Wind Generated Currents In Shallow Water

When wind blows over water it exerts a shear stress on the water surface. Although the wind shear stress is usually very small, its effect, when integrated over a large body of water, can be catastrophic (Dean and Dalrymple, 1991, p. 157). This stress can be estimated using any one of several formulas in the literature. Two of the more commonly used equations are those by Wu (1969) and Van Dorn (1953).

Wu's equation as presented in the RMA2 Users Guide (U.S. Army Corps of Engineers – Waterways Experiment Station, 1996, pp. 118-119) is:

$$\tau = \rho_{air}\ k_w\ W^2 \text{ ------------------------------------ } \quad \text{<1>}$$

where:

$\tau \equiv$ wind stress exerted on the water surface by the wind,

$\rho_{air} \equiv$ mass density of air (0.001226 g/cm^3 at STP)

$$k_w \equiv \begin{cases} \text{Wind Speed less than 100 cm/s} = 1.25/((\text{Wind Speed}/100)^{0.2}) \text{ x } 0.001 \\ 100 \text{ cm/s} < \text{Wind Speed} < 1500 \text{ cm/s} = ((\text{Wind Speed}/100)^{0.5}) \text{ x } 0.001 / 2 \\ \text{Wind Speed greater than 1500 cm/s} = 0.0026 \end{cases}$$

and

$W \equiv$ sustained wind speed in cm/s at a 10 m elevation above the water surface.

Van Dorn's equation (Dean and Dalrymple, 1991, p. 157):

$$\tau = \rho_{water}\ k_{vd}\ W^2 \text{ --------------------------------- } \quad \text{<2>}$$

where:

$\tau \equiv$ wind stress (in N/m^2) exerted on the water surface by the wind,

$\rho_{water} \equiv$ mass density of water =

(~ 62.3 lb$_m$/ft^3= 998 kg/m^3 at 68 degrees F, 0 ppt salinity),

(~ 64.0 lb$_m$/ft^3= 1025 kg/m^3 at 68 degrees F, 35 ppt salinity),

$$k_{vd} \equiv \begin{cases} 1.2 \text{ x } 10^{-6}\ ,\ W < W_C \\ 1.2 \text{ x } 10^{-6} + 2.25 \text{ x } 10^{-6}\left(1 - \dfrac{W_C}{W}\right)^2,\ W > W_C\ , \\ \text{where } W_C = 5.6 \text{ m/s} \end{cases}$$

and

W ≡ sustained wind speed (in m/s) at a 10 m height above the water surface.

The water surface layer is accelerated in the direction of the wind stress. Once this surface layer has attained a velocity, a stress is exerted on the adjacent layer. The development of the velocity profile continues until the bed is reached. This process takes some period of time, the magnitude of which depends on the level of turbulence in the water (which in turn depends on the wind speed, wave heights, etc.). This process is shown schematically in Figure 1.

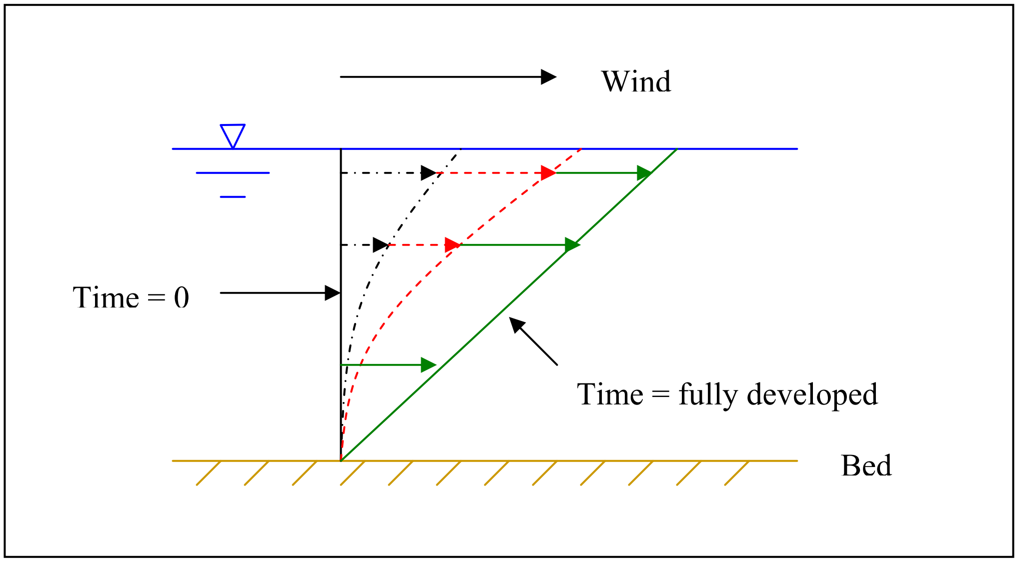

Figure 1. Development of Velocity Profile Due to Wind Stress (Not including Set-up)

At the same time the velocity profile shown in Figure 1 is being developed a second wind generated process is also occurring. This process is the development of water surface setup in the direction of the wind stress. The amount of setup is also dependent on a number of factors but it is interesting to note that the flow generated by the setup is in the direction opposite to the wind stress as shown in Figure 2.

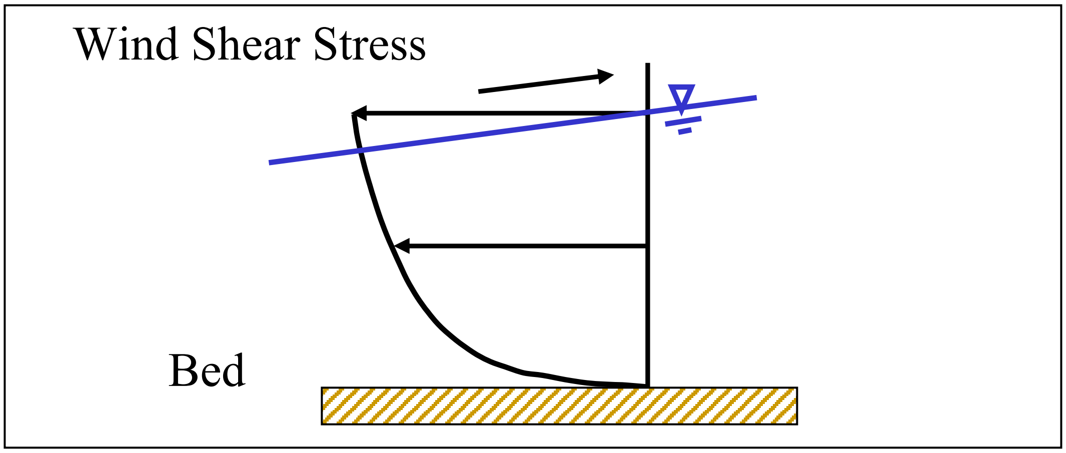

Figure 2. Velocity Profile Due To Wind Generated Setup Only.

When these two wind generated currents are superimposed the result is as shown in Figure 3.

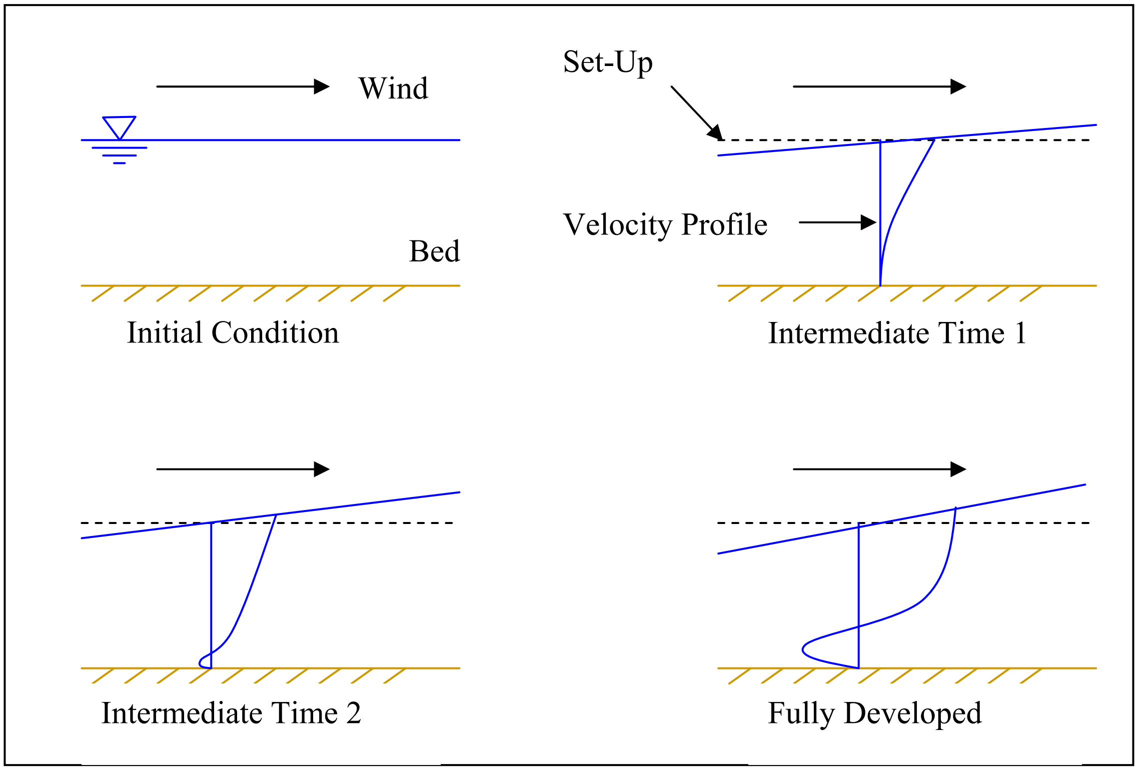

Figure 3. Velocity Profile Due to Wind Stress and Wind Generated Set-up.

Once the velocity profile has reached an equilibrium or ultimate state condition (something that may or may not occur during a hurricane) a relationship between the wind shear stress acting on the water surface and the water shear stress acting on the bed can be obtained (see e.g. Reed, 1957, Dean and Dalrymple, 1991, p. 158, Shore Protection Manual, 1977, p. 3-149).

$$\tau_{\text{bed}} = k\ \tau_{\text{surface}} \quad \text{---------------------------------} \quad <3>$$

where

$\tau_{\text{bed}} \equiv$ shear stress exerted on the bed by the water flow near the bed,
$k \approx 0.2$, and
$\tau_{\text{surface}} \equiv$ shear stress exerted on the water surface by the wind.

The local scour produced by such a three dimensional current profile is difficult to predict and will depend to a large extent on the structure shape. A bridge pier with a large footer (pile cap) near the bed will respond to the velocities near the bed (i.e. those that are directed opposite to the direction of the wind stress). For waterline pile cap piers the scour will respond more to the surface currents that are in the direction of the wind stress (although the secondary flows produced by the structure will be attenuated by the reverse flow in the lower portion of the water column).

Since most local scour prediction equations use depth-averaged current velocity [based on a fully developed (equilibrium) logarithmic profile], an "equivalent" logarithmic profile must be computed for the current in the lower portion of the water column (i.e. the depth-averaged value for the hypothetical logarithmic current that would produce the same bed shear stress as the current generated by the wind). This can be done as follows:

If the fully developed logarithmic velocity profile (for hydraulically rough beds) is expressed as:

$$v = 2.5\ u_* \ln\left(\frac{30\ z}{k\ D_{50}}\right), \quad \text{----------------------------} \quad <4>$$

where:

$v \equiv$ horizontal velocity as a function of the vertical coordinate, positive in the direction of the wind shear stress,
$u_* \equiv \sqrt{\dfrac{\tau_{\text{bed}}}{\rho_{\text{water}}}}$,
$\ln \equiv$ natural alogrithm (log to the base e),
$z \equiv$ the vertical coordinate with the origin at the bed,
$k \equiv$ relative bed roughness (relative to the sediment grain size, D_{50}), and
$D_{50} \equiv$ median grain diameter.

Integration of this expression over the water depth, y_0, results in the following expression:

$$\begin{aligned} V &= 2.5\, u_* \ln\left(\frac{11\, y_0}{k\, D_{50}}\right) \\ &= 2.5 \sqrt{\frac{\tau_{\text{bed}}}{\rho_{\text{water}}}} \ln\left(\frac{11\, y_0}{k\, D_{50}}\right), \end{aligned} \qquad \text{--------------------} \; <5>$$

where:

$V \equiv$ depth averaged velocity, and
$y_0 \equiv$ water depth.

Substitution of the expression for bed shear stress in terms of wind stress on the water surface in Equation 3 into Equation 5 yields:

$$V = 2.5 \sqrt{\frac{0.2\ \tau_{\text{surface}}}{\rho_{\text{water}}}} \ln\left(\frac{11\, y_0}{k\, D_{50}}\right), \text{--------------} \; <6>$$

Note that V is now the depth averaged velocity (for a logarithmic velocity) that would produce the same shear stress on the bed as that produced by the wind generated current.

With some “engineering judgment”, the velocity computed with Equation 6 can be used in the local scour equations to estimate local scour depths. In most cases this velocity will produce conservative local scour depths (i.e. deeper scour depths than will actually occur). Care must be exercised in deciding on the wind velocity to use in Equation 1. Equation 1 was developed for open water situations that are free of obstructions. In many situations there may be only a limited number of wind directions that are unobstructed (in some cases there will be no unobstructed directions). Tall trees on the banks of rivers or high density buildings cause (wind) flow separation and this flow does not reattach for distances of 30 or more obstruction heights down wind. In the wake region the flow velocity is attenuated and the direction variable. This, of course, is why wind rows of trees are planted along side crop fields to reduce soil erosion by wind.

The purpose of this document is not to provide an in-depth treatment of this subject but rather to 1) point out the three-dimensionality of wind generated currents and the inappropriateness of using two-dimensional depth-averaged computer models for their prediction, and 2) to provide some guidance as to how local scour depths can be estimated for wind generated currents.

REFERENCES

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