

# FLEXURAL RELIABILITY OF RC BRIDGE GIRDERS STRENGTHENED WITH CFRP LAMINATES

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## ABSTRACT

This paper presents the development of a resistance model for reinforced concrete bridge girders flexurally strengthened with externally bonded CFRP laminates. The resistance model is used to calculate the probability of failure and reliability index of CFRP strengthened cross-sections. The first order reliability method is employed to calibrate the flexural resistance factor for a broad range of design variables. The study shows that the addition of CFRP improves reliability somewhat because the strength of CFRP laminates has a lower coefficient of variation than steel or concrete. However, the brittle nature of CFRP laminates necessitates a reliability index that is greater than that generally implied in AASHTO-LRFD (1998). This leads to a resistance factor that is slightly lower than currently accepted for reinforced concrete sections in flexure.

## KEYWORDS

*Bridge rehabilitation, composite materials, reliability, Monte Carlo simulation, FORM.*

## INTRODUCTION

The technique whereby carbon fiber reinforced polymer laminates are externally bonded to reinforced or prestressed concrete girders is becoming more established as an alternative to traditional structural rehabilitation methods. Extensive research has shown that externally bonded CFRP laminates improve both short term (Ritchie et al. 1991, Saadatmanesh and Ehsani 1991, Jones and Swamy 1992, Triantafillou and Pelvis 1992, Arduini and Nanni 1997) as well as long term behavior (Shahawy and Beitelman 1999) of concrete girders. Based on the research conducted so far, ACI committee 440 is currently developing design guidelines for external strengthening of concrete structures using fiber reinforced polymer systems. With the exception of a few studies, most of the research conducted on CFRP strengthened structures has been done in a deterministic manner, and the statistical variations associated with the main design variables have been largely ignored.

Reliability-based techniques can be used to account for the randomness in important variables that affect the strength of CFRP strengthened concrete girders. The use of such methods in structural engineering has greatly increased in the past few years as reliability-based models have become better understood and more widely accepted. The recent surge in applications of the theory of reliability to structural engineering problems may be attributed to two main reasons. First, design codes have, and still are, being changed from the *Allowable Stress Design* approach to the *Strength Design* approach. Strength Design provisions in modern design codes are calibrated through reliability-based methods to ensure that the probability of failure,  $P_f$ , does not exceed a target level (Nowak 1995 and Kariyawasam et al. 1997). This approach allows designers to more rationally evaluate the possibility of structural collapse as opposed to Allowable Stress Design, which usually results in hidden reserve strength. The second reason

driving the increasing popularity of structural reliability is that it makes possible a new trend in thought whereby structural systems are characterized in a probabilistic manner rather than using deterministic strength to achieve a more rational balance between safety and life cycle costs (Val et al. 1997, Thoft-Christensen 1998, and Estes and Frangopol 1999).

One of the earliest studies of the reliability of concrete structures strengthened with CFRP was conducted by Pelvis et al. (1995). In their approach, a virtual design space comprised of a number of random parameters was created and used to study flexural reliability of RC beams strengthened with CFRP. Pelvis et al. proposed the use of a reduction factor for CFRP material strength,  $0cF''$ , together with a general resistance factor,  $\phi$ , for overall member flexural strength. The developed reliability model was used to calibrate the resistance factors for a variety of design situations. Although material level reduction factors are adopted by several design codes, their use constitutes a divergence from current U.S. practices in which a single overall albeit behavior specific reduction factor is used. Furthermore, the choice of design factors implied that the study is limited to only reinforced concrete beams in buildings.

In this paper, the reliability of reinforced concrete bridge girders strengthened with CFRP laminates is investigated. The study focuses on cross-sectional flexural behavior and has two specific goals: a) determine resistance models for RC cross sections rehabilitated with CFRP laminates, and b) develop appropriate design factors. The objectives of the paper are achieved through the following tasks:

1. Create a pool of bridge designs that cover a wide range of design parameters. The pool is comprised of a number of reinforced concrete bridges with different spans designed according to AASHTO-LRFD (1998). Each of the bridge designs is assumed to have suffered

various degrees of damage to the main steel reinforcement and is then strengthened back to its original design strength through externally bonded CFRP laminates.

2. Perform Monte Carlo simulations on each of the designed and rehabilitated bridges and use the resulting randomly generated data sets to develop a resistance model for cross-sectional flexural strength.
3. Determine the probability of failure of the designed sections and the reliability index,  $\beta$ , using a computer program which implements the first order reliability method (FORM).
4. Calibrate the flexural resistance factor,  $\phi$ .

Since flexural behavior is the focus of the paper, it is implicitly assumed that other modes of failure such as shear failure, laminate peel-off, and bond failure between laminates and concrete do not control behavior. Such modes of failure can be precluded by additional strengthening or through special detailing (Shahawy and Beitelman 1999).

## DESIGN OF BRIDGES

A broad range of realistic designs are required to investigate reliability and recommend resistance factors for reinforced concrete girders strengthened with CFRP laminates. Three simply supported bridges with the following spans are considered: 22,860mm (75ft), 18,288mm (60ft), and 13,716mm (45ft). The bridges are designated as RC75, RC60, and RC45, respectively. An interior girder for each bridge was chosen for this study and designed for flexure according to AASHTO-LRFD (1998).

The designed cross sections are then assumed to have lost a significant portion of the main reinforcing steel (possibly due to corrosion, vandalism, or collision by a truck). Three levels of damage are considered; namely a loss of 10%, 20% and 30% of the main steel. Rehabilitation schemes are then designed to return the damaged bridge sections to their original strength by externally bonding CFRP laminates to the beam stems. The CFRP laminates are wrapped around the stem of the beams and attached using epoxy adhesives. This technique has been shown to be successful for repair purposes as it reduces the likelihood of laminate peel-off or debonding (Shahawy and Beitelman 2000).

Although arbitrary, the chosen damage levels are realistic in that rehabilitation of the girders using CFRP is a feasible and practical alternative. The damaged bridges are referred to hereafter by appending 1310, D20, or D30 (which correspond to 10, 20, or 30% damage) to the bridge designation listed above. For example, for the 60-ft span design, the damaged bridges are designated RC60-D 10, RC60-1320, and RC60-1330 for 10, 20, and 30% damage levels respectively. Altogether, the number of bridge designs chosen for the reliability study is twelve. The inventory is comprised of 3 undamaged bridges, each having three variations reflecting the three damage levels considered.

### Bridge Geometry, Material Properties, and Loading

Figure 1 shows the cross section used for all bridge spans. The supported road is 10059mm wide (33ft 2in). The bridge cross-section is comprised of a 191mm-thick (7.5in) slab monolithic with five girders spaced at 2134mm. The concrete compressive strength is assumed to be  $f'_c=27.6\text{MPa}$  (4ksi) whereas the steel yield strength is  $f_y= 414\text{MPa}$  (60ksi).

It is assumed that the laminates used for strengthening have 0.23 yarns/mm (6 yarns/inch) in the longitudinal direction and 0.19 yarns/mm (5 yarns/inch) in the transverse direction, and each yarn consists of 12000 fibers. This laminate configuration is one of the configurations successfully used by the Florida Department of Transportation (FDOT) for repair purposes (Shahawy and Beitelman 1999). The tensile strength of CFRP fibers is assumed to be  $\sigma_{fiber} = 3.65 \text{ GPa}$  (530ksi). However, the laminate strength is different than the fiber strength, which must be adjusted to account for size and stress gradient effects.

Laminate strength can be estimated either analytically or through coupon tests. Coupon tests usually involve small specimens and therefore do not adequately capture size effects. They also do not explicitly account for the effect of stress gradients. The authors investigated the use of a method based on the Weibull theory for brittle material to estimate the strength of unidirectional CFRP laminates using the statistical properties of the constituent fibers (Okeil et al. 2000). The technique accounts for both the size effect and the existence of stress gradients, and has been verified through comparisons to experimental data. Application of this method to the designed bridges results in the laminate strengths listed in Table 1. The small variations in the design tensile strength of the CFRP laminates are due to differences in the volume of carbon fibers in the different designs; i.e. size effect. The theory also provides the coefficient of variation, COV, for the laminate strength, which turns out to be 2.2%.

The design bending moments for the interior girder are calculated for the dead loads and live loads according the AASHTO-LRFD specifications (AASHTO 1998). The maximum of "*Lane Load/Standard Truck*" and "*Lane Load/Tandem Load*" cases is considered as shown in Fig. 2. The truck or tandem portion of the live load moments is increased by an impact factor of 33%.

## Design of Cross Sections

The designed cross sections of the interior girder of each bridge are shown in Fig. 3. The pure carbon thickness (in the CFRP laminates) required to return the damaged girders to their original strength is calculated using the fiber section model described in the following section and are given in Table 2. It is worthwhile to note that the expected failure mode of all the rehabilitated cross-sections is steel yielding followed by rupture of the CFRP. Concrete crushing is unlikely because of the presence of the concrete deck, which acts as a flange for the girders.

The stress in the main reinforcing bars due to service loads (dead, live, and impact;  $M_{service}=M_D + M_{L+IM}$ ) is also shown in the table and is well below the yield stress for all cases. It is important to ensure that the service steel stress is well below yielding since overloading the steel can lead to a reduction in the effective stiffness of the member and can result in excessive permanent deformations, both of which create severe serviceability problems. El-Tawil et al (2000) discuss the maximum appropriate steel stress level for this type of serviceability check. Table 2 also shows the ratio of the flexural capacity provided by CFRP to the flexural capacity provided by steel reinforcement. For the 30% damage level, the CFRP laminates are providing about 45% of the moment capacity due to the steel reinforcement for all three spans.

## FIBER SECTION ANALYSIS

The fiber section technique is used to calculate the cross-sectional moment-curvature response of the designed bridges. As shown in Fig. 4, fiber section analysis of a composite cross section entails discretization of the section into many layers (fibers) for which the constitutive models are based on uniaxial stress-strain relationships. Each region represents a fiber of material running longitudinally along the member and can be assigned one of several constitutive models

representing concrete, CFRP, or reinforcing steel. The axial force and bending moment acting on a cross-section are evaluated as stress resultants through an iterative process that ensures compatibility and equilibrium within the cross-section. The iterative solution method used with the fiber section technique is documented elsewhere (El-Tawil et al. 2000).

### Constitutive Properties of Component Materials

The assumed constitutive properties for the component materials are shown in Fig. 5. The stress-strain response of CFRP is taken to be elastic-perfectly brittle whereas the stress-strain curve for steel is elastic-plastic with a post yield strain hardening of 1%. A nonlinear stress-strain relationship is assumed for concrete fibers in compression. Concrete is assumed to crack when it reaches its tensile strength calculated according to the ACI 318 Code (1999). Concrete tension stiffening is also accounted for as shown in Fig. 5(b). Details of the constitutive properties implemented in the model can be found in El-Tawil et al. (2000).

### CFRP Initial Condition

Rehabilitation of concrete structures using CFRP laminates usually takes place while the structure is subjected to a certain level of loading (taken equal to the dead load in this study). Therefore, CFRP laminates are not strained while concrete and steel are both strained at the time of strengthening. The analysis method takes into account this situation as shown in Fig. 4. Just prior to strengthening the cross section with CFRP laminates, the cross section is subjected to a threshold moment  $M_{in}$  resulting in the corresponding strain gradient shown in Fig. 4(c). Knowing that CFRP strains must be zero at this stage, and that subsequently applied moments

(beyond  $M_{in}$ ) will not result in identical strains in adjacent CFRP and concrete fibers, the following equation is applied:

$$\varepsilon_{CFRP,i} = \varepsilon_i - \varepsilon_{in,i}^{CFRP} \quad (1)$$

As shown in Figure 4(d),  $\varepsilon_i$  is the strain in the CFRP fibers corresponding to a moment higher than  $M_{in}$  and calculated assuming that the strain in adjacent concrete and CFRP fibers is identical.  $\varepsilon_{in,i}^{CFRP}$  are the strains in concrete fibers adjacent to CFRP fibers at the threshold moment  $M_{in}$ .  $\varepsilon_{CFRP,i}$  are the adjusted CFRP strains for a moment greater than  $M_{in}$ .

#### Prestressed and Composite Cross Sections.

In addition to reinforced concrete cross-sections, the developed fiber section model is also capable of handling prestressed (or partially prestressed) cross sections and non-monolithic concrete decks. The effects of prestressing are taken into account through a two-stage process, which first satisfies compatibility and equilibrium for the initial prestressing conditions then analyzes the cross-section for the applied loads. Non-monolithic decks are cast onsite after concrete girders have been placed and been subjected to some dead and construction loads. The loading sequence associated with placement of non-monolithic decks is taken into account during the moment-curvature calculations using a process similar to that described above for CFRP laminates and is shown in Figs. 4(b) and 4(c).

## Convergence and Verification of Analysis

Convergence studies conducted using the developed model showed that employing more than 60 fibers to discretize reinforced concrete and prestressed concrete sections does not result in a significant improvement in accuracy. This number of fibers was therefore used in all analyses. The developed fiber model is verified by comparing analytical results to test data from the experimental work reported by Shahawy and Beitelman (1999). Table 3 gives the reinforcing steel and concrete properties for each of the studied beams and summarizes the results of the verification study. The designation of the beams denotes the number of CFRP layers used and nominal concrete strength; e.g. W-3L5 is strengthened with 3 layers and has a nominal concrete compressive strength of  $f'_c = 5$  ksi. The failure mode predicted by the analysis (yielding of steel reinforcement followed by rupture of CFRP laminates) was observed in the tests and the model is capable of accurately tracing the moment curvature response all the way up to failure. It is clear from Table 3 that the flexural capacities ( $M$ ) obtained from the analyses and the values observed from the tests are in good agreement. The average difference is -3.0% and the maximum difference is -6.8%. The flexural capacity as predicted by the analysis is on the conservative side for all beams except Beam W-4L5 where the analysis predicts a slightly higher  $M_{max} (\Delta M_{max} / M_{max} = +2.6\%)$ .

## MONTE CARLO SIMULATIONS

Sixty thousand data sets were randomly generated for the designed bridges. The data sets were generated by assuming appropriate probabilistic distributions for the uncertainties of the various parameters. A review of the literature on bridge and building structures was performed to identify the statistical properties of the various parameters affecting flexural behavior. Table 4

lists the range of statistical properties found in the literature. The table shows the bias (mean/nominal), coefficient of variation (COV =standard deviation/mean), and distribution type assumed by other researchers including Lu et al. (1994), Nowak et al. (1994), Pelvris et al. (1995), Thoft-Christensen (1998), Val et al. (1998), Crespo-Minguillon and Casas (1999), Estes and Frangopol (1999), Stewart and Val (1999). It is obvious from Table 4 that there is no clear consensus on specific values for many of the parameters involved. The bias and coefficient of variation adopted in the current study are also listed in Table 4. Based on the survey, all parameters were assumed to have a normal distribution except for the CFRP laminates, which were assumed to be a Weibull material. It is important to note that CFRP laminates have a relatively low bias and COV compared to steel or concrete. Both analytical and experimental results confirm this observation (Bullock 1974, Harlow and Phoenix 1981, Batdorf 1994, Bakht et al. 2000).

Each of the generated random data sets was analyzed using the fiber section model discussed previously. The resulting moment-curvature relationships for one of the designed RC cross section is shown in Fig. 6(a), where for clarity only 50 curves are drawn. Figure 6(b) shows an idealized moment-curvature relationship for an RC section strengthened with CFRP which is identified by major keypoints; cracking point, threshold moment point (point at which CFRP is bonded), yield point (point at which main steel yields), and ultimate point. Of interest in the reliability study is the flexural capacity (ultimate point) of the cross section. Figure 7 shows the distribution of the flexural resistance for Bridges RC60, RC60-D10, RC60-D20, and RC60-D30 combined. A Chi-squared goodness-of-fit study showed that all the distributions could be substituted with normal statistical distributions with reasonable accuracy.

Table 5 shows the results of the Monte Carlo simulations for all the bridges considered in the study. The information in Table 5 is referred to in the literature as the *Resistance Model* (Nowak and Collins 2000). Resistance models are especially helpful in reliability studies when the problem is highly nonlinear, as in this case where parameters such as steel yielding, concrete cracking and crushing, CFRP rupture, and CFRP initial condition all contribute to making the ultimate flexural strength a nonlinear function.

## RELIABILITY STUDY

### Reliability Index

The performance of a structure in flexure, shear, ...etc., can be represented by a limit state function, also known as a performance function, Z. In its simplest form, the limit state function is the difference between the random resistance of the member, R, and the random load effect acting on the member, Q.

$$Z=R-Q \quad (2)$$

A general limit state function involves a number of random variables,  $X_1, X_2, \dots, X_n$ , representing dimensions, material properties, loads ... etc. Accordingly, Z becomes a random vector  $g(\cdot)$  where

$$Z = g(X_1, X_2, \dots, X_n) \quad (3)$$

Such a general limit state function represents a failure surface, which divides the design space into safe and unsafe designs as can be seen in Fig. 8 for a simple 2-dimensional design space.

The probability of failure,  $P_f$ , for the limit state under consideration can be represented by the Reliability Index,  $\beta$ . The relationship between the reliability index and the probability of failure is

$$P_f = \Phi(-\beta) \quad (4)$$

where  $\Phi(\cdot)$  is the Cumulative Distribution Function (CDF) of the limit state function under consideration. The reliability index can be determined using the following expression

$$\beta = \frac{\mu_z}{\sigma_z} \quad (5)$$

where  $\mu_z$  and  $\sigma_z$  are respectively the mean value and the standard deviation of the Probability Density Function (PDF) of Z. Design codes are developed to result in structures with a  $P_f$  corresponding to a reliability index between 3.0 and 3.75. The range of  $\beta$  is due to many factors such as the importance of the structure, the expected mode of failure, the ratio of live loads to dead loads, ...etc. (Allen 1992).

### First Order Reliability Method (FORM)

The mean and standard deviation ( $\mu_z$ ,  $\sigma_z$ ) of the joint PDF of Z in Eq. 3 are needed to determine the reliability index. Determining these values is not straight forward, especially for the case of a complex limit state function. Several methods are used to determine the reliability index (Ayyub and McCuen 1997). The First Order Reliability Method (FORM) is chosen to study the reliability of the analyzed cross sections. FORM is based on a first order Taylor series expansion of the limit state function, which approximates the failure surface by a tangent plane at

the point of interest. According to FORM, the mean and variance of Z are evaluated and given as

$$\mu_Z \equiv g(\mu_1, \mu_2, \dots, \mu_n) \quad (6)$$

$$\sigma_Z^2 \equiv \sum_{i=1}^n \sum_{j=1}^n \left( \frac{\partial Z}{\partial X_i} \right)_\mu \left( \frac{\partial Z}{\partial X_j} \right)_\mu \text{covariance}(X_i, X_j) \quad (7)$$

where the partial derivatives  $\left( \frac{\partial Z}{\partial X_i} \right)_\mu$  and  $\left( \frac{\partial Z}{\partial X_j} \right)_\mu$  are evaluated at the mean of the basic random variables. In the case of uncorrelated variables, Eq. 7 reduces to

$$\sigma_Z^2 \equiv \sum_{i=1}^n \left( \frac{\partial Z}{\partial X_i} \right)_\mu^2 \sigma_{(x_i)}^2 \quad (8)$$

The most probable failure point is the mean. In the design space, this point is located on the failure surface Z (Eq. 3) such that distance from the origin of the design space to the tangent plane to the failure surface is shortest (see Fig. 8). To locate such a point, an iterative process is needed. The iterations are executed on transformed standard normally distributed random vectors. A detailed description of the process can be found in Estes and Frangopol (1998).

## Results

Due to the complexity of the problem, the limit state function used in this study is simplified as

$$Z = \alpha M_R - (M_D + \eta M_L) \quad (9)$$

Tables 4 and 5 give the bias and COV of the load and resistance models used in this study. The uncertainty of the analysis model is accounted for by using the random variable,  $a$ , which, according to Ellingwood et al. (1980), has a bias = 1.01 and COV = 4.5% for RC sections in flexure. Comparisons of fiber section model results with experimental data show that these values are reasonable for RC sections strengthened with CFRP (Okeil et al. 2000). The live load moment is also treated to account for the dynamic impact and the number of loaded lanes. According to Nowak and Collins (2000) the static live load moment is increased by 10% to account for impact caused by two trucks for all lanes. The COV of the joint live load and dynamic load is suggested to be taken as 18%. The live load is also increased by 5% to account for the heavy traffic volume assumed in this study (ADTT=5000 and 2 loaded lanes). The uncertainty of girder distribution factors (DF) is also accounted for by the random variable,  $\eta$ . The bias and COV of  $\eta$  were taken as 0.924 and 13.5%, respectively according to Kennedy et al. (1992).

The calculated reliability index values are listed in Table 5 for all 12 cases. An examination of Table 5 shows that the reliability index for CFRP strengthened beams increases with the increase of CFRP ratio in the cross section. This is expected since the failure mode is governed by rupture of the CFRP laminates which have a lower COV than steel or concrete. Nevertheless, it is important to point out that it is desirable for RC sections strengthened with CFRP laminates to have greater reliability because failure is more brittle.

## RESISTANCE FACTOR, $\phi$

Target  $\beta$

LRFD design codes typically set the strength requirement in the following form

$$\phi R \geq \gamma_{Q_i} Q_i \quad (10)$$

where the resistance factor,  $\phi$ , and the load factors,  $\gamma_{Q_i}$ , are calibrated to ensure that a target reliability index is achieved. The target reliability index is maintained around 3.5 for structures designed according to AASHTO-LRFD (1998). The reliability study showed that  $\beta$  ranges between 3.37 and 3.44 for the unstrengthened RC sections, which is close to the code target. On the other hand, the reliability index for CFRP strengthened sections is greater than that for RC sections and ranges from 3.47 to 3.83. Although addition of CFRP improves the reliability index, the brittle nature of CFRP behavior imposes more stringent demands on the target reliability index.

Allen (1992) investigated the rationale behind defining a target reliability index. According to his proposed approach, the choice of  $\beta$  depends on the component behavior, system behavior, inspection level, and traffic category. He suggests that the target reliability index should be increased by  $\Delta\beta = 0.25$  for components that fail suddenly with little warning but maintain their post-failure capacity, and by  $\Delta\beta = 0.5$  for components that suffer a sudden and complete loss of capacity. RC sections provide ample warning if properly designed since steel yielding results in significant ductility. The introduction of CFRP laminates changes the behavior and causes failure (rupture of CFRP) to happen at smaller deformations. However, the section maintains a post

failure capacity equal to that of the unstrengthened cross section (see Fig. 6). Following Allen's argument, it is proposed to calibrate Eq. 10 such that the target reliability index for RC sections flexurally strengthened with CFRP  $\beta_{RC-CFRP}^{target}$  is

$$\beta_{RC-CFRP}^{target} = \beta_{RC} + 0.25 \quad (11)$$

where PRC is the reliability index of the unstrengthened RC cross section. Calibrating Eq. 10 can be done by either changing the reduction factor,  $\phi$ , the load factors,  $\gamma_{Qi}$ , or both. Since the used load factors are uniform for other AASHTO-LRFD (1998) provisions, it is more convenient to recalibrate the reduction factor.

Figure 9 shows the effect of changing the reduction factor,  $\phi$ , for Bridges RC60-D10, RC60-D20, and RC60-D30. The figure is a relationship between  $\phi$  and the sum of the squares of  $(\beta - \beta_{RC-CFRP}^{target})^2$ . The lowest point on the relationship curve is the optimum point that would maintain the smallest error for all cases and is determined by nonlinear regression. Results of the optimization are given in Table 6. It can be seen that if all damaged cases are considered a  $\phi$  of 0.902 is needed to maintain  $\beta_{RC-CFRP}^{target}$ . When considering each damage level separately,  $\phi$  would be 0.881, 0.904, and 0.918 for the 10%, 20%, and 30% damage levels, respectively. Maintaining  $\phi$  at 0.9 would violate  $\beta_{RC-CFRP}^{target}$  for the low damage levels (10%). To reach a safe design that encompasses a wide range of damage levels,  $\phi$  is taken as 0.85. The next section shows the effect of using  $\phi = 0.85$  on the design of a wide range of damage levels and L/D ratios.

## **Effect of $\phi = 0.85$**

To study the impact of using  $\phi = 0.85$  for the design of RC sections strengthened with CFRP, the cross sections in Fig. 3 are used. Each cross-section is subjected to a range of  $M_L/M_D$  ratios varying from 0.0 (dead load only) to 4.0 (very high live load). The reliability index,  $\beta$ , is determined for all  $M_L/M_D$  ratios and damage levels using FORM. Figure 10 is a plot of the effect of  $M_L/M_D$  on the reliability index. It can be seen that using  $\phi = 0.85$  results in cross-sections with a reliability index,  $\beta$ , that conforms to what current codes normally target for a wide range of  $M_L/M_D$ .  $\beta$  falls below acceptable limits only in the case of unrealistically high  $M_L/M_D$  ratios. The figure again shows that the use of more CFRP enhances the reliability of the cross section because of the low COV of CFRP materials as discussed earlier.

## **SUMMARY AND CONCLUSIONS**

The flexural reliability of bridge girders flexurally strengthened with CFRP laminates has been investigated. A detailed nonlinear analysis model that accounts for material nonlinearities and the condition at time of CFRP placement is developed. Monte Carlo simulations were performed using the developed model to determine the resistance models for bridge girder cross sections strengthened with CFRP. A study using the developed resistance models showed that the reliability index of the strengthened cross sections is greater than that of the reinforced concrete sections and increases with increasing CFRP ratio. This is attributed to the low coefficient of variation for CFRP ultimate strength, which is lower than the coefficient of variation of the strength of steel or concrete. Although the reliability index improves with addition of CFRP, the flexural resistance factor is recommended as  $\phi = 0.85$ , which is lower than

that recommended by AASHTO-LRFD for RC sections under flexure. The reduced  $\phi$  value is calibrated to result in a larger target reliability index than is normally specified in recognition of the brittle nature of CFRP behavior.

This study focused solely on flexural behavior of cross-sections strengthened with CFRP. Further research is needed to investigate the probabilistic nature of other modes of failure including shear resistance of beams strengthened with CFRP laminates as well as peel-off and debonding of laminates.

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Table 1: Usable tensile stress used in design of RC Bridges

| Bridge Case | Damage | $\sigma_{CFRP}$ |
|-------------|--------|-----------------|
|             |        | GPa [ksi]       |
| RC75        | 10%    | 1.99 [289.2]    |
|             | 20%    | 1.97 [285.7]    |
|             | 30%    | 1.95 [283.5]    |
| RC60        | 10%    | 2.01 [291.4]    |
|             | 20%    | 1.98 [287.8]    |
|             | 30%    | 1.97 [285.7]    |
| RC45        | 10%    | 2.02 [292.7]    |
|             | 20%    | 2.00 [289.3]    |
|             | 30%    | 1.98 [287.4]    |

Table 2: Design summary of bridge cross sections

| Bridge | Steel<br>Damage<br>Level | CFRP thickness<br>$t_{CFRP}$ (mm) | $\frac{M_{CFRP}}{M_{steel}}$ | Steel Stress at<br>Service<br>% of $f_y$ |
|--------|--------------------------|-----------------------------------|------------------------------|--|
|        |                          |                                   |                              |  |
| RC75   | 0%                       | --                                | --                           | 66.2                                     |
|        | 10%                      | 0.145                             | 0.12                         | 72.4                                     |
|        | 20%                      | 0.280                             | 0.26                         | 80.0                                     |
|        | 30%                      | 0.420                             | 0.44                         | 89.5                                     |
| RC60   | 0%                       | --                                | --                           | 65.2                                     |
|        | 10%                      | 0.135                             | 0.12                         | 71.1                                     |
|        | 20%                      | 0.265                             | 0.27                         | 78.3                                     |
|        | 30%                      | 0.390                             | 0.45                         | 87.3                                     |
| RC45   | 0%                       | --                                | --                           | 64.8                                     |
|        | 10%                      | 0.130                             | 0.13                         | 70.3                                     |
|        | 20%                      | 0.247                             | 0.27                         | 77.1                                     |
|        | 30%                      | 0.365                             | 0.45                         | 85.4                                     |

Table 3: Comparison of Failure Moments

| Specimen | Material Strength, MPa [ksi] |                                |                                 |         | Flexural Capacity (M»..)  |                        |                 |
|----------|------------------------------|--------------------------------|---------------------------------|---------|---------------------------|------------------------|-----------------|
|          | Yield Stress<br>( $f_y$ )    | Concrete Strength<br>( $f_c$ ) | CFRP Strength<br>( $\sigma_f$ ) | COV (%) | Experiment kN-rn [kip-in] | Analysis kN-m [kip-in] | Difference* (%) |
| W-1L5    |                              | 35.9 [5.2]                     | 2200[314.4]                     |         | 211.4 [1871]              | 205.0 [1813]           | -3.1            |
| W-2L5-A  |                              | 37.2 [5.4]                     | 2140[310.4]                     |         | 259.5 [2300]              | 243.5 [2155]           | -6.3            |
| W-2L5-B  | 441[64]                      | 35.1 [5.1]                     | 2140[310.4]                     | 2.2     | 259.9 [2300]              | 242.2 [2143]           | -6.8            |
| W-3L5    |                              | 35.1[5.11]                     | 2120[308.1]                     |         | 282.5 [2500]              | 278.5 [2464]           | -1.4            |
| W-4L5    |                              | 35.1 [5.1]                     | 2110[306.4]                     |         | 305.1 [2700]              | 313.1 [2770]           | +2.6            |
|          |                              |                                |                                 |         | Average                   |                        | -3.0            |

\* (+) indicates unconservative prediction, (-) indicates conservative prediction

Table 4: Statistical properties of variables involved in the study

| Variable                                     | Other Researchers |           | Current Study |           | Distribution Type |                           |
|--|-------------------|-----------|---------------|-----------|-------------------|---------------------------|
|  |                   | Bias      | COV (%)       | Bias      | COV (%)           |                           |
| Dimensions ( $h, d, b$ )                     |                   | 1.00-1.03 | 0.5-7.0       | 1.00      | 3.0               | Normal                    |
| Area of steel ( $A_s$ )                      |                   | 1.00      | 0.0-4.0       | 1.00      | 1.5               | Normal - Deterministic    |
| Concrete strength ( $f'_c$ )                 |                   | 0.81-1.25 | 9.0-21.0      | 1.10      | 18.0              | Normal - LogNormal        |
| Rebars yield stress ( $f_y$ )                |                   | 1.00-1.22 | 8.0-13.0      | 1.10      | 12.5              | Normal - Beta - LogNormal |
| CFRP failure strain ( $\epsilon_{u,CFRP}$ )* | Analytical        | 1.33      | 7.4-10.0      | 1.10      | 2.2               | Weibull                   |
|  | Experimental      | --        | 2.2-5.1       | --        | -                 |                           |
| Model Uncertainty (a)                        |                   | 1.01-1.10 | 4.5-12.0      | 1.01      | 4.5               | Normal                    |
| Uncertainty of Girder DF ( $\eta$ )          |                   | 0.89-1.02 | 9.1-14.0      | 0.924     | 13.5              | Normal                    |
| Dead Load (D)                                |                   | 1.00-1.05 | 8.2-25.0      | 1.05      | 10.0              | Normal                    |
| Live Load (L)                                | Buildings         | 1.20      | 9.0-25.0      | --        | --                | Extreme Event I           |
|  | Bridges           | 1.25-1.52 | 12.0-41.0     | 1.35-1.38 | 18.0              | Normal - Modified Normal  |

\* analytical results used by Pelvis et al. (1995) and in this study; experimental results are reported in Baicht et al. (2000).

Table 5: Results of Monte Carlo simulation (moment units in kN.mm)

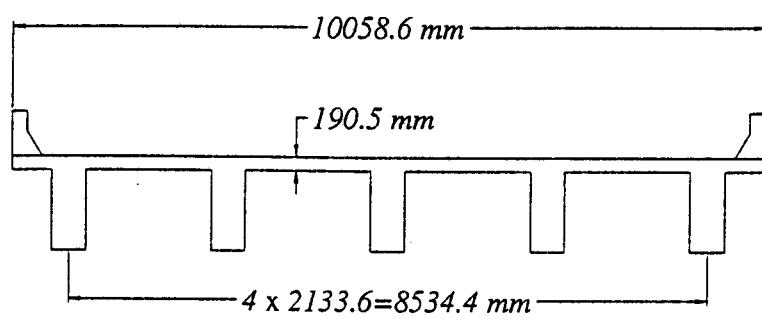
| Case     | $M_L/M_D$ | $M_n$              | $M_R$              |       |            | Reliability Index<br>$\beta$ |
|----------|-----------|--------------------|--------------------|-------|------------|------------------------------|
|          |           |                    | Value              | Bias  | COV<br>(%) |                              |
| RC45     |           |                    | $2.59 \times 10^6$ | 1.149 | 9.73       | 3.37                         |
| RC45-D10 |           | $6 \times 10^6$    | $2.57 \times 10^6$ | 1.138 | 8.62       | 3.47                         |
| RC45-D20 | 1.78      | $2.26 \times 10^6$ | $2.56 \times 10^6$ | 1.136 | 7.66       | 3.59                         |
| RC45-D30 |           |                    | $2.55 \times 10^6$ | 1.132 | 6.71       | 3.69                         |
| RC60     |           |                    | $4.49 \times 10^6$ | 1.151 | 9.80       | 3.41                         |
| RC60-D10 |           |                    | $4.41 \times 10^6$ | 1.130 | 8.68       | 3.49                         |
| RC60-D20 | 1.28      | $3.90 \times 10^6$ | $4.42 \times 10^6$ | 1.132 | 7.69       | 3.65                         |
| RC60-D30 |           |                    | $4.39 \times 10^6$ | 1.126 | 6.79       | 3.75                         |
| RC75     |           |                    | $7.38 \times 10^6$ | 1.150 | 9.80       | 3.44                         |
| RC75-D10 |           |                    | $7.24 \times 10^6$ | 1.127 | 8.68       | 3.53                         |
| RC75-D20 | 0.92      | $6.42 \times 10^6$ | $7.23 \times 10^6$ | 1.126 | 7.74       | 3.69                         |
| RC75-D30 |           |                    | $7.21 \times 10^6$ | 1.123 | 6.80       | 3.83                         |

**Table 6: Optimum  $\phi$  to achieve  $\beta_{\text{RC-CFRP}}^{\text{target}}$**

| Case          |     | Optimum $\phi$ |
|---------------|-----|----------------|
|               | D10 | 0.883          |
| RC45          | D20 | 0.902          |
|               | D30 | 0.916          |
|               | D10 | 0.879          |
| RC60          | D20 | 0.904          |
|               | D30 | 0.917          |
|               | D10 | 0.881          |
| RC70          | D20 | 0.905          |
|               | D30 | 0.921          |
| All cases     |     | 0.881          |
| All D20 cases |     | 0.904          |
| All D30 cases |     | 0.918          |
| All cases     |     | 0.902          |

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**Figure 1: Cross section of 5-girder bridge**

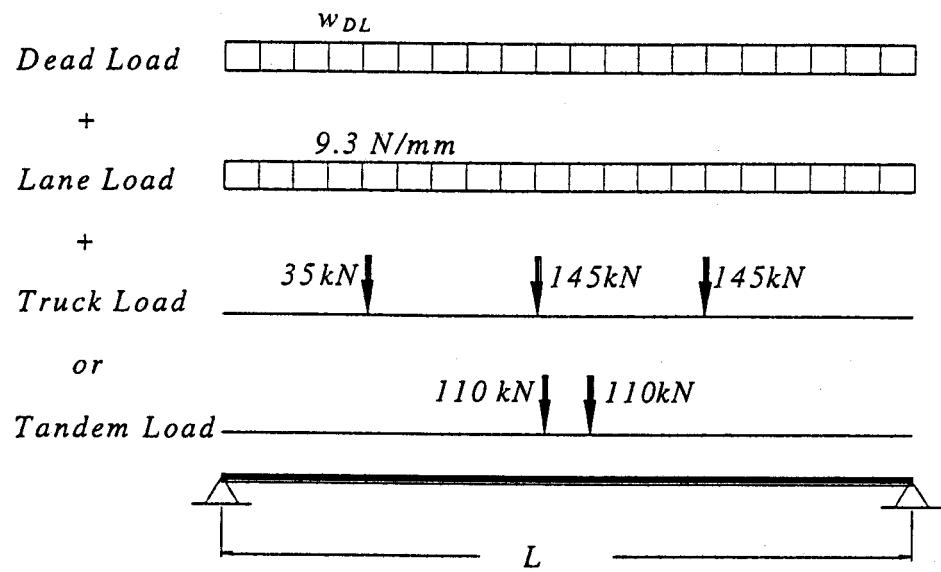
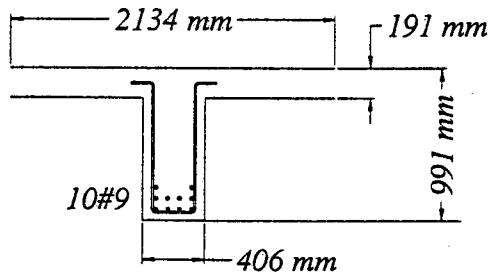
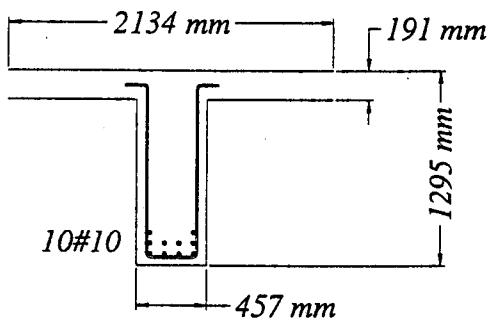


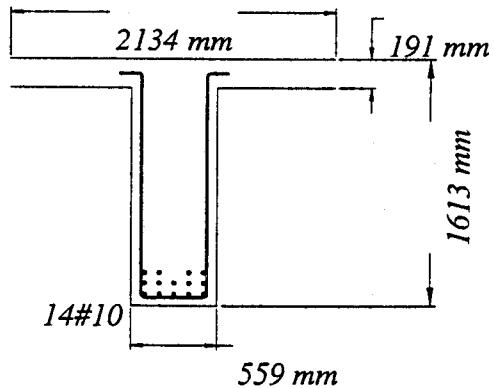
Figure 2: Loading cases considered in design of bridges



(a) RC45



(b) RC60



(c) RC75

Figure 3: Cross sections of undamaged interior bridge girders.

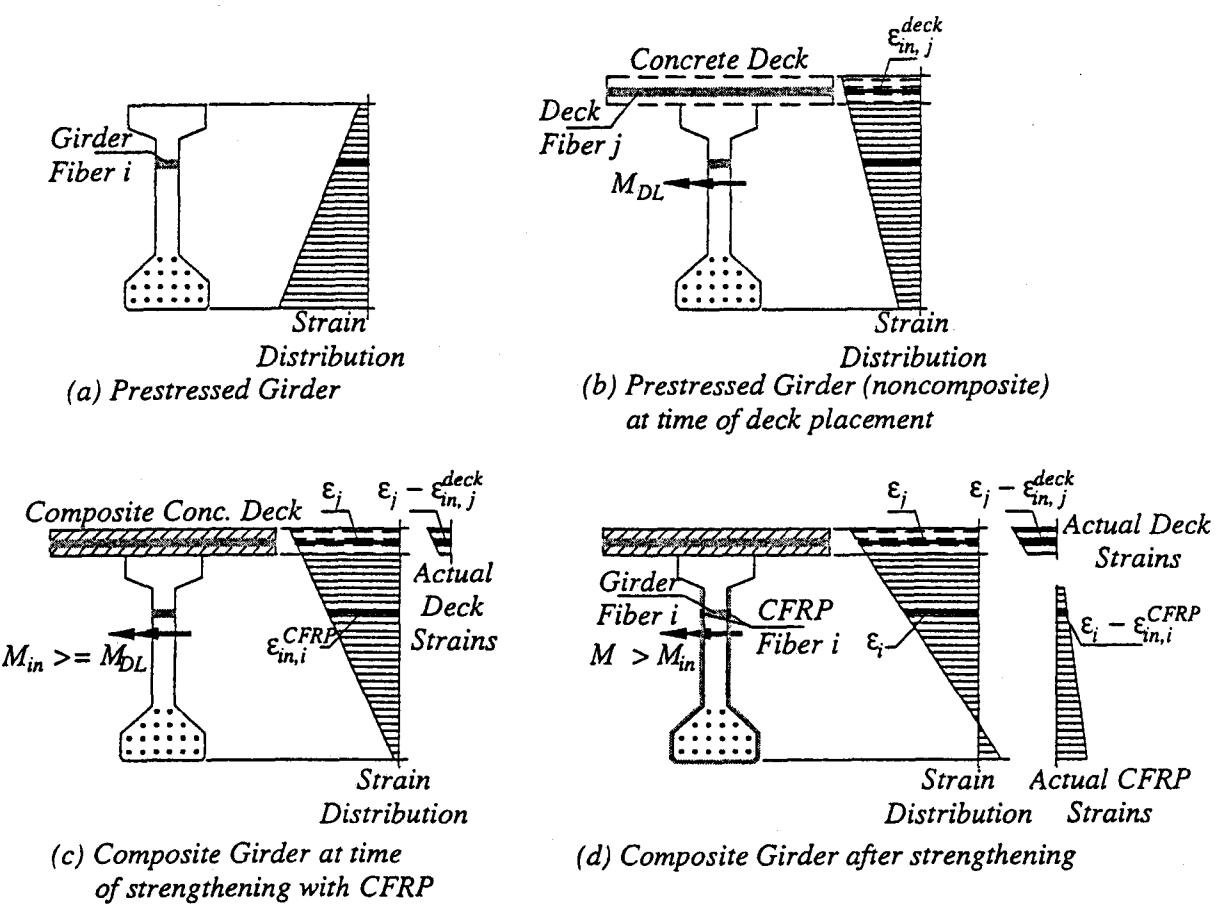
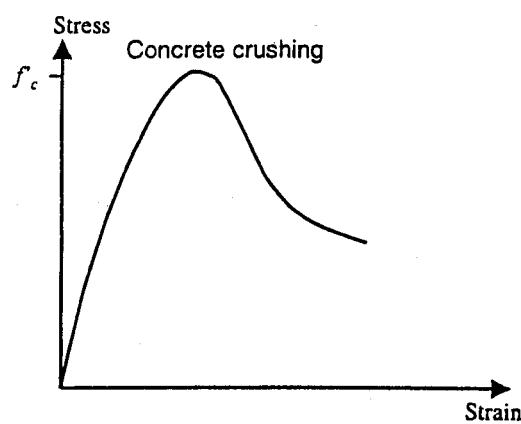
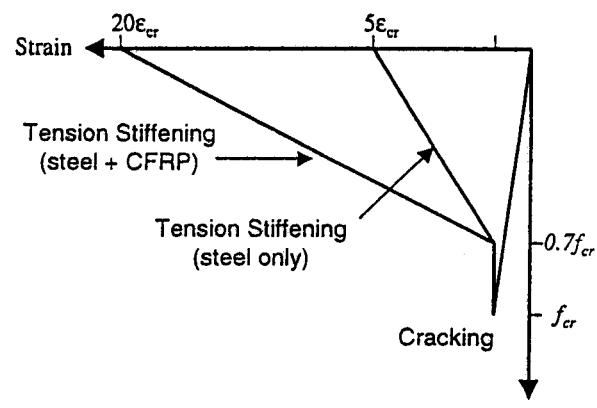


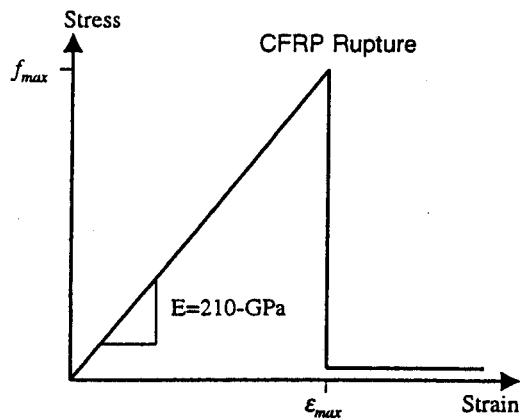
Figure 4: Sequence of analysis for girders with CFRP-strengthened girders with composite decks.



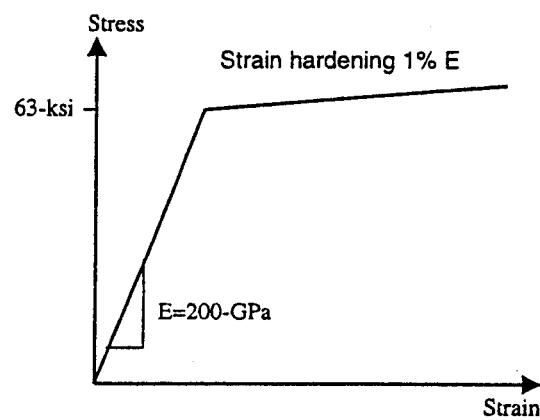
(a) Concrete in compression



(b) Concrete in tension

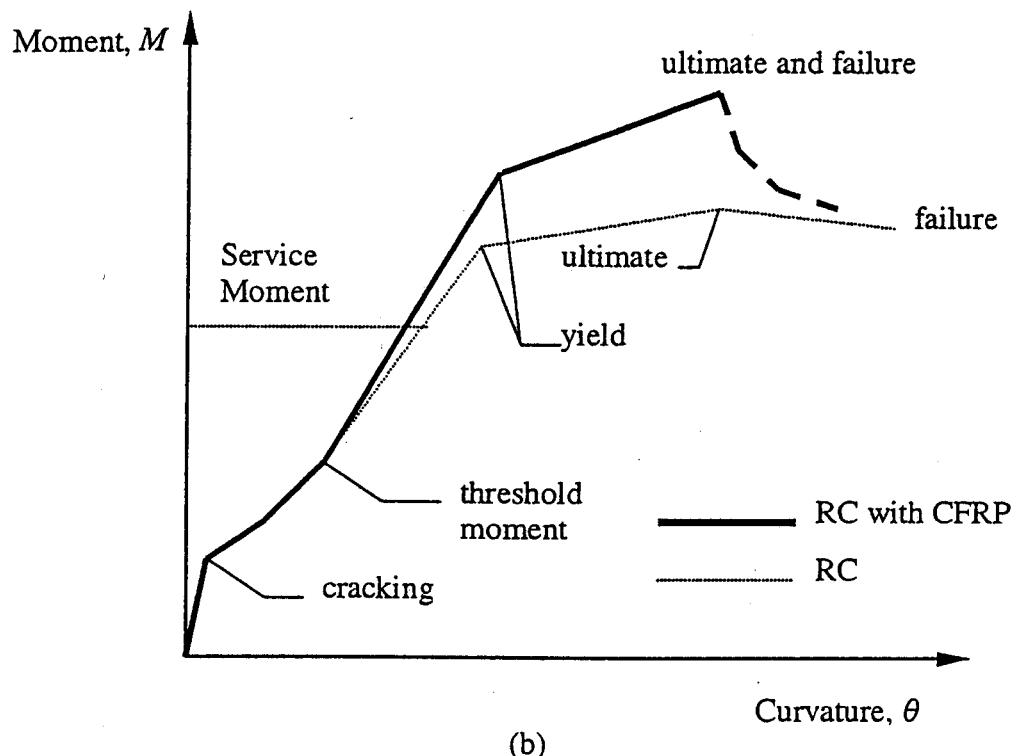
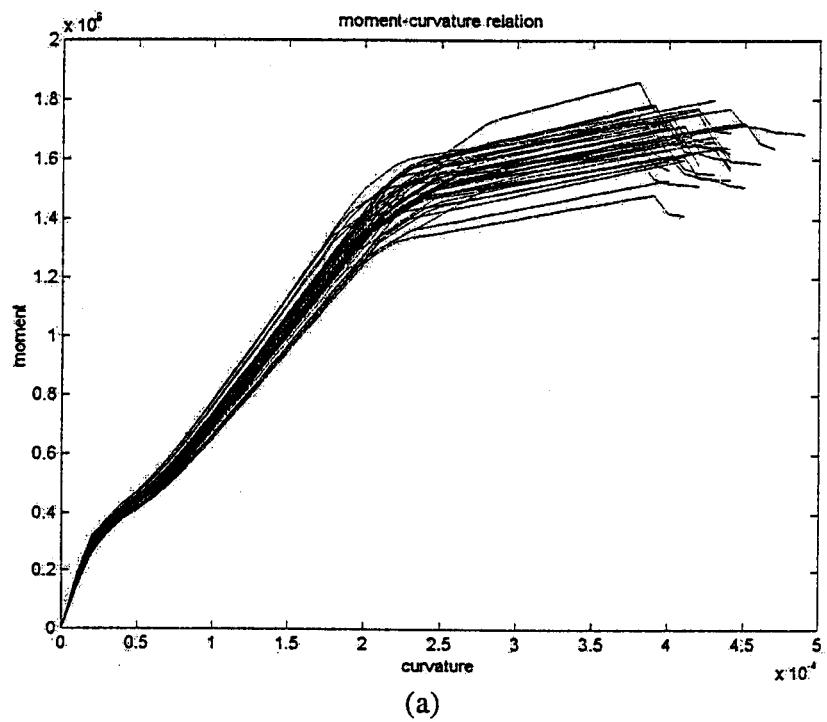


(c) CFRP in tension

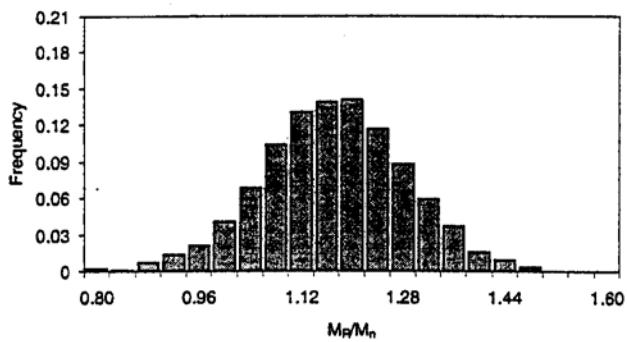


(d) Steel in tension or compression

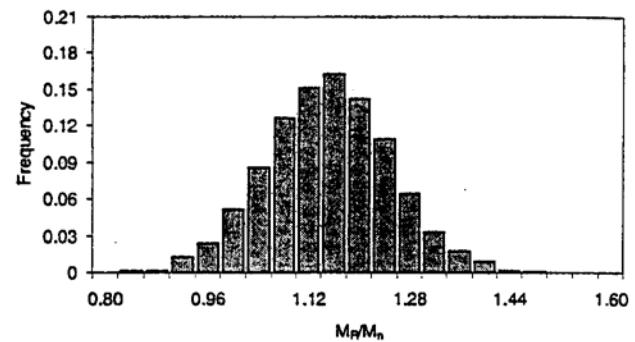
Figure 5: Monotonic constitutive models for component materials



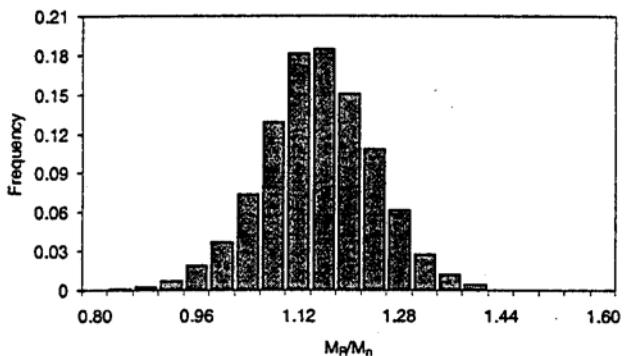
**Figure 6:** Moment-curvature relationship (a- as obtained from Monte Carlo simulation (50 cases shown), b- idealized  $M-\theta$ )



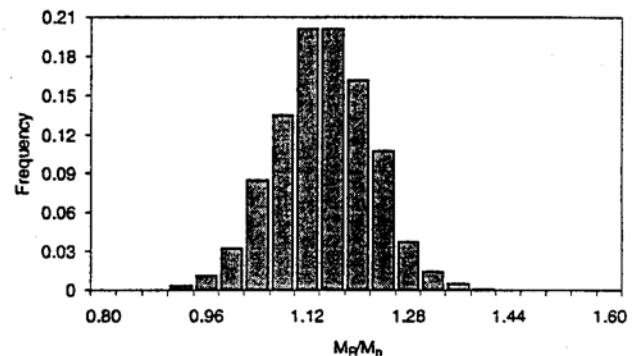
(a) RC60



(b) RC60-D10

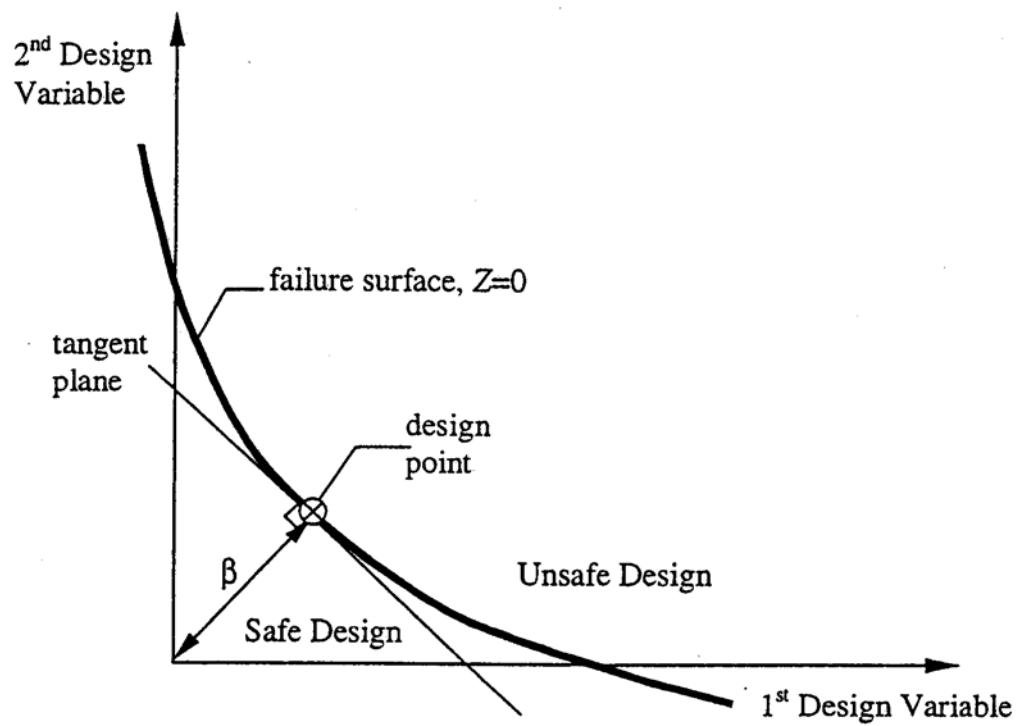


(c) RC60-D20

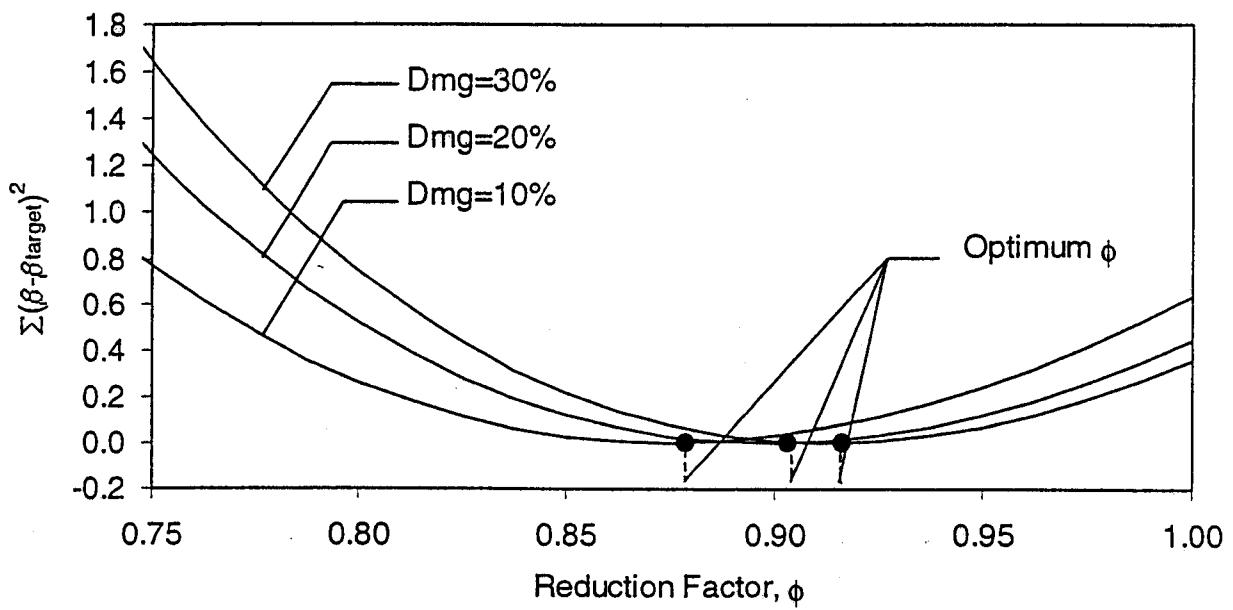


(d) RC60-D30

**Figure 7: Histograms of flexural resistance for Bridges RC60, RC60-D10, RC60-D20, and RC60-D30.**



**Figure 8: A simple reduced design space showing the design point, reliability index,  $\beta$ , and limit state function, Z.**



**Figure 9:** Determining reduction factor,  $\phi$ . (Bridges RC60-D10, RC60-D20, and RC60-D30)

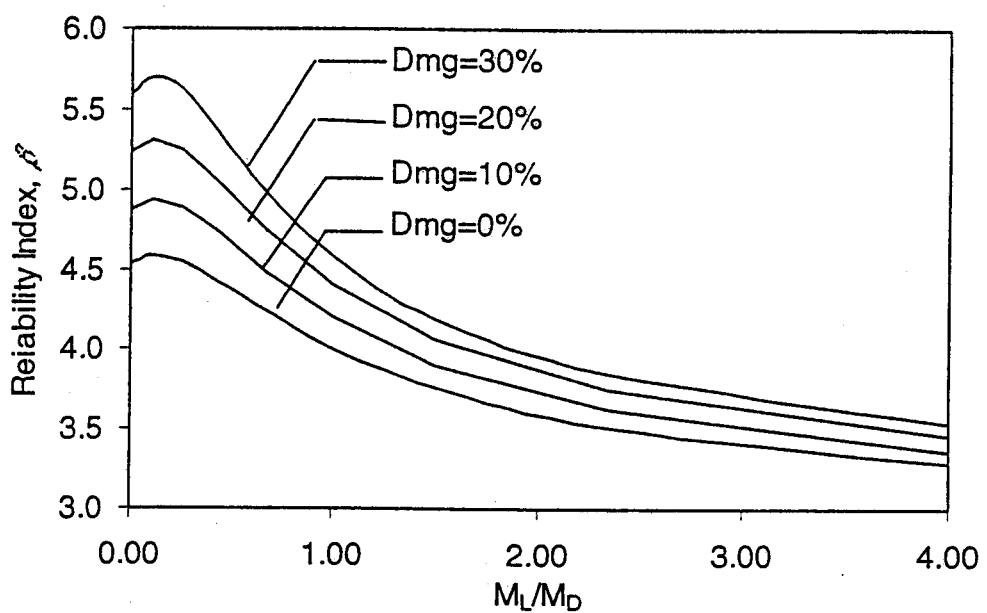


Figure 10: Effect of  $M_L/M_D$  on Reliability Index,  $\beta$ . (Bridge RC45,  $\phi = 0.85$ )