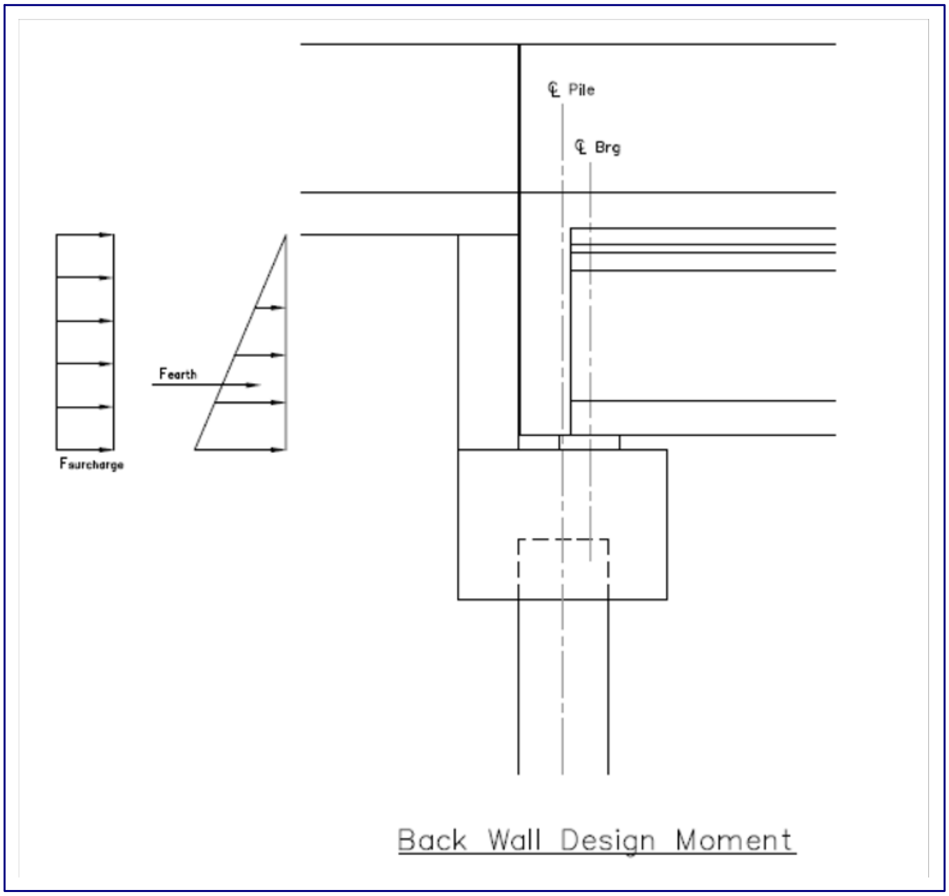
Load factor for horizontal earth (EH)........ $\gamma_{p.max} := 1.5$

Load factor for live load surcharge (LS)... $\gamma_{LS} := 1.75$

B. Back Wall Design

B1. Design Moments

Calculate Design Moments for Backwall per foot of back wall



The moment for the back wall design can be calculated by taking the applied lateral loads at the location of the resultant passive force and designing the back wall as simply supported between the End Bent and Approach Slab.

Lateral Earth Force........................... $F_{earth} := \dfrac{|k_a| \cdot \gamma_{soil} \cdot h_{BW}^{\,2}}{2} = 0.23 \cdot \dfrac{kip}{ft}$

Lateral Earth Moment Arm................ $y_{earth} := \dfrac{h_{BW}}{3} = 1.2\,ft$

Lateral Surcharge Force.................... $F_{surcharge} := \Delta_p \cdot h_{BW} = 0.51 \cdot \dfrac{kip}{ft}$

Lateral Surcharge Moment Arm......... $y_{surcharge} := \dfrac{h_{BW}}{2} = 1.8\,ft$

Bending Moment Calculation............ $P_1 := F_{earth}$

$P_2 := F_{surcharge}$

$a := y_{earth}$

$b := y_{surcharge}$

Bending Moment Calculation cont'd.... $M_1 := \frac{\gamma_{p.max} \cdot P_1 \cdot (h_{BW} - a) + \gamma_{LS} \cdot P_2 \cdot b}{h_{BW}} \cdot a = 0.81 \cdot \frac{kip \cdot ft}{ft}$

$M_2 := \frac{\gamma_{p.max} \cdot P_1 \cdot a + \gamma_{LS} \cdot P_2 \cdot (h_{BW} - b)}{h_{BW}} \cdot b = 1.01 \cdot \frac{kip \cdot ft}{ft}$

$M_{Str1} := \max(M_1, M_2) = 1.01 \cdot \frac{kip \cdot ft}{ft}$

$M_1 := \frac{P_1 \cdot (h_{BW} - a) + P_2 \cdot b}{h_{BW}} \cdot a = 0.49 \cdot \frac{kip \cdot ft}{ft}$

$M_2 := \frac{P_1 \cdot a + P_2 \cdot (h_{BW} - b)}{h_{BW}} \cdot b = 0.6 \cdot \frac{kip \cdot ft}{ft}$

$M_{Srv1} := \max(M_1, M_2) = 0.6 \cdot \frac{kip \cdot ft}{ft}$

Back wall design moments:

Strength... $Mr_{BW} := M_{Str1} \cdot L_{BW} = 1.01 \cdot kip \cdot ft$

Service.. $M_{BW} := M_{Srv1} \cdot L_{BW} = 0.6 \cdot kip \cdot ft$

B2. Flexural Design

Factored resistance $M_r = \phi \cdot M_n$

$$M_r = \phi \cdot A_s \cdot f_s \cdot \left[d_s - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot b} \right) \right]$$

where $M_r := Mr_{BW} = 1.01 \cdot kip \cdot ft$

$b := L_{BW} = 12 \cdot in$

Initial assumption for area of steel required

Size of bar.......................... $bar := \text{"4"}$

Proposed bar spacing........... $spacing := 12 \cdot in$

Bar area............................. $A_{bar} = 0.200 \cdot in^2$

Bar diameter......................... $dia = 0.500 \cdot in$

Area of steel provided per foot of back wall $A_s = 0.20 \cdot in^2$

Distance from extreme compressive fiber to centroid of reinforcing steel.... $d_s := t_{BW} - cover_{sub} - \frac{dia}{2} = 8.75 \cdot in$

Assume $f_s := f_y$ [LRFD 5.7.2.1]

Stress block factor........................... $\beta_1 := \min\left[\max\left[0.85 - 0.05 \cdot \left(\frac{f_{c.sub} - 4000psi}{1000psi}\right), 0.65\right], 0.85\right] = 0.78$

Distance between the neutral axis and compressive force........................... $c := \frac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot \beta_1 \cdot b} = 0.28 \cdot in$

Solve the quadratic equation for the area of steel required

Given $M_r = \left[\phi \cdot A_s \cdot f_s \cdot \left[d_s - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot b}\right)\right]\right]$

$A_{s.reqd} := \mathrm{Find}(A_s) = 0.03 \cdot in^2$

Check assumption that $f_s = f_y$.......... $Check\_f_s := \mathrm{if}\left(\frac{c}{d_s} < 0.6, \text{"OK"}, \text{"Not OK"}\right) = \text{"OK"}$

The area of steel provided, $A_s = 0.20 \cdot in^2$, should be greater than the area of steel required, $A_{s.reqd} = 0.03 \cdot in^2$. If not, decrease the spacing of the reinforcement. Once A_s is greater than $A_{s.reqd}$, the proposed reinforcing is adequate for the design moments.

Moment capacity provided..................... $M_{r.prov} := \phi \cdot A_s \cdot f_s \cdot \left[d_s - \frac{1}{2} \cdot \left(\frac{A_s \cdot f_s}{0.85 \cdot f_{c.sub} \cdot b}\right)\right] = 7.78 \cdot kip \cdot ft$

B3. Crack Control by Distribution Reinforcement [LRFD 5.7.3.4]

Concrete is subjected to cracking. Limiting the width of expected cracks under service conditions increases the longevity of the structure. Potential cracks can be minimized through proper placement of the reinforcement. The check for crack control requires that the actual stress in the reinforcement should not exceed the service limit state stress (LRFD 5.7.3.4). The stress equations emphasize bar spacing rather than crack widths.

The maximum spacing of the mild steel reinforcement for control of cracking at the service limit state shall satisfy.........

$$s \le \frac{700 \cdot \gamma_e}{\beta_s \cdot f_{ss}} - 2 \cdot d_c$$

where

$$\beta_s = 1 + \frac{d_c}{0.7(h - d_c)}$$

Exposure factor for Class 1 exposure condition.......................................

$$\gamma_e := 1.00$$ **[SDG 3.10]**

Overall thickness or depth of the component.............................

$$t_{BW} = 12 \cdot in$$

Distance from extreme tension fiber to center of closest bar..............................

$$d_c := cover_{sub} + \frac{dia}{2} = 3.25 \cdot in$$

$$\beta_s := 1 + \frac{d_c}{0.7(t_{BW} - d_c)} = 1.53$$

The neutral axis of the section must be determined to determine the actual stress in the reinforcement. This process is iterative, so an initial assumption of the neutral axis must be made.

Guess value $x := 1.4 \cdot in$

Given $$\frac{1}{2} \cdot b \cdot x^2 = \frac{E_s}{E_{c.sub}} \cdot A_s \cdot (d_s - x)$$

$$x_{na} := Find(x) = 1.36 \cdot in$$

Tensile force in the reinforcing steel due to service limit state moment......................

$$T_s := \frac{M_{BW}}{d_s - \frac{x_{na}}{3}} = 0.87 \cdot kip$$

Actual stress in the reinforcing steel due to service limit state moment......................

$$f_{s.actual} := \frac{T_s}{A_s} = 4.33 \cdot ksi$$

Required reinforcement spacing..........

$$s_{required} := \frac{700 \cdot \gamma_e \cdot \frac{kip}{in}}{\beta_s \cdot f_{s.actual}} - 2 \cdot d_c = 99.23 \cdot in$$

Provided reinforcement spacing..........

$$spacing = 12 \cdot in$$

The required spacing of mild steel reinforcement in the layer closest to the tension face shall not be less than the reinforcement spacing provided due to the service limit state moment.

$$LRFD_{5.7.3.4} := \begin{vmatrix} \text{"OK, crack control for M is satisfied"} \quad \text{if } s_{required} \ge spacing \\ \text{"NG, crack control for M not satisfied, provide more reinforcement"} \quad \text{otherwise} \end{vmatrix}$$

$LRFD_{5.7.3.4}$ = "OK, crack control for M is satisfied"

B4. Minimum Reinforcement

The minimum reinforcement requirements ensure the moment capacity provided is at least 1.2 times greater than the cracking moment.

Modulus of Rupture...............................

$$f_r := -0.24 \cdot \sqrt{f_{c.sub} \cdot ksi} = -562.8 \cdot psi$$

Section modulus...................................

$$S := \frac{b \cdot t_{BW}^2}{6} = 288 \cdot in^3$$

Flexural cracking variability factor...........

$$\gamma_1 := 1.6$$

Ratio of specified minimum yield strength to ultimate tensile strength of the reinforcement.......................................

$$\gamma_3 := 0.67$$

Cracking moment.................................

$$M_{cr} := f_r \cdot S \cdot \gamma_1 \cdot \gamma_3 = -14.48 \cdot kip \cdot ft$$

Required flexural resistance............

$$M_{r.reqd} := \min\left(\left|M_{cr}\right|, 133 \cdot \% \cdot M_r\right) = 1.35 \cdot kip \cdot ft$$

Check that the capacity provided, $M_{r.prov} = 7.8 \cdot ft \cdot kip$, exceeds minimum requirements, $M_{r.reqd} = 1.3 \cdot ft \cdot kip$.

$$LRFD_{5.7.3.3.2} := \begin{vmatrix} \text{"OK, minimum reinforcement for moment is satisfied"} \quad \text{if } M_{r.prov} \ge M_{r.reqd} \\ \text{"NG, reinforcement for moment is less than minimum"} \quad \text{otherwise} \end{vmatrix}$$

$LRFD_{5.7.3.3.2}$ = "OK, minimum reinforcement for moment is satisfied"

B5. Shrinkage and Temperature Reinforcement [LRFD 5.10.8]

Size of bar ("4" "5" "6" "7") $bar_{st} := bar$

Shrinkage reinforcement provided........... $bar_{spa.st} := spacing$

Bar area... $A_{bar} = 0.20 \cdot in^2$

Bar diameter................................... $dia = 0.500 \cdot in$

Minimum area of shrinkage and temperature reinforcement.....................

$$A_{ST} := \max\left[\begin{bmatrix} \left(0.11 \dfrac{in^2}{ft}\right) \\ \min\left[\begin{bmatrix} \left(0.6 \dfrac{in^2}{ft}\right) \\ \left[\dfrac{1.3 \cdot h \cdot t_{BW} \cdot \dfrac{kip}{in \cdot ft}}{2 \cdot (h + t_{BW}) \cdot f_y}\right] \end{bmatrix}\right] \end{bmatrix}\right] = 0.11 \cdot \frac{in^2}{ft}$$

Maximum spacing for shrinkage and temperature reinforcement......................

$$spacing_{ST.reqd} := \min\left(\frac{A_{bar}}{A_{ST}}, 3 \cdot t_{BW}, 18 \cdot in\right) = 18 \cdot in$$

The bar spacing should be less than the maximum spacing for shrinkage and temperature reinforcement

$$LRFD_{5.10.8} := \begin{vmatrix} \text{"OK, minimum shrinkage and temperature requirements"} & \text{if } bar_{spa.st} \le spacing_{ST.reqd} \\ \text{"NG, minimum shrinkage and temperature requirements"} & \text{otherwise} \end{vmatrix}$$

$LRFD_{5.10.8} =$ "OK, minimum shrinkage and temperature requirements"

C. Summary of Reinforcement Provided

Moment reinforcement (each face)

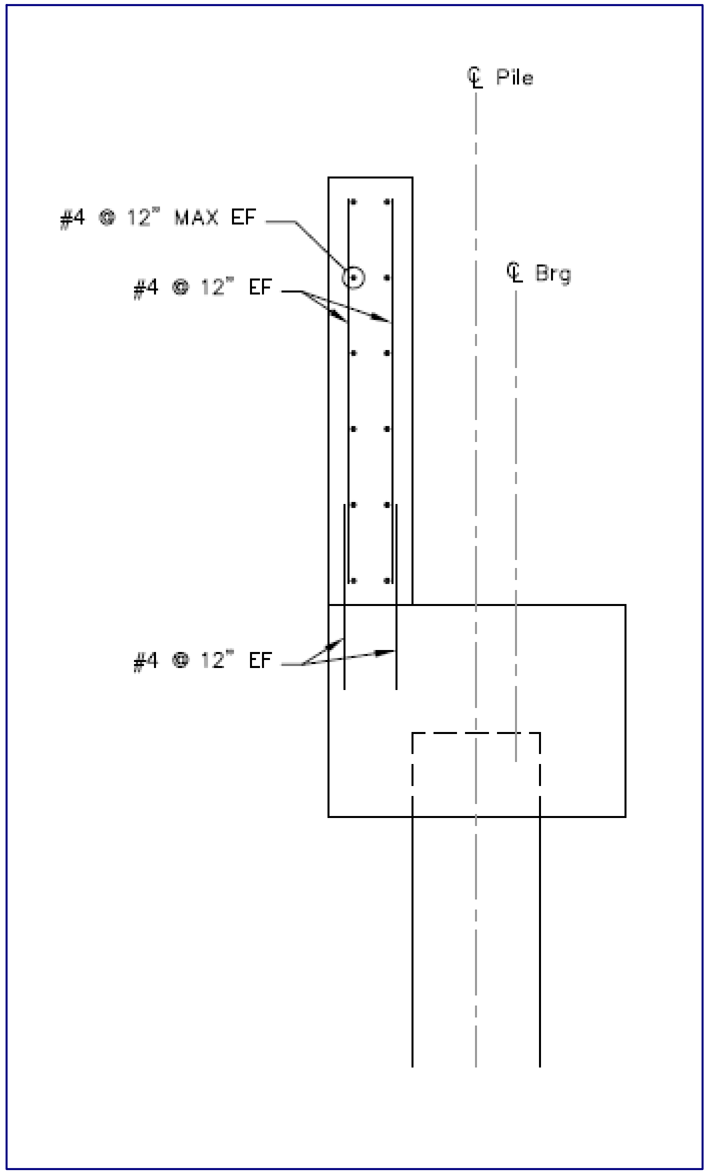
Bar size............ $bar = "4"$

Bar spacing....... $spacing = 12 \cdot in$

Temperature and Shrinkage

Bar size............ $bar_{shrink.temp} = "4"$

Bar spacing........ $bar_{spa.st} = 12 \cdot in$



Redefine Variables