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Report No. BD545 RPWO #30  
FINAL REPORT

Date: January 2008

Contract Title: Prevention of Splitting Failure at Ends of Prestressed Beams during Fabrication  
UF Project No. 00051118  
Contract No. BD545 RPWO #30

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**PREVENTION OF SPLITTING FAILURE AT ENDS OF PRESTRESSED  
BEAMS DURING FABRICATION**

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**Technical Report Documentation Page**

1. Report No. BD545 RPWO #30		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle  Prevention of Splitting Failure at Ends of Prestressed Beams during Fabrication				5. Report Date January 2008	
				6. Performing Organization Code	
7. Author(s) R. A. Cook and M. J. Reponen				8. Performing Organization Report No. 00051118	
9. Performing Organization Name and Address University of Florida Department of Civil Engineering 345 Weil Hall / P.O. Box 116580 Gainesville, FL 32611-6580				10. Work Unit No. (TRAIS)	
				11. Contract or Grant No. BD545 RPWO #30	
12. Sponsoring Agency Name and Address Florida Department of Transportation Research Management Center 605 Suwannee Street, MS 30 Tallahassee, FL 32301-8064				13. Type of Report and Period Covered Final Report	
				14. Sponsoring Agency Code	
15. Supplementary Notes					
16. Abstract <p>The purpose of this research project was to determine the cause of vertical cracks that sometimes occur near the ends of the bottom flange of prestressed AASHTO, Florida Bulb-T, and Florida U beams. These vertical cracks were primarily observed on long-span beams and occurred during the transfer of the prestressing force to the concrete. The project initiated with mail surveys submitted to three FDOT prestressed beam manufacturers in Florida to determine the extent and types of end cracking each manufacturer was experiencing during the production process of their AASHTO, Florida Bulb-T, and Florida U beams. Site visits followed the surveys to observe the beam production process and to discuss when each type of crack forms with field personnel.</p> <p>The primary recommendation resulting from this research was that the coefficient of friction be reduced at the ends of the beams in order to reduce the tension stresses caused by friction as the beam displaces and cambers. A possible method for accomplishing this is to provide embedded steel base plates at the ends of the beams. Requirements for manufacturer's to install embedded steel plates were implemented by the FDOT. Two years following this implementation, follow-up studies with the manufacturers indicated that the addition of the embedded plate has been effective in eliminating vertical end cracking.</p> <p>In addition to the work originally planned for this project, an analytical vertical crack sensitivity model was developed to evaluate the sensitivity of the beams to friction and other effects that could potentially contribute to vertical cracking. This prototype crack sensitivity model is based on a linear elastic spring model that incorporates the effects of static and dynamic friction at the ends of the beams during the detensioning process.</p>					
17. Key Word prestressed, beams, cracks, fabrication, detensioning				18. Distribution Statement No restrictions	
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of Pages 237	22. Price

## **ACKNOWLEDGMENTS**

The authors acknowledge and thank the Florida Department of Transportation for providing the funding for this research project. The authors wish to thank Gate Concrete Products Company, Dura-Stress, Inc., and Standard Concrete Products, Inc. for participation throughout the course of the project.

## **EXECUTIVE SUMMARY**

The purpose of this research project was to determine the cause of vertical cracks that sometimes occur near the ends of the bottom flange of prestressed AASHTO, Florida Bulb-T, and Florida U beams. These vertical cracks were primarily observed on long-span beams and occurred during the transfer of the prestressing force to the concrete. These cracks form at the base of the beam just a few inches from the end and propagate vertically upward towards the web region of the beam.

This research project initiated with mail surveys submitted to three FDOT prestressed beam manufacturers in Florida to determine the extent and types of end cracking each manufacturer was experiencing during the production process of their AASHTO, Florida Bulb-T, and Florida U beams. Site visits followed the surveys to observe the beam production process. The site visits also provided the opportunity to talk with manufacturer's engineers and technicians about the different types of cracks that occur during the production process and when, where, and how each type of crack forms.

The primary recommendation resulting from this research was that the coefficient of friction be reduced at the ends of the beams in order to reduce the tension stresses caused by friction as the beam displaces and cambers. A possible method for accomplishing this is to provide embedded steel base plates at the ends of the beams. Requirements for manufacturers to install embedded steel plates were issued in FDOT Temporary Design Bulletin C05-13 on August 19, 2005.

The final phase of this project was to perform a follow-up investigation to determine if the recommendation for minimizing the possibility of vertical cracking by reducing the coefficient of friction at the ends of the beams proved to be effective. Based on December 2007

responses from the three FDOT prestressed beam manufacturers that participated in this project, the addition of the embedded steel plate at the ends of the beams has accomplished this.

In addition to the work originally planned for this project, an analytical vertical crack sensitivity model was developed to evaluate the sensitivity of the beams to friction and other effects that could potentially contribute to vertical cracking. This prototype crack sensitivity model incorporates the effects of both static and dynamic friction, the detensioning sequence, temperature, debonded strands, beam length, number of beams, spacing between beams, spacing between end beams and bulkheads, and concrete strength at time of detensioning.

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# **CHAPTER 1 INTRODUCTION**

## **Project Overview**

The purpose of this research project was to determine the cause of vertical cracks that sometimes occur near the ends of the bottom flange of prestressed AASHTO, Florida Bulb-T, and Florida U beams. These vertical cracks were primarily observed on long-span beams and occurred during the transfer of the prestressing force to the concrete. These cracks form at the base of the beam just a few inches from the end and propagate vertically upward towards the web region of the beam.

Initial considerations regarding the cause of this type of cracking included the effect of the friction force that develops between the ends of the beam and the formwork during the detensioning process. Figure 1-1 shows a beam with eccentric reinforcement prior to detensioning and during detensioning as camber occurs due to the eccentric prestressing force. As indicated in Figure 1-1, the reactions at the end of the cambered beam produce friction forces that tend to cause vertical cracking as the beam attempts to displace during the detensioning process.

This research project initiated with mail surveys submitted to three FDOT prestressed beam manufacturers in Florida to determine the extent and types of end cracking each manufacturer was experiencing during the production process of their AASHTO, Florida Bulb-T, and Florida U beams. Site visits followed the surveys to observe the beam production process. The site visits also provided the opportunity to talk with manufacturer's engineers and technicians about the different types of cracks that occur during the production process and when, where, and how each type of crack forms.

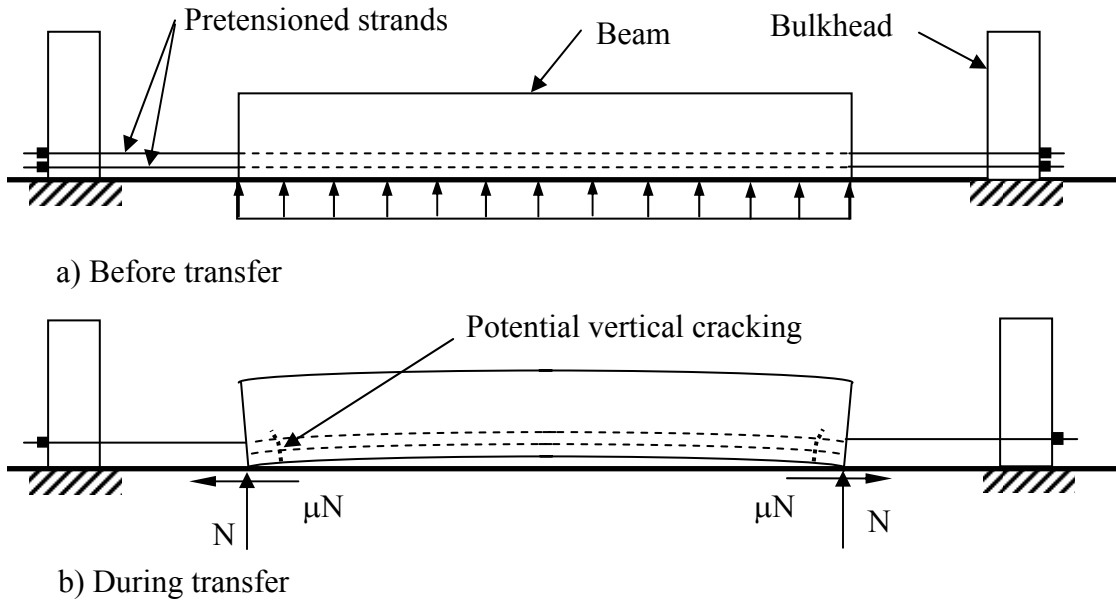


Figure 1-1. Friction Developed in Transfer of Eccentric Prestress

The primary recommendation resulting from this research was that the coefficient of friction be reduced at the ends of the beams in order to reduce the tension stresses caused by friction as the beam displaces and cambers. A possible method for accomplishing this is to provide embedded steel base plates at the ends of the beams. Requirements for manufacturer's to install embedded steel plates were issued in FDOT Temporary Design Bulletin C05-13 on August 19, 2005.

The final phase of this project was to perform a follow-up investigation to determine if the recommendation for minimizing the possibility of vertical cracking by reducing the coefficient of friction at the ends of the beams proved to be effective. Based on December 2007 responses from the three FDOT prestressed beam manufacturers that participated in this project, the addition of the embedded steel plate at the ends of the beams has accomplished this.

In addition to the work originally planned for this project, an analytical vertical crack sensitivity model was developed using MathSoft's MathCAD Version 12 to evaluate the sensitivity of the beams to friction and other effects that could potentially contribute to vertical

cracking. Information on this model is contained in Chapter 4 with sample results presented in Chapter 5 and the full model in Appendix B. The prototype crack sensitivity model incorporates the effects of both static and dynamic friction, the detensioning sequence, temperature, debonded strands, beam length, number of beams, spacing between beams, spacing between end beams and bulkheads, and concrete strength at time of detensioning. Field measurements of beam end movements during detensioning were measured for three 139 ft long 72" Florida Bulb-T beams and compared to the predicted values from the MathCAD model. The crack sensitivity model provided reasonable results but could not exactly predict the end movements of the prestressed beams in the field measurements likely due to non-simultaneous cutting and dynamic effects.

### **Prestressed Concrete Background**

Prestressed beams are formed by stretching steel strands with hydraulic jacks across a casting bed that can be as long as 800 feet (Nilson 1987, Naaman 2004). The strands are then anchored with chucks to bulkheads (see Figure 1-2) at both ends of the casting bed. Beams are then cast along the length of the casting bed with a set of prestressing strands running through all of the beams. When the concrete hardens the prestressing strands become bonded to the concrete. The portion of the strands between the beams that do not have concrete bonded to them is referred to as free strands (Kannel, French, Stolarski 1998). When the concrete compressive strength reaches the project specified release strength, the free strands are then cut one at a time and the force within each prestressing strand transfers to the concrete beams, placing the beams in a state of compression. This compression force causes the concrete beams to axially shorten and camber. Unlike post-tensioned strands, prestressed strands require a certain distance to fully transfer their force through bond to the concrete. The distance required is known as the transfer length of the prestressing strand. ACI 318-02 defines the transfer length

( $l_t$ ) as equal to one third the effective stress in the steel strand ( $f_{se}$ ) multiplied by the diameter of the strand ( $d_b$ ). The transfer length is an important concept because the transferred force varies from zero at the end of the beam, to the full prestress force at the transfer length (Nilson 1987). Because a transfer length is required to transfer the prestress force to the beam, the ends of prestressed beams are vulnerable to cracking if tension strains develop in the end region concrete. Tension pull and friction are two sources of tension strain at the end of a prestressed beam. Tension pull and friction are discussed below.



Figure 1-2. Typical Bulkhead and Strand Anchorage Chucks

When some of the free strands are cut, the prestressed beams on the casting bed axially shorten and camber (Naaman 2004). The axial shortening provides the largest movement while the camber produces a small additional amount of end movement due to the rotation of the end face of the beam. As the beams shorten and rotate the uncut free strands are forced to stretch to accommodate this movement (Mirza, Tawfik 1978). This stretch creates a tension force in the uncut free strands in addition to the prestress force (Mirza, Tawfik 1978). This additional force is referred to as "tension pull" in this report (see Figure 1-3). Temperature change in the free strands between the time of beam casting and the time of strand detensioning changes the tension pull magnitude. If the temperature of the free strands decreases, the tension pull is increased. If

the temperature of the free strands increases, the tension pull is decreased. The tension pull transfers into the concrete beam over the transfer length required for the given tension pull magnitude. The transfer length required for a given tension pull magnitude is referred to as the reverse transfer length (Kannel, French, Stolarski 1998).

As shown in Figure 1-1, friction between the bottom of the concrete beam ends and the steel casting bed is a force that can create tension strain at the ends of a prestressed beam. The static friction force ( $F_s$ ) is the coefficient of static friction ( $\mu_s$ ) multiplied by the normal force ( $N$ , one half of the beam weight) while the dynamic friction force ( $F_d$ ) is the coefficient of dynamic friction ( $\mu_d$ ) multiplied by the normal force ( $N$ ). The static friction force must be overcome before any movement can occur. The dynamic friction force is the friction force a body feels while it is in motion. If at any time the force causing motion of the body becomes less than the dynamic friction force, motion ceases. For motion to occur again, the static friction force must once again be overcome. In the case of a prestressed beam, the friction acts at both ends of the beam as the beam cambers. Given a constant coefficient of friction, the greater the beam length the larger the concentrated friction force at the bottom end edges of the beam becomes.



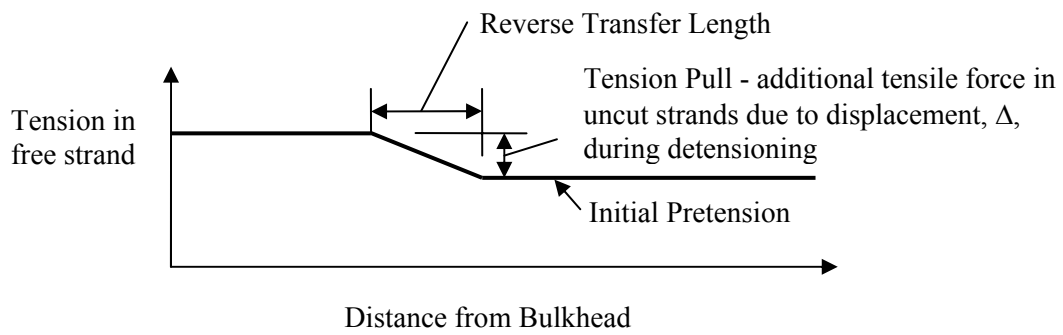
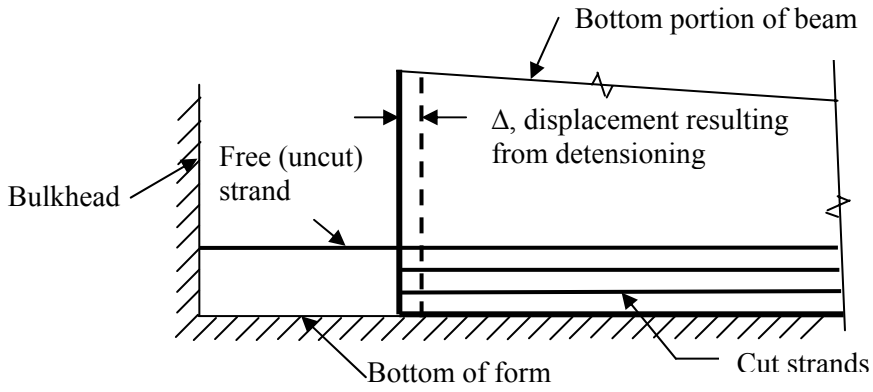


Figure 1-3. Tension Pull - increased tension in free strands during detensioning

## CHAPTER 2 END CRACKING LITERATURE REVIEW

### **Introduction**

The following summarizes two studies previously conducted on end cracking in prestressed beams. The first study conducted by Mirza and Tawfik focused on vertical end cracking on 73' AASHTO Type III beams (Mirza, Tawfik 1978). The second study conducted by Kannel, French, and Stolarski investigated vertical, angular, and horizontal end cracking on 45", 54", and 72" I-beams with draped strands and steel bearing plates (Kannel, French, Stolarski 1998).

### **Review of Mirza and Tawfik 1978**

In order to determine how to prevent vertical cracking in the AASHTO Type III beams, Mirza and Tawfik first experimented on 45' to 50' long rectangular beams. The goal was to determine the primary causes of the vertical cracking. It was theorized that the vertical end cracking was caused by the restraining force in the uncut strands as the beam was being detensioned. As strands are cut, the beam shortens and the uncut strands, because they are still attached to the beam and the bulkhead, are forced to stretch. This stretch creates a resisting force that is transferred to the concrete beams. The magnitude of the resisting force is dependent upon the length of the strands between the beams. The beams were cast in three sets of two, with each set having a different length of strand between the beam ends and the bulkheads. By attaching strain gages to the steel strands and by using dial gages on the beam ends, the total resisting force in the uncut strands was determined. The experiment showed that although the resisting force per strand increases throughout the cutting process, the total resisting force reaches a maximum

at a point when approximately half the strands have been cut. This is the point when the cracks were observed to form. It was also observed that the crack widths decreased during the cutting of the second half of the strands. Because the cracks were within a few inches of the beam ends it was concluded that the resisting force must be transferred to the concrete over a short distance, and that this distance was less than the compression transfer length of the cut strands.

An analytical model was created by idealizing the beams and the uncut strands as a series of linear-elastic springs with the beams being quite stiff in comparison to the uncut strands. Using a stiffness analysis, the resisting force in the uncut strands was determined after each strand was cut. These analytical values were compared to experimental values and found to be on average 10 to 20 percent larger in the middle range of the cutting order. The analytical model determined that simultaneous release of the strands resulted in the lowest tensile stresses in the concrete beams. It also determined that if non-simultaneous release did occur it was best to cut the longer strand (between the bulkhead and the beam) before cutting the shorter strand (between the two beams). The analytical model did not incorporate the effects of friction between the beam and the casting bed.

In order to combat the resisting force in the uncut strands, the AASHTO Type III beams were modified in three ways as a result of this project: fifteen inch long steel bearing plates (Figure 2-1) were installed on the bottom of the beam ends, two three foot long Grade 40 rebars were added in the bottom flange of the beam ends, and additional prestressing strands were debonded for each beam. After making these modifications, vertical cracks were no longer observed in the AASHTO Type III prestressed beams. To prevent vertical cracking in general, Mirza and Tawfik recommended making the length of the prestressing strands between the bulkhead and the prestressed beams at least 5% of the bed length. If this length could not be

provided, they recommended debonding additional prestressing strands for a debonded length equal to or greater than the compression transfer length. Debonding reduces the resisting force in the uncut strands by reducing the average stiffness of the uncut strands. A debonded strand also helps by moving a portion of the resisting force to an interior region of the beam where the prestress force has been fully developed and the concrete can handle the resisting force without cracking.



Figure 2-1. Steel Bearing Plate

### **Review of Kannel, French, Stolarski 1998**

The study conducted by Kannel, French, and Stolarski investigated vertical, angular, and horizontal end cracking on 45", 54", and 72" I-beams with draped strands and steel bearing plates at the ends of the beams. The study included field surveys of cracking at a particular prestressed manufacturer. An ABAQUS 3-D Finite Element model of a half beam was created to model the stresses in the concrete at the end region of the beam during the detensioning process. Multiple strand cutting patterns were chosen for analysis to determine the relationship

between end cracking and strand cutting pattern. The favorable strand cutting patterns were then tested on full scale 45", 54", and 72" prestressed I-beams.

The study indicated that the vertical end cracks formed as a result of tension stresses created in the end of the beam from the uncut strands. In the initial work in this study the cracks formed during release of the draped strands which were all detensioned first. Additional work indicated that if two straight strands were cut before every six draped strands were cut the vertical crack would not form. Angular cracks formed in the bottom flange when using the initial cutting procedure where the straight strands were cut from the outside face of the flange toward the interior of the flange placing the outside face in compression, while the interior of the flange was under tension from the uncut strands. It was determined that changing the strand cutting pattern to better balance the tensile and compressive forces on the bottom flange cross section would reduce the occurrence of the angular crack (this was accomplished by staggering the cutting of interior and exterior strand columns). The horizontal web cracks noted in the field surveys were not the subject of the study but were discussed briefly in Appendix B of the report. This discussion identified these cracks as those recognized by Gergely and Sozen (Gergely and Sozen 1967) in a study to develop rules for providing stirrups in the end of the beam. In the Kannel, French, and Stolarski study, this type of crack was shown to occur independently of the strand cutting pattern. It was suggested that increasing the slope of the flange/web transition over the first 18" of the beam could reduce the occurrence of this horizontal crack.

Kannel, French, and Stolarski determined that end cracks in general form due to two things, the restraining force from the uncut strands and the tension stresses created by the strand cutting pattern. To reduce the occurrence of end cracking in prestressed beams Kannel, French, and Stolarski had three primary suggestions: change the strand cutting pattern, debond

additional prestressing strands, and increase the slope on the top surface of the bottom flange.

Implementation of the first two of these recommendations in the field proved successful.

## CHAPTER 3 MANUFACTURER SURVEY AND FIELD INSPECTIONS

### **Manufacturer Survey**

Surveys were sent to three FDOT prestressed beam manufacturers in Florida to determine the extent and types of end cracking that each manufacturer was experiencing during the production process of their AASHTO, Florida Bulb-T, and Florida U-beams. A sample of the returned survey can be seen in Appendix A.

### **Field Inspection Introduction**

Site visits following the surveys provided an opportunity to both observe cracks that had occurred and to discuss cracking tendencies with site engineers and technicians about the different types of end cracks and when, where, and how each type of crack forms. Three Florida prestressed concrete manufacturers were visited from January 2005 to February 2006. AASHTO, Florida Bulb-T, and Florida U-beams in the manufacturer's storage areas and on the casting beds were visually inspected for end cracking. The prestressed beams were of various lengths and consisted of various numbers and types of prestressing strands. The detensioning process of AASHTO Types 3 and 4, and 72" Florida Bulb-T prestressed beams was also observed.

Based on observations of the detensioning process, most of the beam movement occurs near the end of the strand cutting process. Not only do beams shorten and camber as strands are cut, but beams also slide as units on the casting bed. The beams next to the bulkheads are most

likely to slide, and this sliding appears to be the result of non-simultaneous strand cutting. For example, the strand on the left side of a beam is cut before the strand on the right side of the beam, and the beam slides to the right.

During the detensioning process, a flagman signals when each prestressing strand is to be cut. Workmen located between the beams on the casting bed then apply their torches to the specified strands. As the seven wire strands are cut, distinctive popping sounds are made as each of the seven wires in a strand break. Sometimes the seven wire “popping” sequence occurs rapidly in some areas while at other times this is delayed as the torch is repositioned. As a result, although the flagman coordinates the individual strand cutting along the casting bed, the end effect is not simultaneous cutting of each strand.

The manufacturers indicated that vertical end cracking tends to occur more frequently on larger, longer span beams. The cracking sometimes appears to occur randomly. For example, the third beam on a casting bed of five beams cracks, yet none of the other beams on the bed have any cracks. Each beam end on the casting bed appears to experience slightly different forces during the detensioning process due to variations in friction, the beam spacing arrangement on the casting bed, non-simultaneous cutting, and accidental additional strand cuts. The following information lists the different types of end cracks found during the site visits to the three Florida prestressed concrete manufacturers.

### **Field Inspection Results**

Six general types of end cracks were evident during the site visits to the three Florida prestressed concrete manufacturers.

**Vertical End Cracks:** This type of cracking was observed on beams during the site visits and based on interviews, had previously occurred in the process of casting “long span” beams.



Figure 3-1 shows an example of a vertical end crack. Figure 3-2 shows a beam where a vertical end crack has been repaired. Figure 3-3 provides an example of how this type of crack occurs on beams with skewed ends.



Figure 3-1 Example of Vertical End Crack



Figure 3-2 Repaired Vertical End Crack



Figure 3-3. Vertical End Crack on Skewed Beam

If not repaired, this type of crack will tend to increase in width in field applications as a result of longitudinal shrinkage and temperature effects coupled with the vertical support loads. The vertical end crack is the object of study for this research project. According to interviewed field personnel, this type of end crack is a maintenance issue that slows down production and also raises questions regarding loss of bond and ingress of chlorides to the prestressing strands in the field application. Field personnel believe that reducing the coefficient of friction between the casting bed and the bottom of the prestressed beam helps reduce the occurrence of the vertical crack.

**Radial Cracks:** Radial cracking is a fan shaped multiple crack pattern that extends over the entire height of the beam (see Figure 3-4). The cracks in Figure 3-4 have been covered with a sealer that highlights their locations. This cracking pattern was observed on 72” Florida Bulb-T, and 78” Florida Bulb-T prestressed beams. The cracks originating in the bottom flange are angled upward, the cracks in the web are approximately horizontal, and the cracks in the top

flange are angled downward. Three or four vertical top flange cracks spaced at about five feet finish off the pattern. The lifting hooks used for these beams were located a few feet in from the top ends of the beam. When removed from the formwork, the dead load support for the beam shifts from being located at the bottom end of the member to being located a few feet in from the top end. The result of this is that a negative moment exists between the lifting hook and the end of the member due to the prestress in the bottom flange with the resulting tensile stresses causing the radial cracking. Although not visually desirable, these cracks close and remain under compression when the beam is lowered and the vertical support is once again located at the bottom end of the member.



Figure 3-4. Radial Cracking

**Angular Cracks:** This crack originates in the sloped part of the bottom flange a few inches from the end of the beam and propagates upward at an angle towards the web. The angular crack in Figure 3-5 has been highlighted with chalk to increase visibility. As discussed in Kannel, French, Stolarski 1998, the angular crack forms due to tension stresses created from the compression forces from the cut strands and the tension forces from the uncut strands. It was determined that changing the strand cutting pattern to better balance the tensile and compressive forces on the bottom flange cross section will reduce the occurrence of the angular crack (Kannel, French, Stolarski 1998).



Figure 3-5. Angular Crack

**Strand Cracks:** This type of crack originates at the end of a prestressing strand and propagates towards the outer surface of the beam (see Figure 3-6). The prestressed strand crack will often run through multiple prestressing strands before reaching the exterior surface of the concrete beam. This crack is likely caused by a combination of the Hoyer Effect, strand rust occurring after casting and during storage, and the strand cutting pattern.

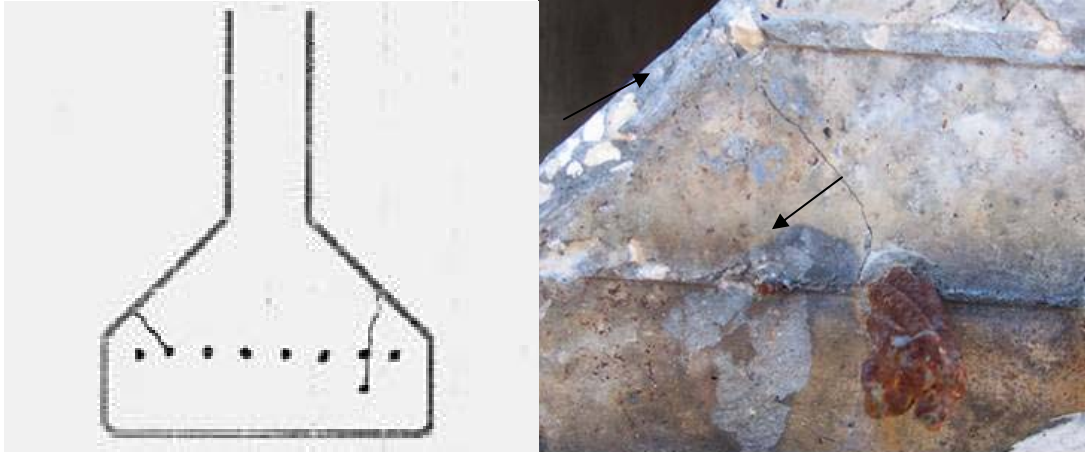


Figure 3-6. Strand Crack

When a load is applied to a prestressing strand, the prestressing strand elongates and the diameter reduces due to the Hoyer effect (primarily a function of Poisson's ratio). In the transfer length region of a prestressed beam, the force within an individual cut prestressing strand varies from zero at the end of the beam to the full prestress value at the compression transfer length. Due to Hoyer Effect, the prestressing strand in the transfer length region wants to expand when the strand is detensioned. This expansion effect creates a bursting force in the concrete (see Figure 3-7). Any rusting of the strand ends occurring after casting further would increase the bursting force at the very end of the beam.



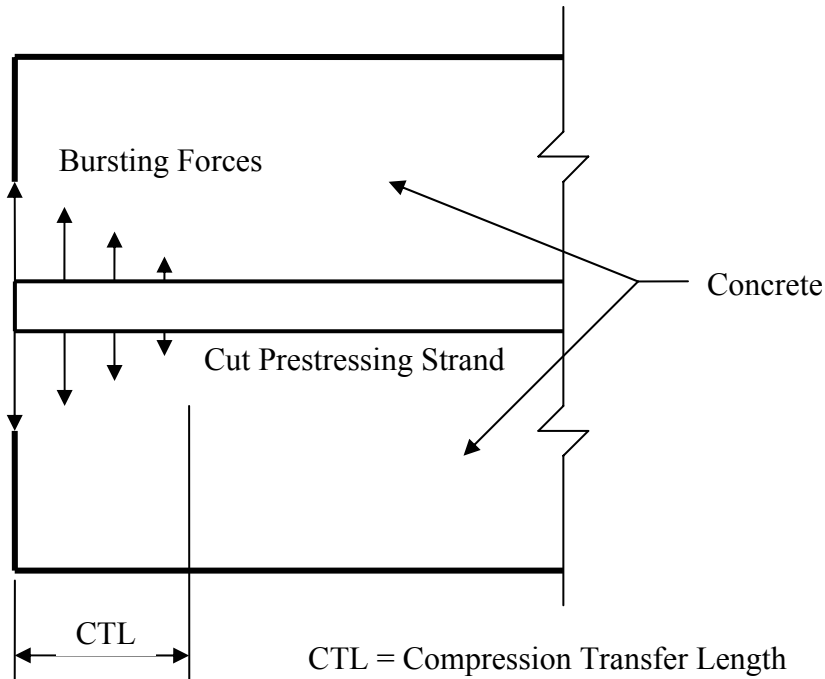


Figure 3-7. Bursting Forces Caused by Prestressing Strands

**Horizontal Top Flange Cracks:** The horizontal top flange crack begins at the end face of the upper flange and moves inward horizontally (see Figure 3-8). Field personnel indicated that this crack is caused by the formwork pressing against the concrete as the beam cambers during detensioning. The horizontal top flange crack can be prevented by providing space between the formwork and the concrete before detensioning begins.



Figure 3-8. Horizontal Top Flange Crack

**Horizontal Web Cracks:** The horizontal web crack begins at the end of the beam very near or at the interface between the web and the bottom flange (see Figure 3-8). The crack extends only a short distance into the member. This type of crack was also identified in Kannel, French, and Stolarski (1998) and as discussed in that report was initially recognized by Gergely and Sozen (1967) in their work related to the development of design recommendations for providing stirrups in the end of the beam.



Figure 3-9. Horizontal Web Crack

### **Field Inspection Summary**

In addition to vertical end cracks, other types of cracks were identified during the field survey of three Florida prestressed concrete manufacturers: radial cracks, angular cracks, strand cracks, horizontal top flange cracks, and horizontal web cracks. The focus of this research project is the vertical end crack (see Figure 3-1) and the following chapters will focus exclusively on the vertical end crack.

## CHAPTER 4 ANALYTICAL VERTICAL CRACK SENSITIVITY MODEL

### **Introduction**

The occurrence of vertical cracking can be affected by many variables:

- friction between the beam and the casting bed
- length of the free strands
- temperature change
- debonding lengths
- number of debonded strands
- number of prestressing strands
- jacking force per strand
- tension strength of the concrete
- cross-sectional area of the beam
- beam length
- beam and free-strand spacing configuration
- modulus of elasticity of the concrete
- modulus of elasticity of the prestressing
- the detensioning process from inner to outer strands

Because there are so many different variables that influence the formation of vertical cracks, it was necessary to determine which variables have the greatest potential effect on vertical crack formation so that the best possible solution could be determined. To accomplish this, MathCAD 12 was used to develop a prototype vertical crack sensitivity model.

The vertical crack sensitivity model developed in this project provides an extension of the linear-elastic spring model for prestressed beam detensioning introduced



by Mirza and Tawfik (1978) that addressed the strand detensioning process but that did not incorporate friction. As noted in Chapter 1, the static friction force must be overcome by strand detensioning before any beam movement can occur (with the beam movement resulting in “tension pull” forces developing in the uncut strands). Once the static friction force is overcome, the dynamic friction force becomes active while sliding occurs. If at any time the force causing motion of the body becomes less than the dynamic friction force, motion ceases. For motion to occur again, the static friction force must once again be overcome. The effects of both static and dynamic friction are included in the analytical detensioning procedure used in the vertical crack sensitivity model.

### **Basis of Vertical Crack Sensitivity Model**

The prototype vertical crack sensitivity model is based on the following general assumptions:

- The exact distribution of stresses in the beam and prestressing strands at the end of the beam can not be determined by basic elastic structural analysis procedures (i.e., plane stress/plane strain on the full beam cross-section). The end of the beam is in a region with a concentrated vertical reaction and concentrated horizontal friction force on the bottom end edge of the beam. These localized forces develop as the strands are detensioned and the beam cambers and attempts to shorten from the eccentric axial forces applied by the detensioned strands (compressive forces concentrated in the beam flange). In addition as camber and shortening occur, tension forces develop in the strands not yet detensioned causing additional localized tension (“tension pull” forces) in the beam flange.
- The basic assumption of the model is that sensitivity to vertical cracking can be estimated by assuming that the bottom flange of the beam acts as an independent structural member using basic elastic structural analysis procedures. In effect, this assumes that a horizontal crack occurs at the intersection of the bottom flange and the web and that this crack extends through the reverse transfer length of the strands. This assumption would appear to be somewhat validated by the noted presence of horizontal web cracks in studies by Gergely and Sozen 1967, Kannel, French, and Stolarski 1998, and the surveys in this report.

- The static friction force must be overcome by strand detensioning before any beam movement can occur. Once the static friction force is overcome, the dynamic friction force becomes active while sliding occurs. If at any time the force causing motion of the body becomes less than the dynamic friction force, motion ceases. For motion to occur again, the static friction force must once again be overcome
- The strands between all the beams on the casting bed are cut exactly at the same time for every strand in the cutting order.
- The strand cutting pattern evenly balances the transferred compression forces from the individual cut strands to the bottom flange of the beam.

The analytical vertical crack sensitivity model presented can not predict the exact stresses occurring in the ends of the prestressed beams. However, the crack sensitivity model incorporates the effects of friction between the ends of the beams and the prestressing formwork during the detensioning process that has not been addressed previously. This is potentially a valuable tool for determining trends that can identify the cause and potential remedies to reduce the occurrence of vertical cracking when the effects of both static and dynamic friction are considered.

During detensioning the friction force between the beam and the bottom form is distributed over an area at the bottom of the beam ends. As strands are cut and the beam camber increases, this distributed area is reduced until the friction force acts nearly as a line load across the very ends of the beam. When the tension pulls at the two ends of a beam are unequal, the acting direction of the static friction force may change and the beam may slide as a unit on the casting bed. This phenomenon is referred to as "global movement." in the analytical model and in this report. In order for global movement to occur, the tension pull at the two ends of a beam must be unequal.

The cracking criterion used in the vertical crack sensitivity model in Appendix B is provided by Eq. 4-1. In Eq. 4-1, the variable " $f_{calc}$ " is the calculated stress on the lower surface of the bottom flange at a distance from the end of the beam equal to the reverse

transfer length (i.e., the location where the tension from the uncut strands is at a maximum). The variable “f” represents the potential cracking tension stress in the bottom flange and is calculated using Eq. 4-2. In Eq. 4-2, “f<sub>ci</sub>” is the compressive strength of the concrete at the time of detensioning.

$$\frac{f_{\text{calc}}}{f} \geq 1 \quad \text{When a crack is likely to occur} \quad (4-1)$$

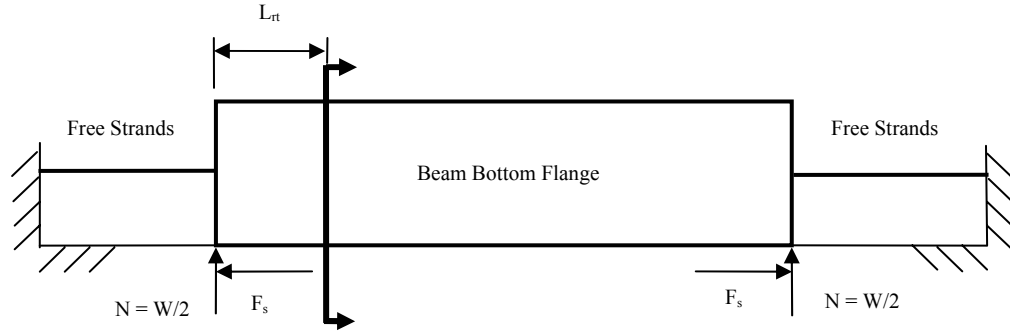
$$f = 5\sqrt{f_{\text{ci}}} \quad \text{Tension stress when cracking is likely to occur} \quad (4-2)$$

The calculated stress in the concrete bottom flange (f<sub>calc</sub>) is based on four factors: the transferred prestress force, the static friction force, the bearing force, and the tension pull (see Figure 4-1). The stress in the concrete is calculated at the bottom of the beam at a distance from the end face of the beam equal to the reverse transfer length. Eq. 4-3 is used to calculate f<sub>calc</sub>.

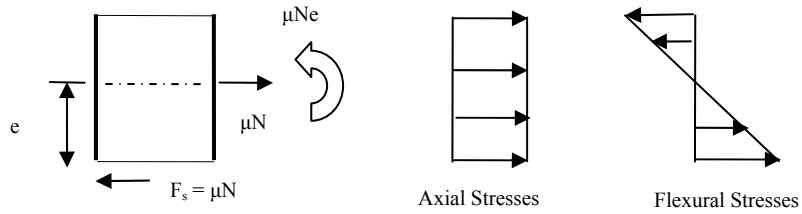
$$f_{\text{calc}} = \left( \frac{\text{CRTL}}{A_{\text{bf}}} - \frac{F_s * e^2}{I_{\text{bf}}} - \frac{F_s}{A_{\text{bf}}} - \frac{N * L_{\text{rt}} * e}{I_{\text{bf}}} - \frac{\text{TP}}{A_{\text{bf}}} \right) * -1 \quad (4-3)$$

where:

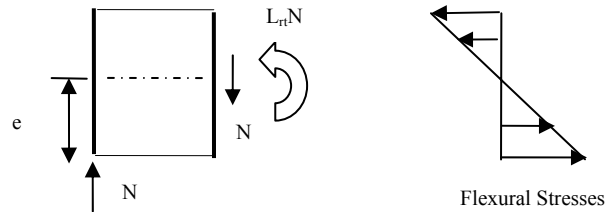
- f<sub>calc</sub> = estimated tension stress in the bottom of the beam at a distance equal to the reverse transfer length from the end of the beam.
- CRTL = Transferred prestress force at a distance from the end face equal to L<sub>rt</sub>
- A<sub>bf</sub> = Area of bottom flange
- F<sub>s</sub> = Static friction force (μN)
- e = distance from the centroid of the bottom flange to the bottom of the beam
- I<sub>bf</sub> = Moment of inertia of bottom flange of the beam relative to the centroid of the bottom flange
- N = Bearing force (one half the weight W of the beam)
- L<sub>rt</sub> = Reverse transfer length (RTL in Appendix B) (Eq 4-6)
- TP = Tension pull



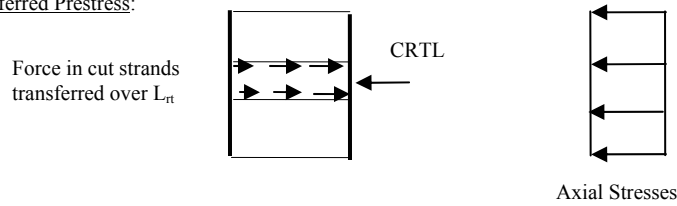
Friction:



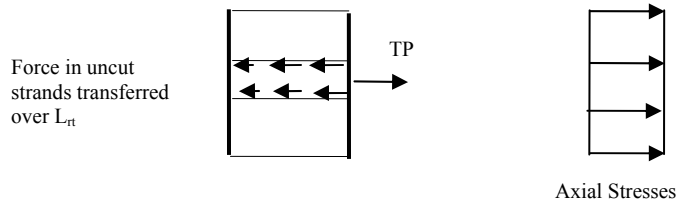
Normal Force:



Transferred Prestress:



Tension Pull:



LEGEND

W = Beam weight

e = Distance from centroid of bottom flange to bottom of beam

$L_{rt}$  = Reverse transfer length (RTL) (Eq 4-6)

CRTL = Transferred prestress force at a distance from the end face equal to  $L_{rt}$

N = Bearing force

$F_s$  = Static friction force ( $\mu N$ )

TP = Tension pull

Figure 4-1. Assumed Stress in Concrete Bottom Flange

## Specific Assumptions Used in Vertical Crack Sensitivity Model

The specific assumptions made in the prototype vertical crack sensitivity model of Appendix B are listed below.

- The modulus of elasticity of the concrete is calculated using Eq. 4-4 (Nawny 1996). The compressive strength of the concrete at the time of cutting is  $f_{ci}$ , and  $\delta$  is the unit weight of the concrete.

$$E = (40000 \sqrt{\frac{f_{ci}}{\text{psi}} + 10^6}) \left( \frac{\delta}{145 \text{pcf}} \right)^{1.5} \text{psi} \quad (4-4)$$

- The unit weight of the concrete is taken as 150 pcf (PCI 1999)
- Temperature strain is superimposed on the free strands only to account for temperature changes between the time of beam casting and the time of detensioning. The thermal coefficient of the prestressing strands is  $6.67 \times 10^{-6}$  in/in/ $^{\circ}$ F (Barr, Eberhard 2005).
- The tension pull created in each uncut free strand set due to beam movements is based on the average lengths of all the free strands in each set. These lengths include any debonding lengths.
- The effects of top flange prestressing strands are ignored.
- The transfer length of a prestressing strand is modeled as shown in Eq. 4-5 (Abrishami, Mitchell 1993). The reverse transfer length is calculated using Eq. 4-6. The variable  $f_{ci}$  (ksi) is the compressive strength of the concrete at the time of cutting. The variable  $D$  (in) is the diameter of the prestressing strand. The variable  $f_j$  (ksi) is the stress in the prestressing strand due to the jacking force. The variable  $f_{TP}$  is the stress in the prestressing strand due to the tension pull.

$$\text{TransferLength} = 0.33 f_j * D \sqrt{\frac{3}{f_{ci}}} \quad (4-5)$$

$$\text{ReverseTransferLength} = 0.33 f_{TP} * D \sqrt{\frac{3}{f_{ci}}} \quad (4-6)$$

- The prestress force is assumed to linearly transfer through bond to the concrete over the compression transfer length (ACI 2002). The tension pull is assumed to linearly transfer through bond to the concrete over the reverse transfer length. For the purposes of concrete elastic shortening the prestress force from a cut strand is assumed to act at a distance from the end face of the beam equal to 2/3rds of the compression transfer length of the strand. For debonded strands, the prestress force

is assumed to act at a distance from the end face of the beam equal to the debonded length plus 2/3rds of the compression transfer length of the strand.

- Each strand cut is divided into 20 calculation steps. These calculation steps allow for beam movements to occur as the strand is weakened during the cutting process.
- Beam movements are considered small compared to the average lengths of the free strands.
- Dynamic beam motions are ignored.
- Strand relaxation is ignored. Maximum relaxation for low-relaxation strand is 3.5% when the strand has been loaded to 80% of the tensile strength (Nilson 1987).
- The prestressing strands and the concrete beams are assumed to be linear elastic throughout the entire detensioning process. The elastic modulus of grade 270 low relaxation strand is taken as 28,500ksi (Portland Cement Association 2002). This assumption is acceptable because the vertical cracks form within the first half of the cutting order, and if the prestressing strands do become inelastic, this occurs during the second half of the cutting order.
- For analysis, the debonded length input needs to be greater than the transfer length of the fully bonded prestressing strands. This is necessary because the model assumes if a strand is debonded that the tension pull in that strand is transferred to a region of the beam beyond the crack-prone end region.
- The reverse transfer length is considered the critical section for the analytical model calculations. This is the point where all of the tension pull has been transferred through bond to the concrete.
- Camber end movement after each strand cut, is added to the axial end movement. Camber end movement after each strand cut is calculated as follows (see Figure 4-2).

$$\text{Incremental camber end movement} = \frac{\text{axial movement from strand cut}}{\text{total axial movement when all strands are cut}} (\text{total end movement from camber})$$

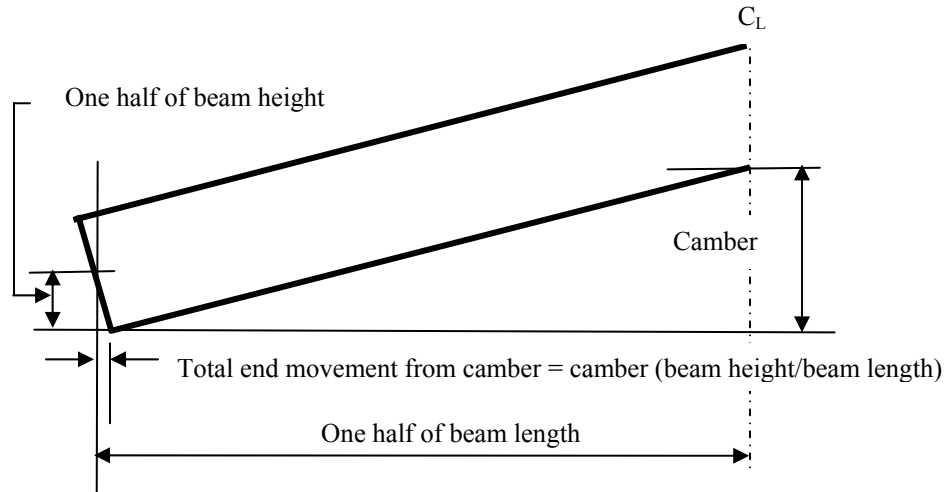


Figure 4-2. End Movement from Camber

### Analytical Model Input Variables

The first input variable for the model consist of the type of beam (BT-72, BT-78, AASHTO 2, AASHTO 3, AASHTO 4, AASHTO 5, AASHTO 6, FUB-48, FUB54, FUB-63, FUB-72) and a custom setting where the user can input a custom beam area. Two or more beams can be chosen for simultaneous analysis. The beams can also be different lengths on the same casting bed. The number of bottom strands, type of strand, and jacking force per strand must then be specified. The choices for type of strand consist of .500" 270ksi, 9/16" 270ksi, and .600" 270ksi strands. The free strand length between all the beams must be specified, with the free strand length for the end beams as the length between the beam face and the abutment. Each debonded strand and its associated debonded length must then be specified. Temperature change in the free strands from the time of beam casting to the time of strand cutting can also be inputted. Finally the coefficient of static and dynamic friction between the bottom of the beams and the casting bed must be specified.

## Analytical Model Flow Chart

The flow chart for the solution procedure used in the analytical model in

Appendix B is outlined in Figure 4-3.

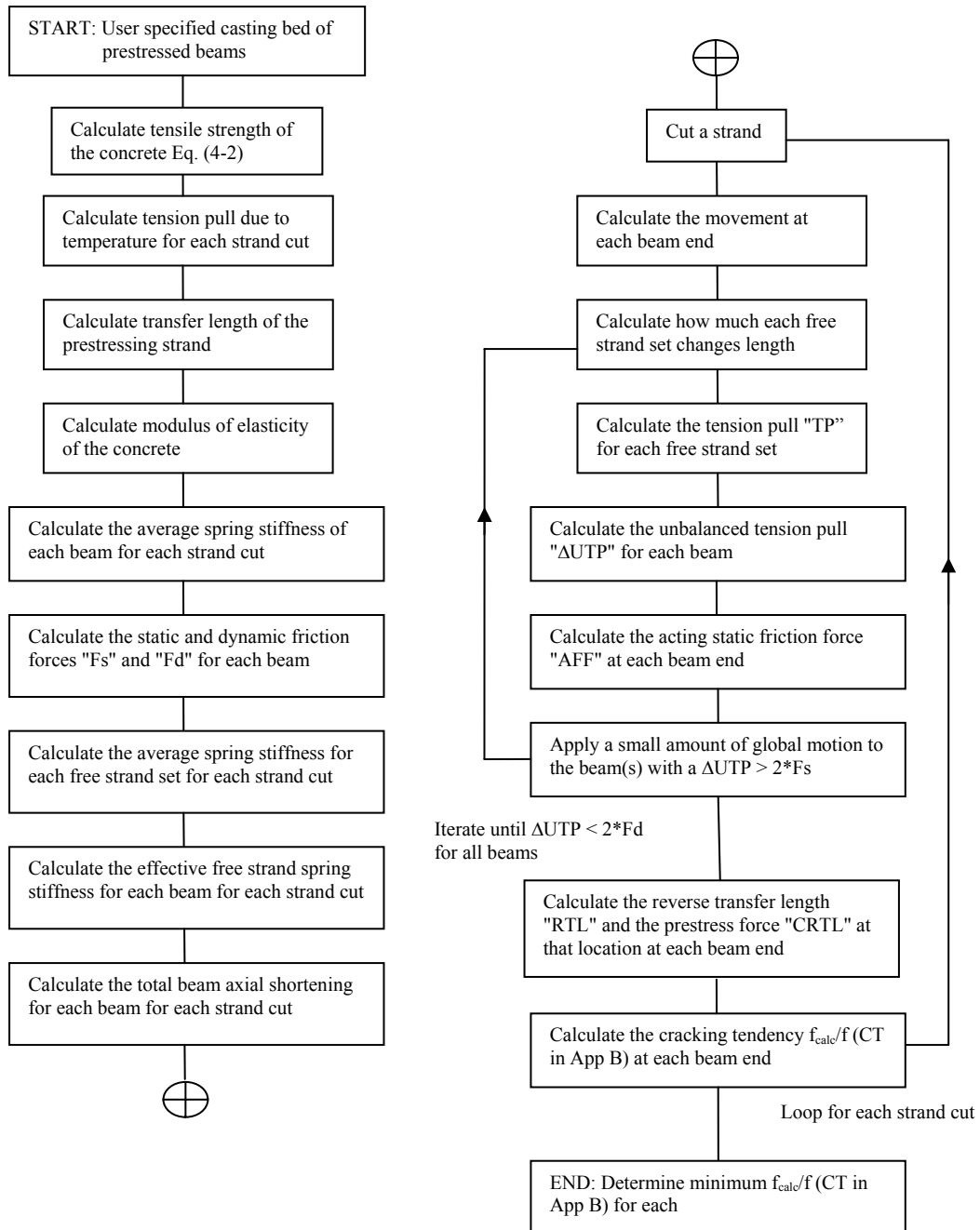


Figure 4-3. Analytical Model Flow Chart



## CHAPTER 5 RESULTS OF VERTICAL END CRACK SENSITIVITY MODEL ANALYSIS

### **Introduction**

Tension strain in the end region of a prestressed beam can be affected by many variables as discussed in Chapter 4. For this reason, it was necessary to determine which variables had the greatest effect on tension strains in the end region, so that the most efficient solution to vertical cracking could be determined. This was accomplished by performing a sensitivity analysis using the MathCad 12 analytical model. Using a test two-beam case, one input variable was altered at a time and the resulting change in the ratio of  $f_{calc}$  (the calculated stress at the bottom of the beam located at a distance equal to the reverse transfer length from the end of the beam, Eq. (4-3)) to  $f$  (the nominal maximum tensile failure stress (Eq. (4-1))) was evaluated. The test case is shown in Figure 5-1 and the input data for the test case is shown in Table 5-1. The  $f_{calc}/f$  results for the generic test case are shown in Figure 5-2. Seven alterations were made to the test case input data shown in Table 5-1 in order to determine which input changes result in the largest  $f_{calc}/f$  output changes. The alterations are the coefficient of friction, number of prestressing strands, concrete strength during detensioning, beam length, temperature change, number of debonded strands, and free strand length. For all seven alternatives of the test case, only the  $f_{calc}/f$  output for beam 1 end 1 for the first twenty strands cuts is shown (see Figure 5-1).

## Test Case

The test case is a 72" Florida Bulb-T configuration (see Figure 5-1). The input data is shown in Table 5-1. The  $f_{calc}/f$  results for the input data shown in Table 5-1 is provided in Figure 5-2.

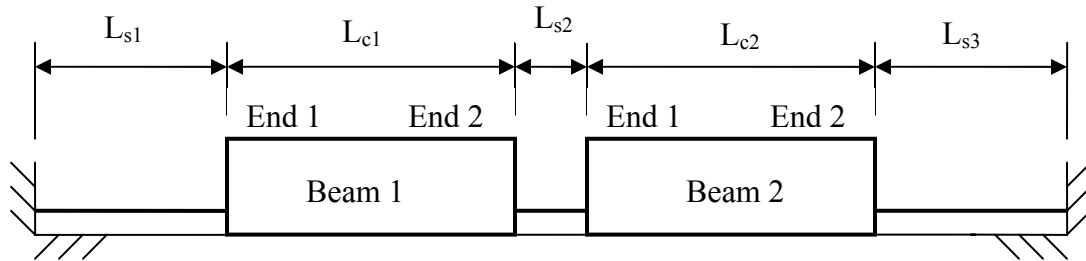


Figure 5-1. Test Case

Table 5-1. Test Case Input Data

Variable	Value	Variable	Value
Type of Beam	BT-72	$L_{s1}$	40ft
$L_{c1}$	140ft	$L_{s2}$	3ft
$L_{c2}$	140ft	$L_{s3}$	40ft
Number of Strands	40		.600
Jacking Force per Strand	44k	Strand Type	270ksi
Concrete Release Strength	8ksi	Debonded Strands	#37 5ft
Unit Weight of Concrete	150pcf		#38 5ft
Temperature Change	0		#39 5ft
Static Coefficient of Friction	0.45	Camber	#40 5ft
Dynamic Coefficient of Friction	0.40		2.5in

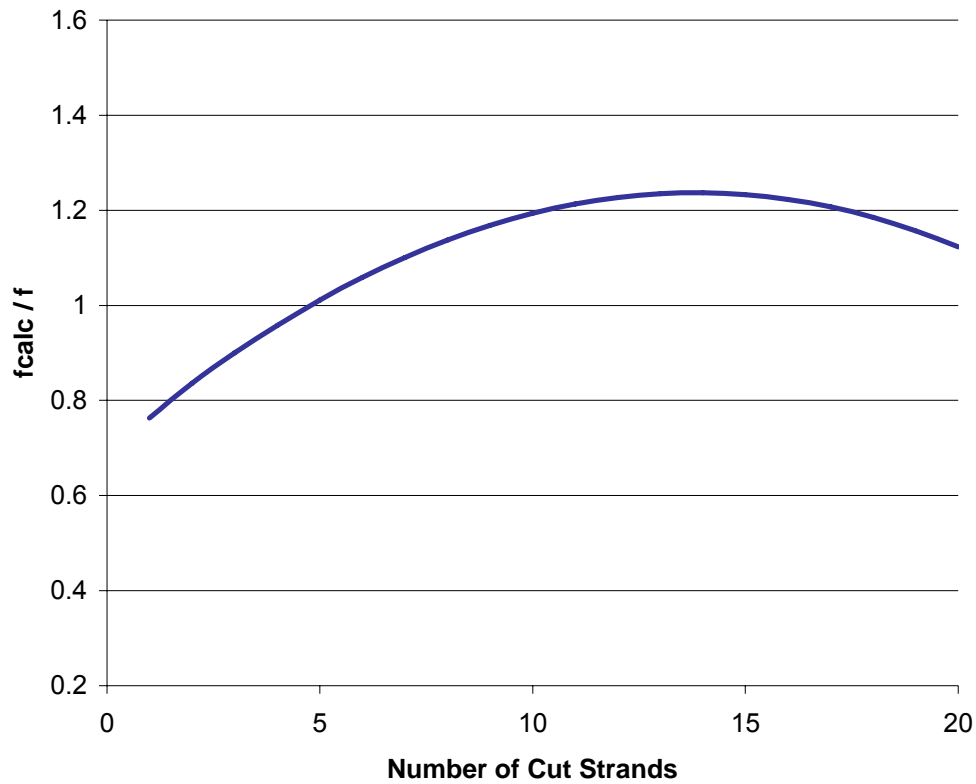


Figure 5-2. Test Case, No Alterations

### Modification 1: Alter the Coefficient of Friction

The first modification is the static and dynamic friction coefficients between the casting bed and the bottoms of the prestressed beams. The  $f_{calc}/f$  output is shown in Figure 5-3 for static friction coefficients of 0.15, 0.25, 0.35, and 0.45. The dynamic friction coefficient is assumed to be 0.05 less than the static friction coefficient in all cases. Figure 5-4 shows the maximum  $f_{calc}/f$  results for various coefficients of friction.

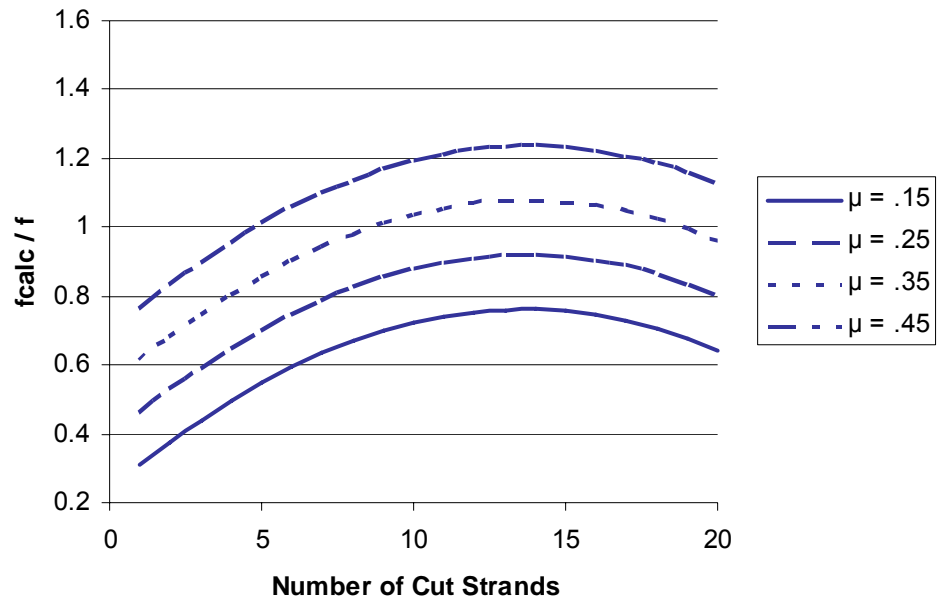


Figure 5-3. Alter the Friction Coefficient

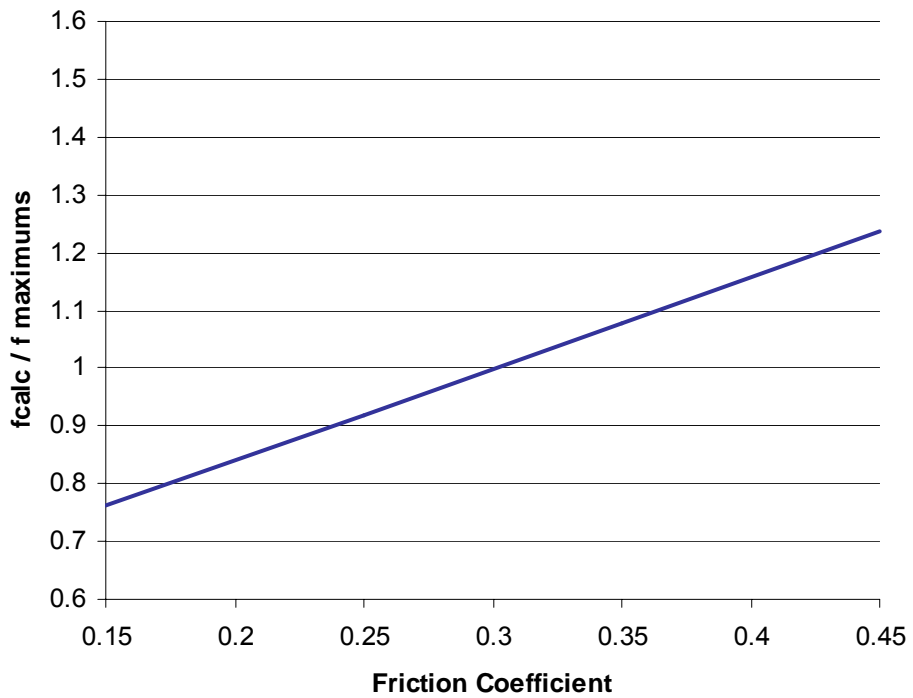


Figure 5-4. Friction Coefficient  $f_{calc}/f$  Maximums

## Modification 2: Alter the Number of Prestressing Strands

The second modification is the total number of prestressing strands. The  $f_{calc}/f$  output is shown in Figure 5-5 for 30, 40, and 50 prestressing strands. Figure 5-6 shows the maximum  $f_{calc}/f$  results for the number of prestressing strands.

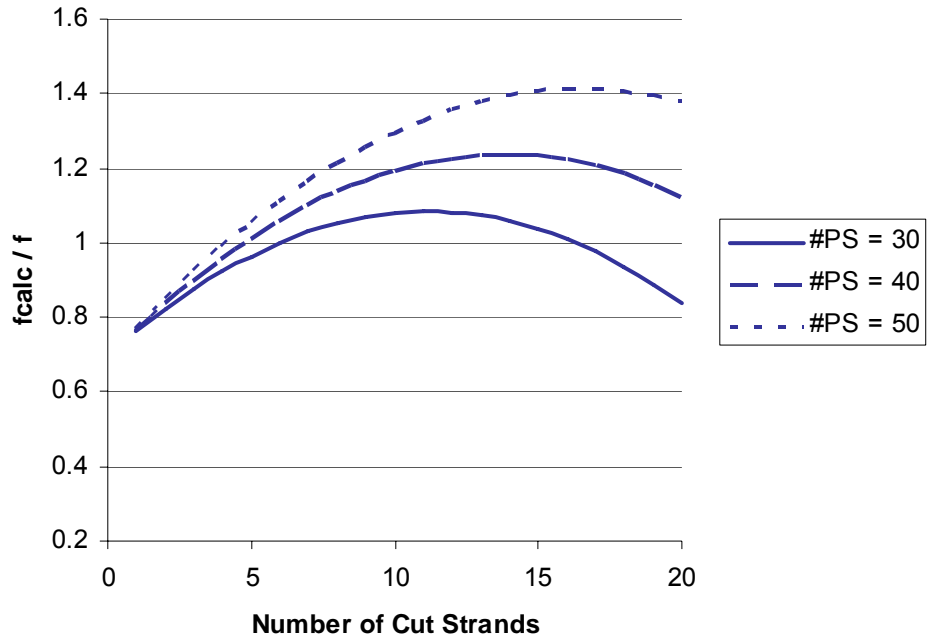


Figure 5-5. Alter the Number of Prestressing Strands

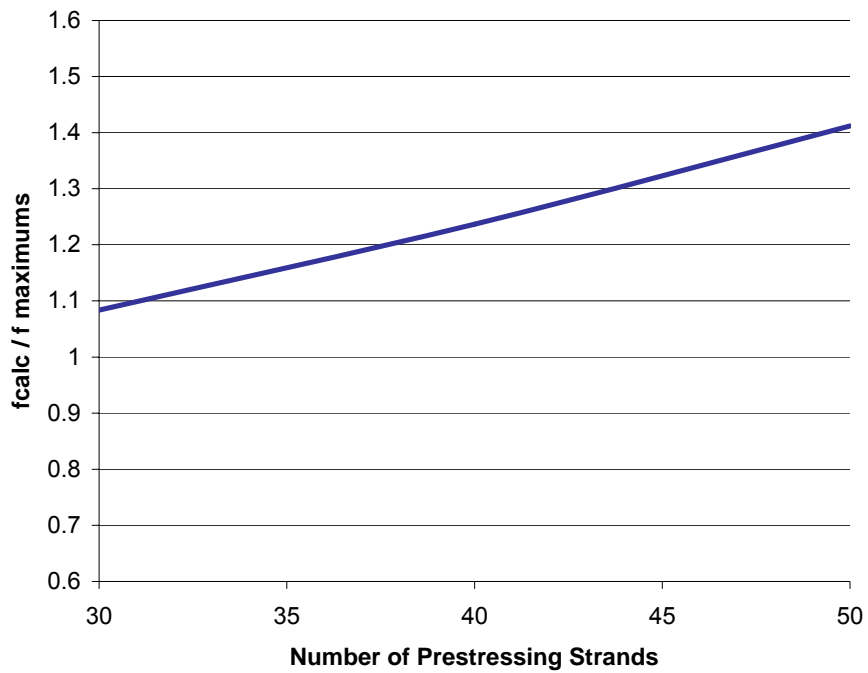


Figure 5-6. Number of Prestressing Strands  $f_{calc}/f$  Maximums

### Modification 3: Alter the Concrete Release Strength

The third modification is the concrete release strength. The  $f_{calc}/f$  output is shown in Figure 5-7 for concrete release strengths of 6ksi, 7ksi, 8ksi, and 9ksi. Figure 5-8 shows the maximum  $f_{calc}/f$  results versus the concrete release strength.

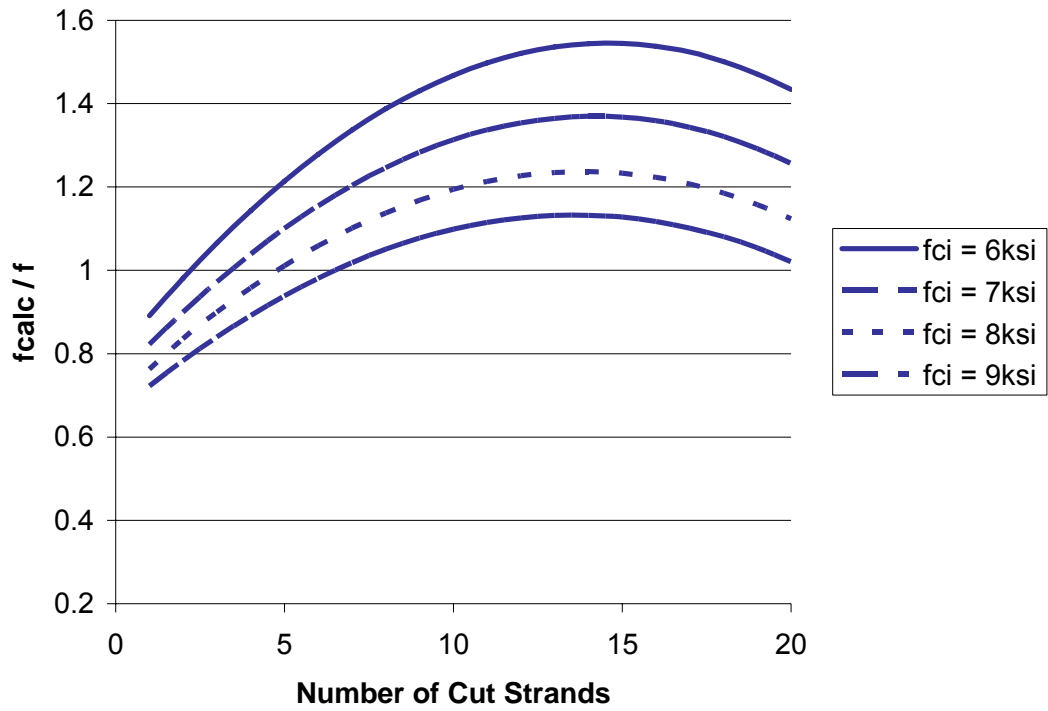


Figure 5-7. Alter the Concrete Release Strength

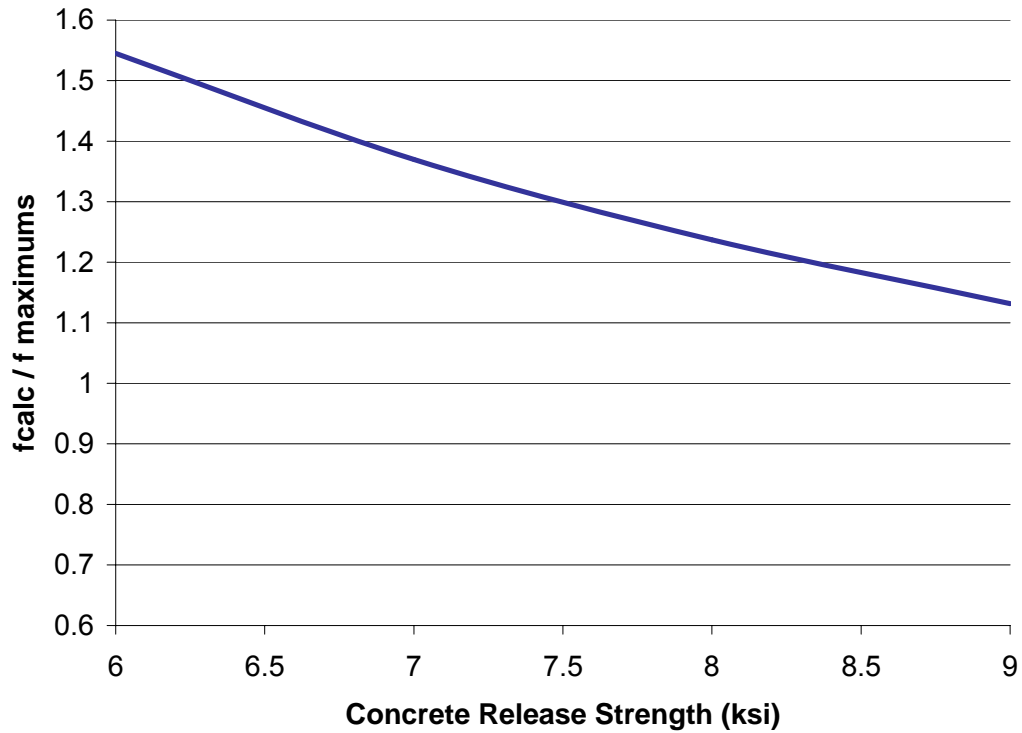


Figure 5-8. Concrete Release Strength  $f_{calc}/f$  Maximums

#### Modification 4: Alter the Beam Lengths

The fourth modification is the length of the prestressed beams. The  $f_{calc}/f$  output is shown in Figure 5-9 for beam lengths of 100ft, 120ft, 140ft, and 160ft. Figure 5-10 shows the maximum  $f_{calc}/f$  results versus beam length.

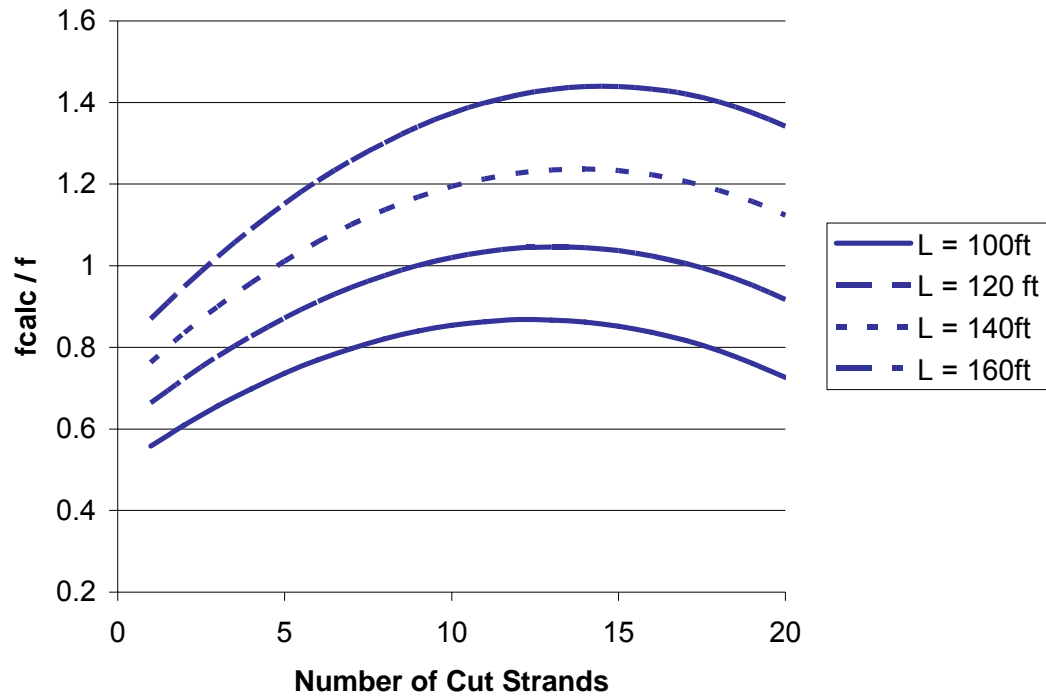


Figure 5-9. Alter the Beam Lengths



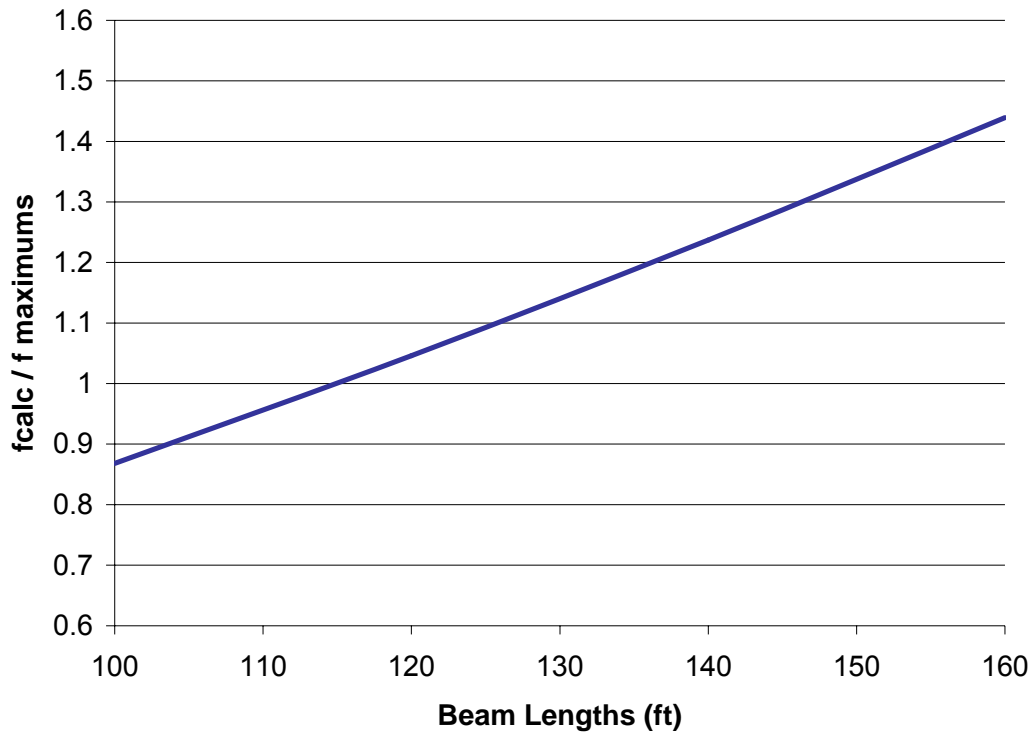


Figure 5-10. Beam Lengths  $f_{calc}/f$  Maximums

### Modification 5: Alter the Temperature Change

The fifth modification is the temperature change in the free strands. A positive temperature change indicates that the temperature at the time of detensioning is lower than the temperature at the time of beam casting. When this occurs, the free strands attempt to shorten, but are prevented by the beams and the bulkheads. A negative temperature change indicates that the temperature at the time of detensioning is higher than the temperature at the time of beam casting. When this occurs, the free strands relax an amount dependent upon the magnitude of the temperature change. The  $f_{calc}/f$  output is shown in Figure 5-11 for temperature changes of  $-40^{\circ}\text{F}$ ,  $-20^{\circ}\text{F}$ ,  $0^{\circ}\text{F}$ ,  $20^{\circ}\text{F}$ , and  $40^{\circ}\text{F}$ .

Figure 5-12 shows the maximum  $f_{calc}/f$  results regarding temperature change.

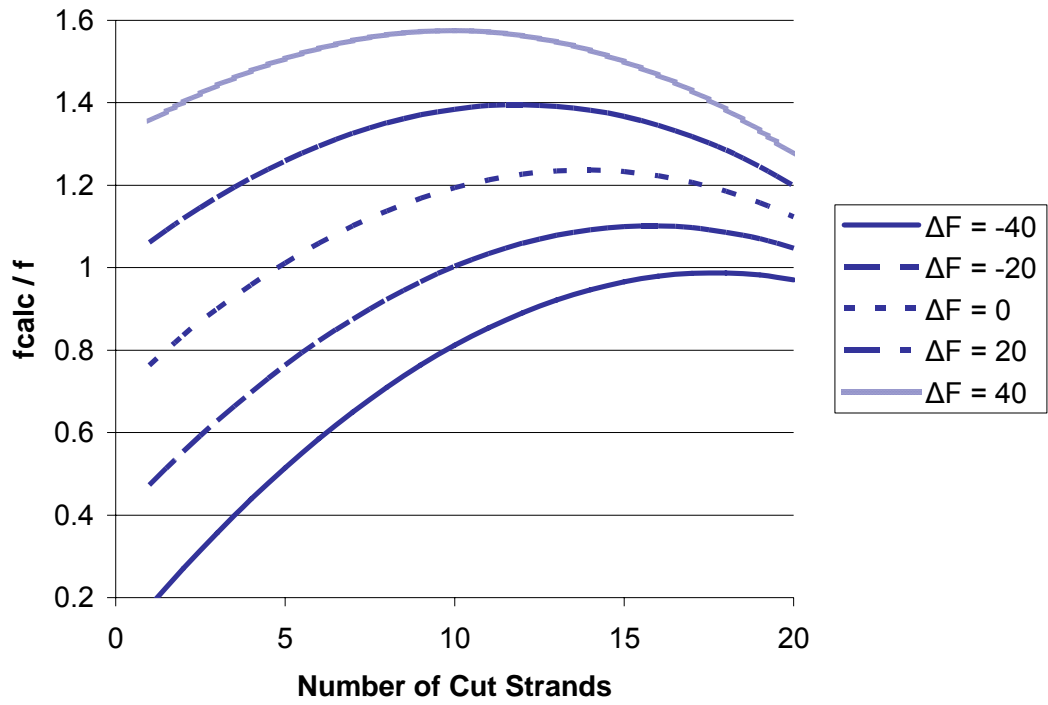


Figure 5-11. Alter the Temperature Change

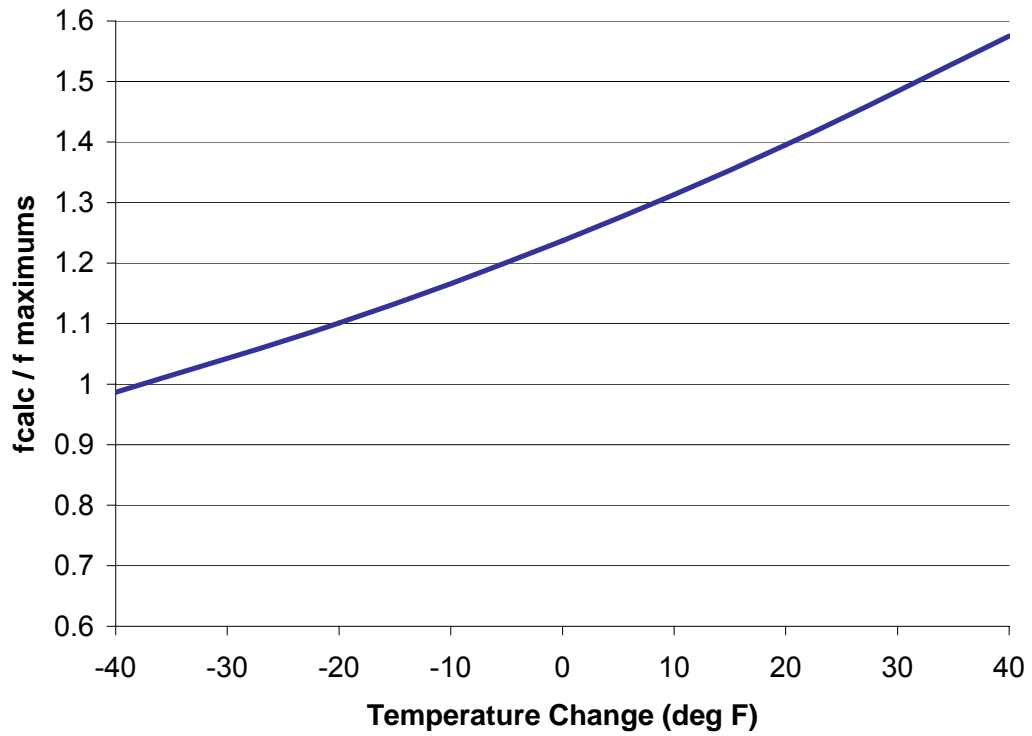


Figure 5-12. Temperature Change  $f_{calc}/f$  Maximums

### Modification 6: Alter the Number of Debonded Strands

The sixth modification is the number of debonded strands. The  $f_{calc}/f$  output is shown in Figure 5-13 for 4, 6, 8, and 10 debonded strands. Figure 5-14 shows the maximum  $f_{calc}/f$  results regarding the number of debonded strands.

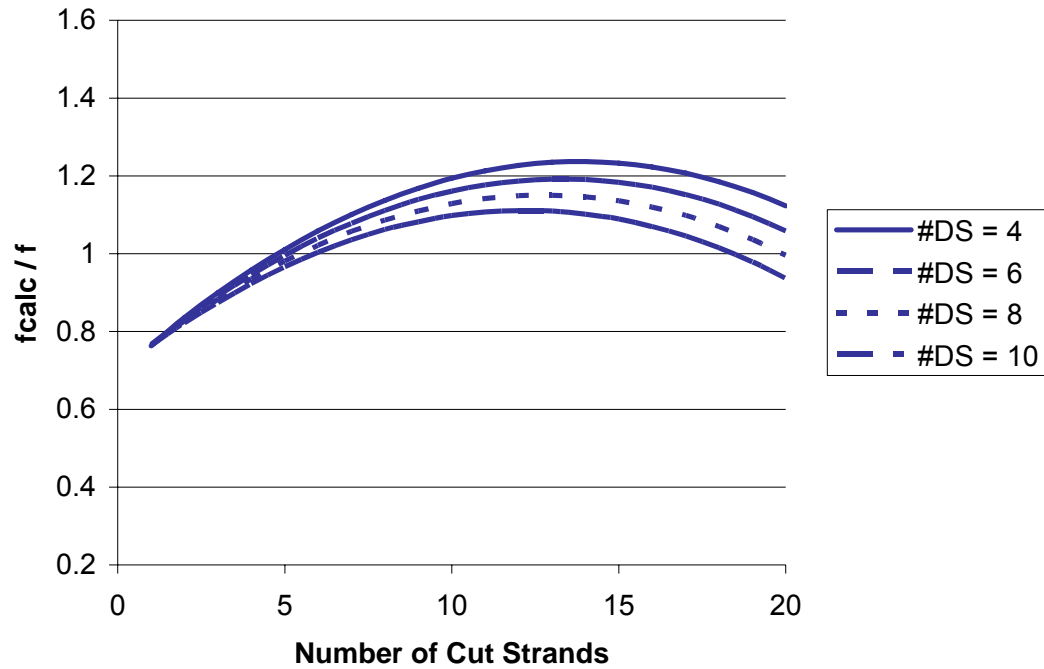


Figure 5-13. Alter the Number of Debonded Strands

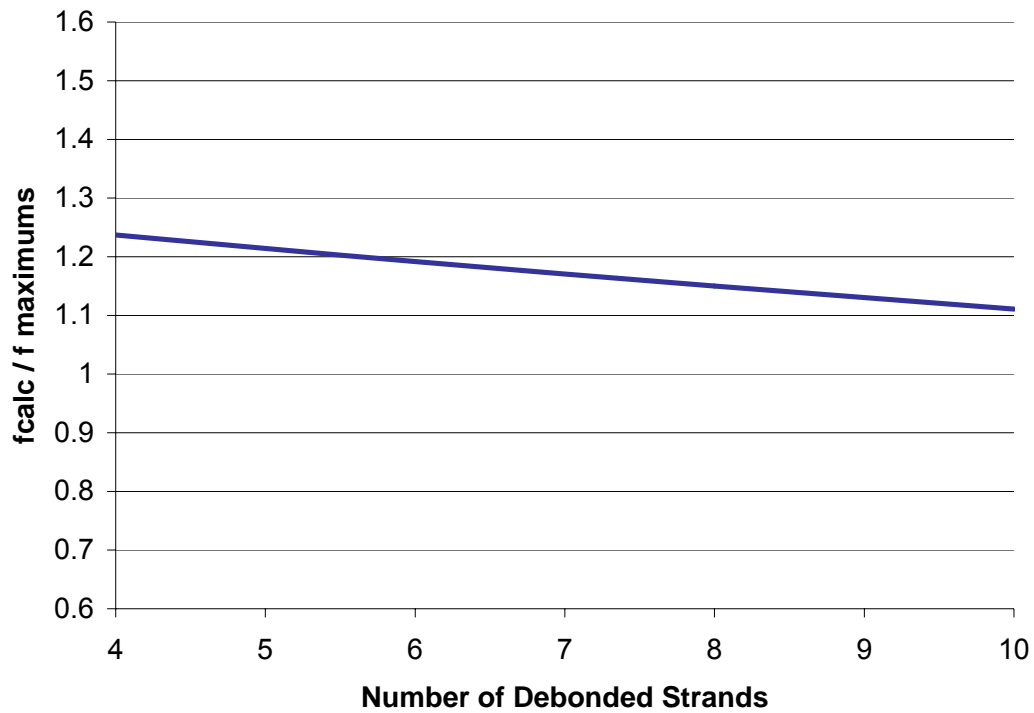


Figure 5-14. Number of Debonded Strands  $f_{calc}/f$  Maximums

### Modification 7: Alter the Free Strand Length

The seventh modification is the free strand length between the beams and the bulkheads. The  $f_{calc}/f$  output is shown in Figure 5-15 for free strand lengths of 25ft, 40ft, 55ft, and 70ft. Figure 5-16 shows the maximum  $f_{calc}/f$  results related to the number of debonded strands..

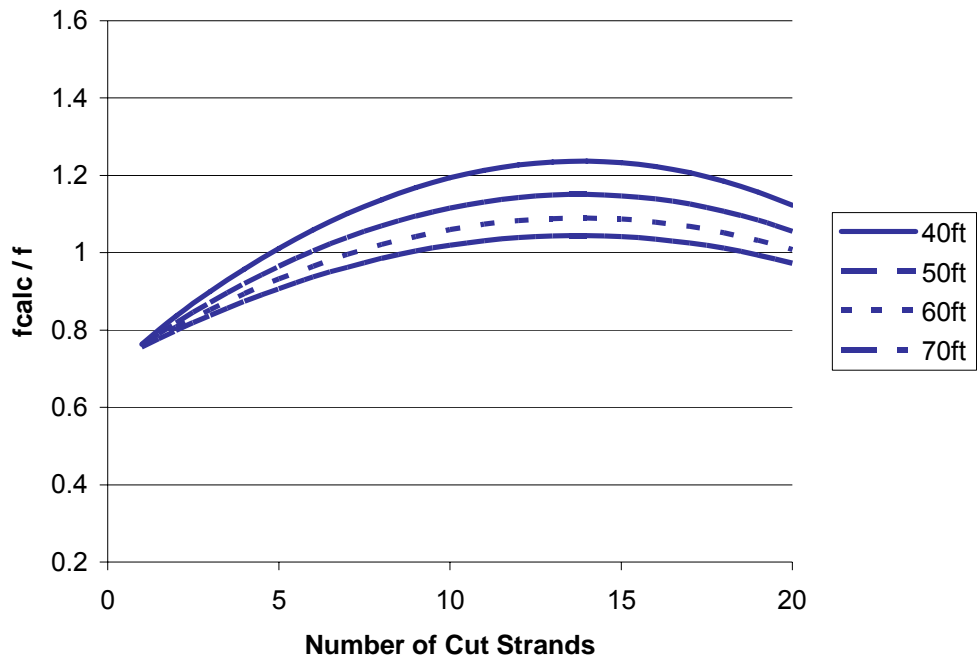


Figure 5-15. Alter the Free Strand Length

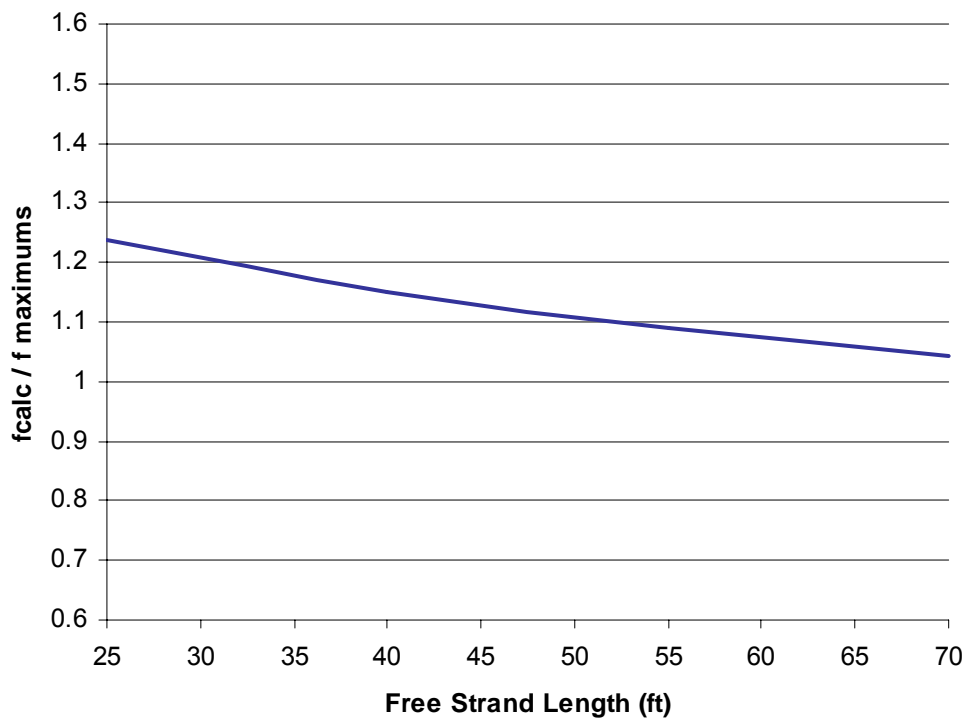


Figure 5-16. Free Strand Lengths  $f_{calc}/f$  Maximums

### **Analytical Model Conclusions**

According to the prototype analytical sensitivity model developed (Appendix B), the following trends have been determined from the test case.

- Increasing the coefficient of friction between the casting bed and the bottom of the beam makes the beam more likely to crack.
- Increasing the number of prestressing strands makes the beam more likely to crack.
- Decreasing the concrete release strength makes the beam more likely to crack.
- Increasing the beam length makes the beam more likely to crack
- A temperature reduction in the free strands from the time of beam casting to the time of strand detensioning makes the beam more likely to crack. A temperature increase in the free strands from the time of beam casting to the time of strand detensioning makes the beam less likely to crack.
- Decreasing the number of debonded strands makes the beam more likely to crack.
- Decreasing the free strand length between the bulkhead and the beam makes the beam more likely to crack. This effect is increased as the number of beams on the casting bed increases.

The variables that have the greatest effect on the tension strains the end region of a prestressed beam experiences are temperature change, friction, concrete release strength, beam length, and number of prestressing strands. The free strand lengths on the casting bed have the next greatest effect. The number of debonded strands has a small effect on the tension strains in the end region of a prestressed beam.

### **Field Data Results**

In February 2006 field data was collected at Gate Concrete in Jacksonville Florida. Beam end movements were measured for the three 139 ft long 72" Florida Bulb-T beams on the casting bed (see Figure 5-17). Measurements of movement were made at both ends of beam 2, the right end of beam 1, and the left end of beam 3, during the strand

cutting process (see Figure 5-17). Measurements were taken visually with a millimeter scale from a reference mark after the desired strands were cut. The field data was then compared to the calculated values from the analytical model in Appendix B. The input values for the analyzed beams are listed in Table 5-2. The movements for the left end of beam 2 are shown graphically in Figures 5-18 and 5-19. The movements for the right end of beam 1 are shown graphically in Figure 5-20. The movements for the left end of beam 3 are shown graphically in Figure 5-21. The cutting pattern and the locations of the debonded strands can be seen in Appendix D.

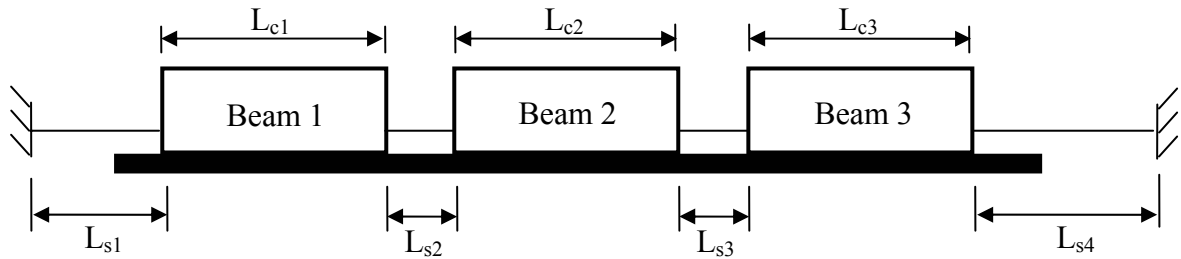


Figure 5-17. 72" Florida Bulb-T Arrangement

Table 5-2. 72" Florida Bulb-T Input Data

Variable	Value	Variable	Value
Type of Beam	BT-72	$L_{s1}$	58' 5"
$L_{c1}$	139' 2 <sup>3/8</sup> "	$L_{s2}$	2' 10"
$L_{c2}$	139' 2 <sup>3/8</sup> "	$L_{s3}$	2' 10"
$L_{c3}$	139' 2 <sup>3/8</sup> "	$L_{s4}$	88' 3"
			.600
Number of Strands	42	Strand Type	270ksi
Jacking Force per Strand	44k	Debonded Strands	4 x 5'
Concrete Release Strength	7360psi		4 x 10'
Unit Weight of Concrete	150pcf		2 x 15'
Temperature Change	NA	Estimated $\mu_d$	0.25
Camber B1 = 3"		Estimated $\mu_s$	0.30
Camber B2 = 2 <sup>5/8</sup> "			
Camber B3 = 3 <sup>1/4</sup> "			

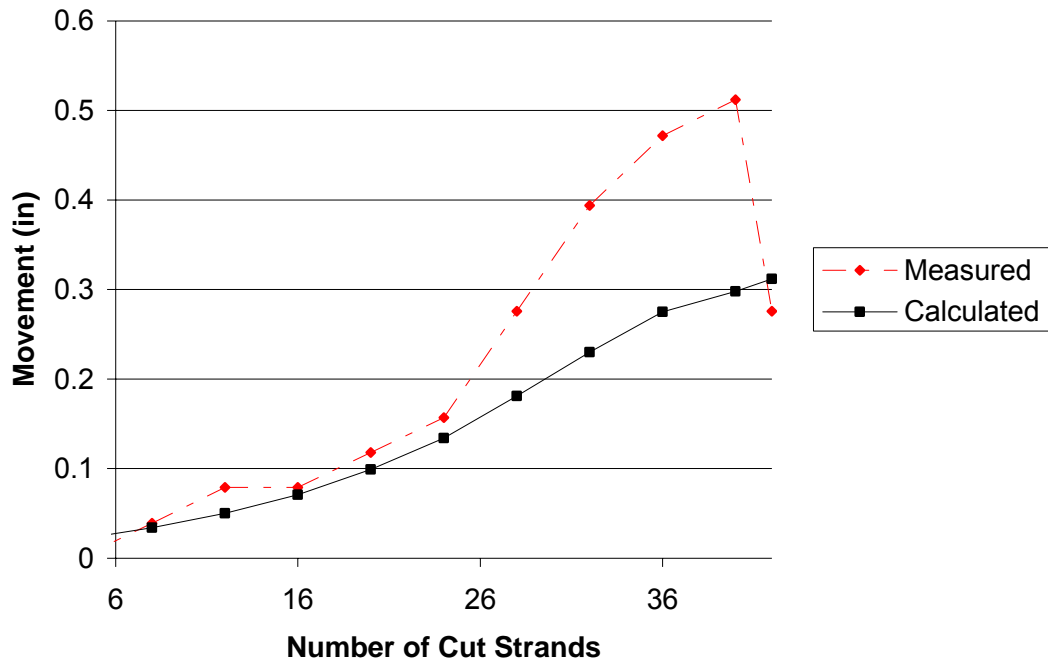


Figure 5-18. Beam 2 Left End Measured vs Calculated

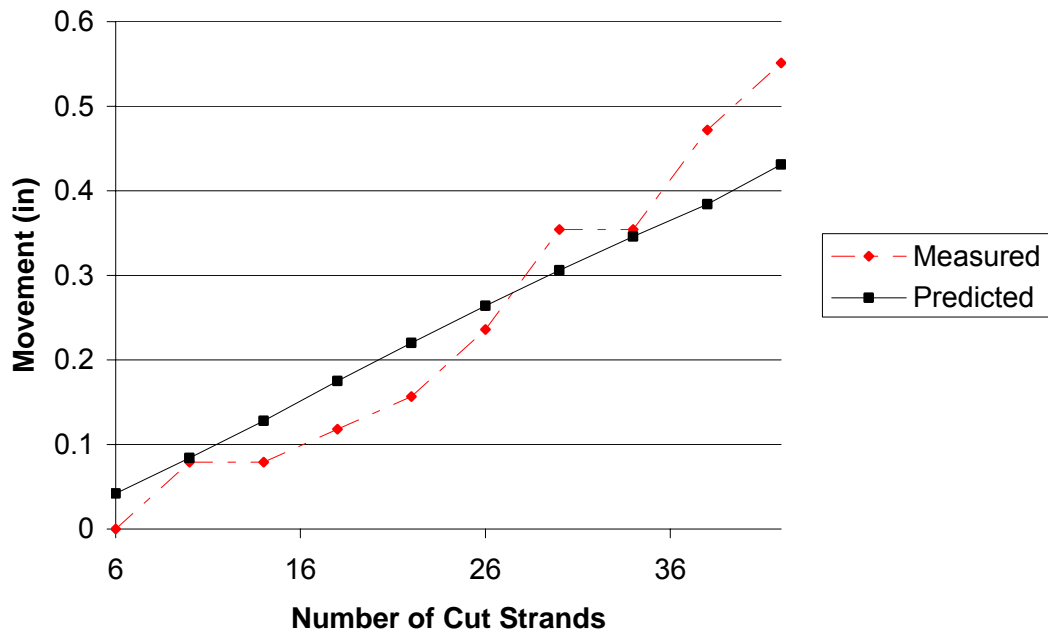


Figure 5-19. Beam 2 Right End Measured vs Calculated



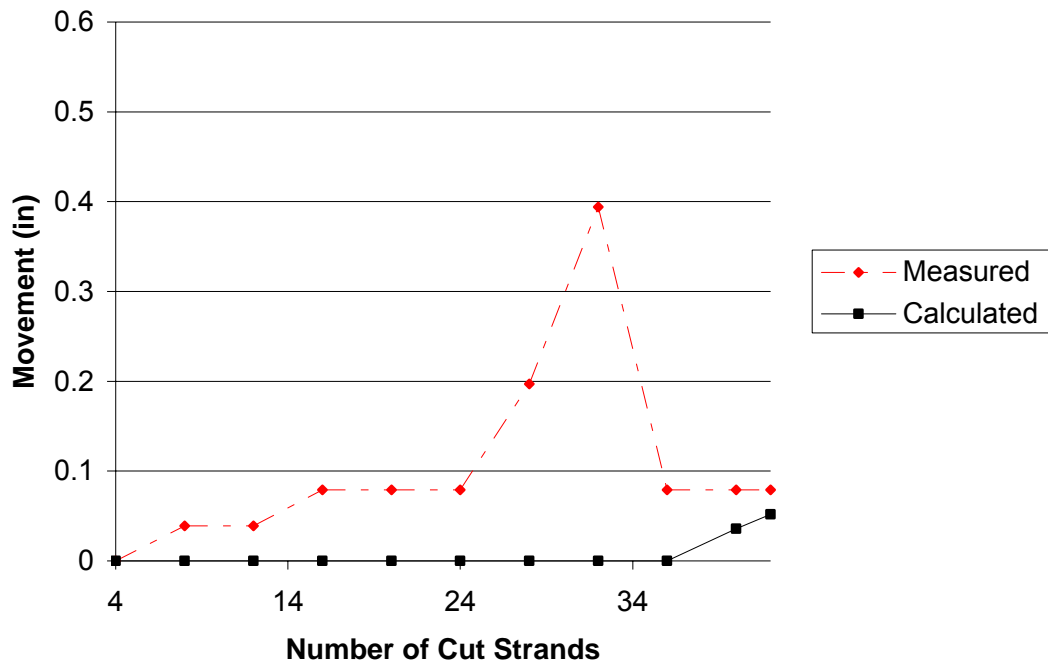


Figure 5-20. Beam 1 Right End Measured vs Calculated

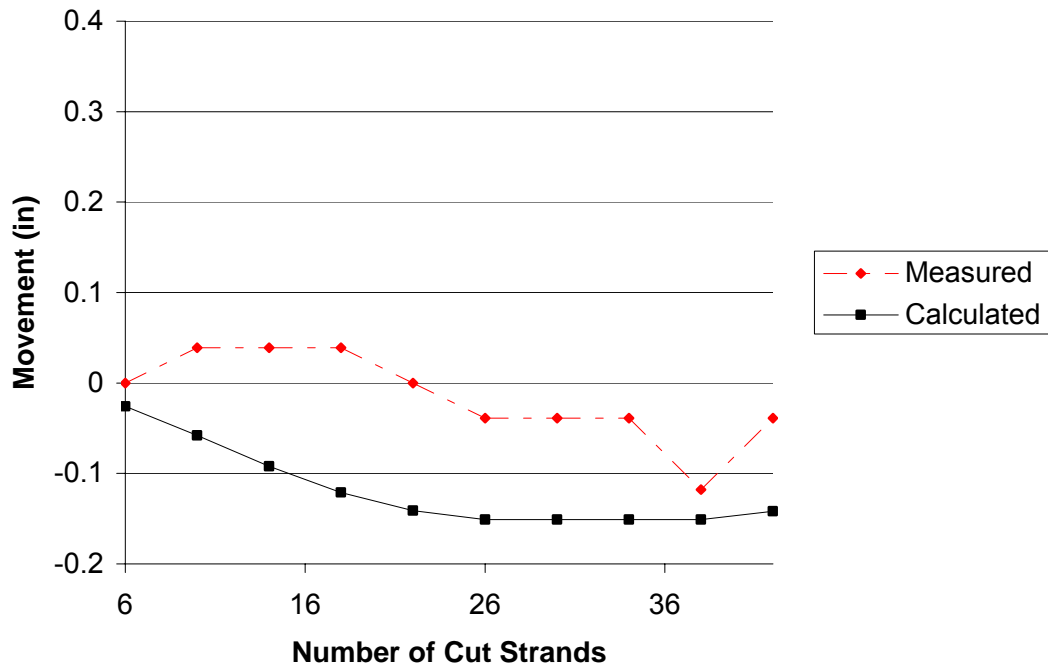


Figure 5-21. Beam 3 Left End Measured vs Calculated

### **Field Data Conclusions**

The field data results for beam 2 are higher than calculated on the left side of the beam (particularly after 25 strands had been cut) and generally lower than calculated on the right side of the beam. There are many possible explanations for this, but most likely the beam experienced global motion to the right due to non-simultaneous strand cutting. The total calculated beam shortening for beam 2 (.827") agrees with the measured total beam shortening (.819"). Beam 1 and beam 3 data show that global motion is a very significant issue. The conclusion that can be drawn from the field data is that without being able to determine which workman will cut their strand the fastest, it is not possible to calculate the actual movements of the beam ends in the field.

## CHAPTER 6 CONCLUSIONS AND RECOMMENDATIONS

The occurrence of vertical cracking can be affected by many variables: friction coefficient between the beam and the casting bed, temperature changes, concrete strength during detensioning, length of the free strands, number of debonded strands, number of prestressing strands, tension strength of the concrete, cross-sectional area of the beam, beam length, beam spacing configuration, etc.. Because there are so many different variables that influence the formation of vertical cracks, it was necessary to determine which variables had the greatest effect on vertical crack formation so that the best possible solution could be determined. The MathCad 12 prototype vertical crack sensitivity analytical model presented in Appendix B was created to determine the best vertical crack solution for a given casting bed of beams. This analytical model was not created to predict the exact stresses in the concrete beams and the steel strands because that is not possible due to non-simultaneous cutting, dynamic effects, and the disturbed region properties of a prestressed beam end. For this reason, no hard and fast rule can be created to eliminate vertical cracking in prestressed beams. However, by performing a test beam sensitivity analysis using the analytical sensitivity model provided in Appendix B, trends were developed and the variables that are most likely to cause vertical cracking were determined.

The analytical sensitivity model determined that the variables that have the greatest effect on vertical cracking are the friction coefficient between the casting bed and the

bottom of the beams, temperature change between the time of beam casting and the time of strand detensioning, concrete release strength, beam length, and the number of prestressing strands. The free strand lengths on the casting bed have the next greatest effect. The number of debonded strands and the lengths of the debonded strands have a small effect on vertical cracking. The trends that were developed with the analytical sensitivity model in Appendix B are listed below.

- Reducing the coefficient of friction between the casting bed and the bottom of the beam decreases the likelihood of vertical end cracking.
- Increasing the number of prestressing strands increases the likelihood of vertical end cracking.
- Decreasing the concrete release strength increases the likelihood of vertical end cracking.
- Increasing the beam length increases the likelihood of vertical end cracking.
- A temperature reduction in the free strands from the time of beam casting to the time of strand detensioning increases the likelihood of vertical end cracking. A temperature increase in the free strands from the time of beam casting to the time of strand detensioning decreases the likelihood of vertical end cracking.
- Decreasing the number of debonded strands increases the likelihood of vertical end cracking.
- Decreasing the free strand length between the bulkhead and the beams increases the likelihood of vertical end cracking.

The conclusion that can be drawn from this research study is that the three most important things to do in order to reduce the occurrence of vertical end cracks are to:

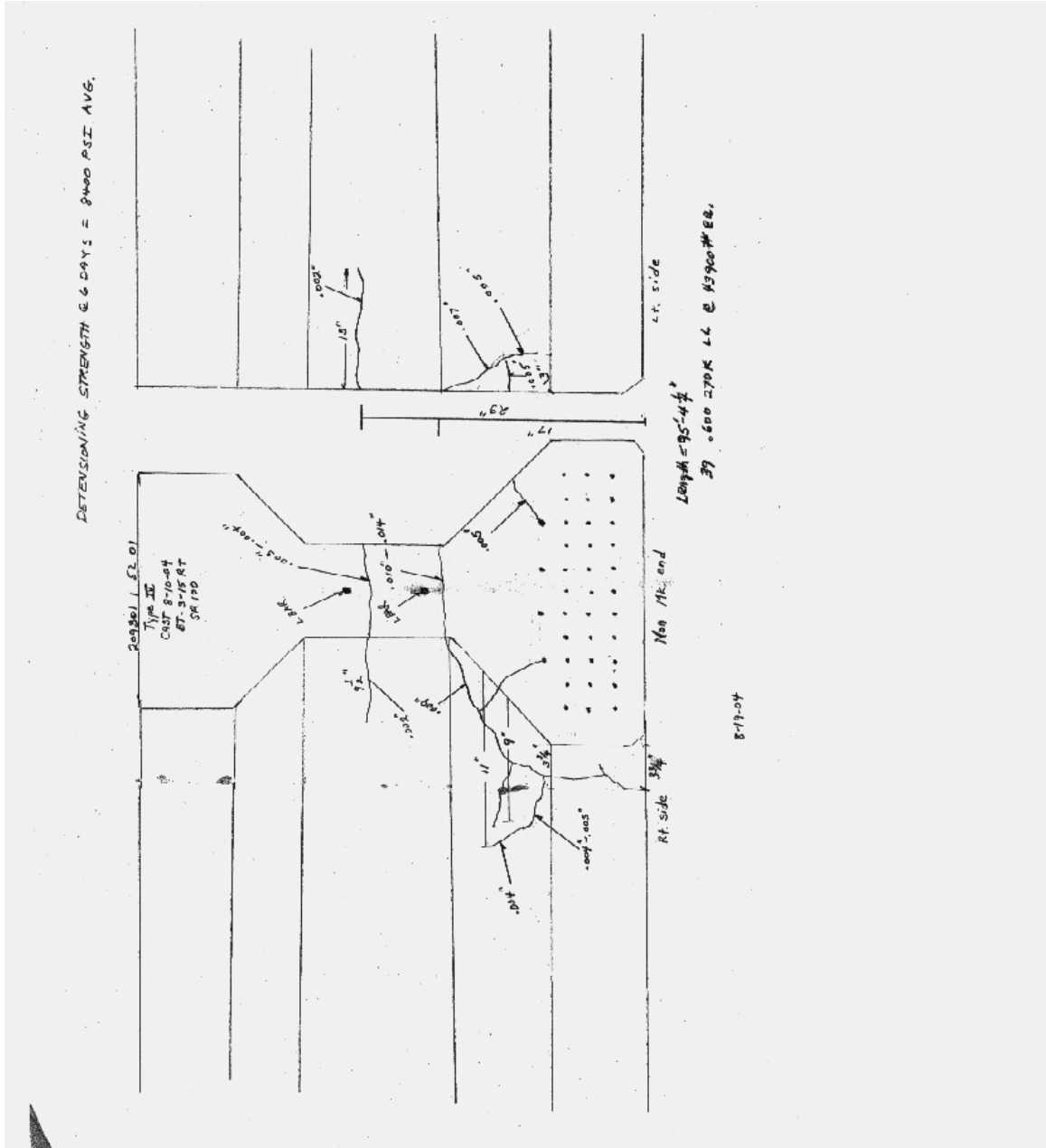
- Lower the coefficient of friction between the casting bed and the bottom of the beams.
- Detension the prestressing strands when the temperature of the free strands is similar or warmer than the temperature of the free strands when the beams were cast.
- Add additional space between the beams.

Lowering the coefficient of friction between the casting bed and the bottom of the beam ends can be accomplished by smoothing the casting bed before each new casting, adding lubricants under the beam ends, and by installing steel bearing plates at the beam ends (see Figure 2-1). The primary recommendation resulting from this project is to install a steel bearing plate at the beam ends in order to significantly reduce the friction force that develops between the end of the beam and the formwork.

## REFERENCES

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APPENDIX A  
 SAMPLE OF RETURNED SURVEY FORMS



DETENSIONING STRENGTH — (8 1/2" AVG.) 6 day break

209301-1152-01

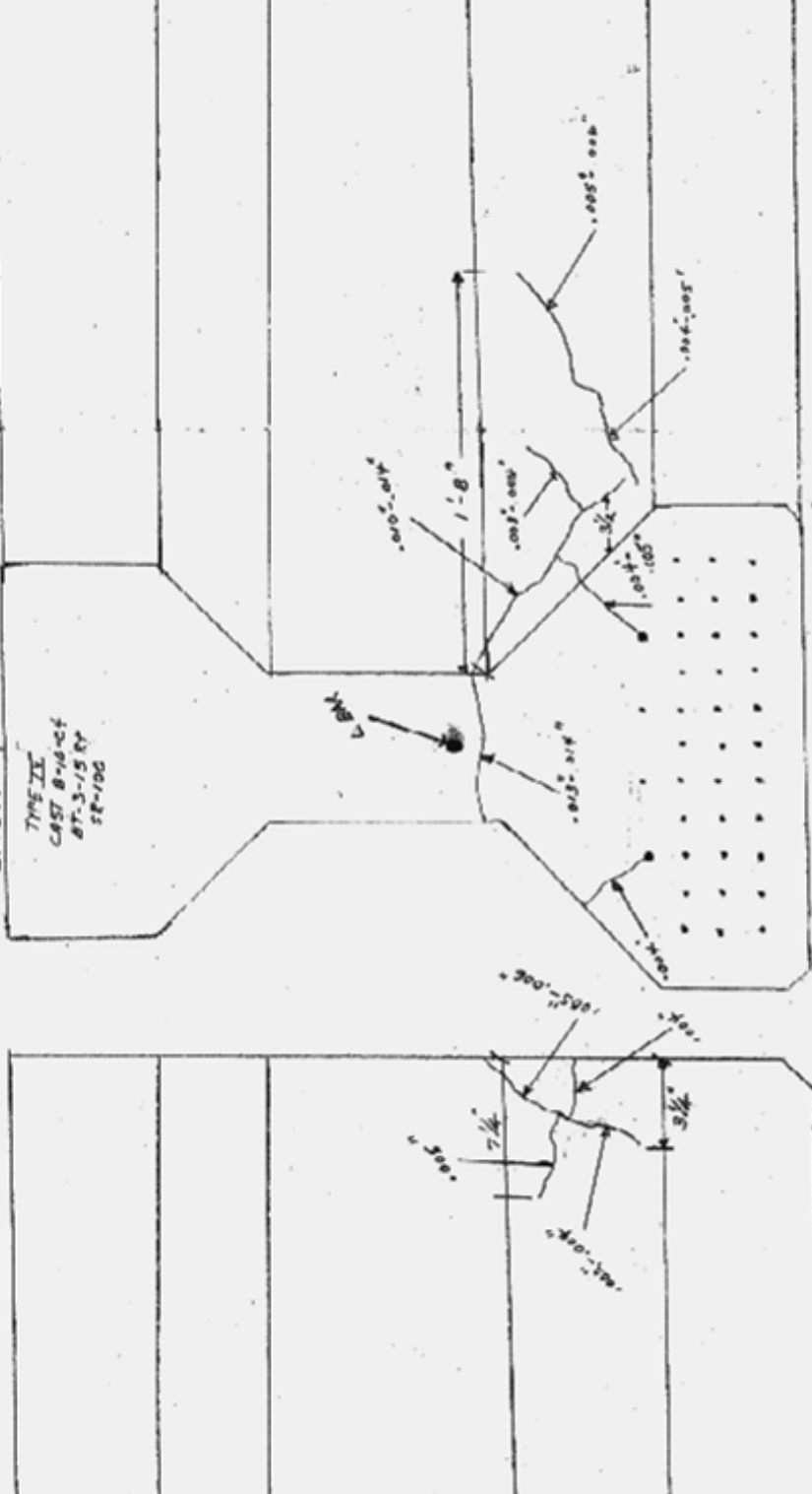
TYPE II  
CAST 8-10-47  
BT-3-15 RT  
ST-100

L. END

RK. END

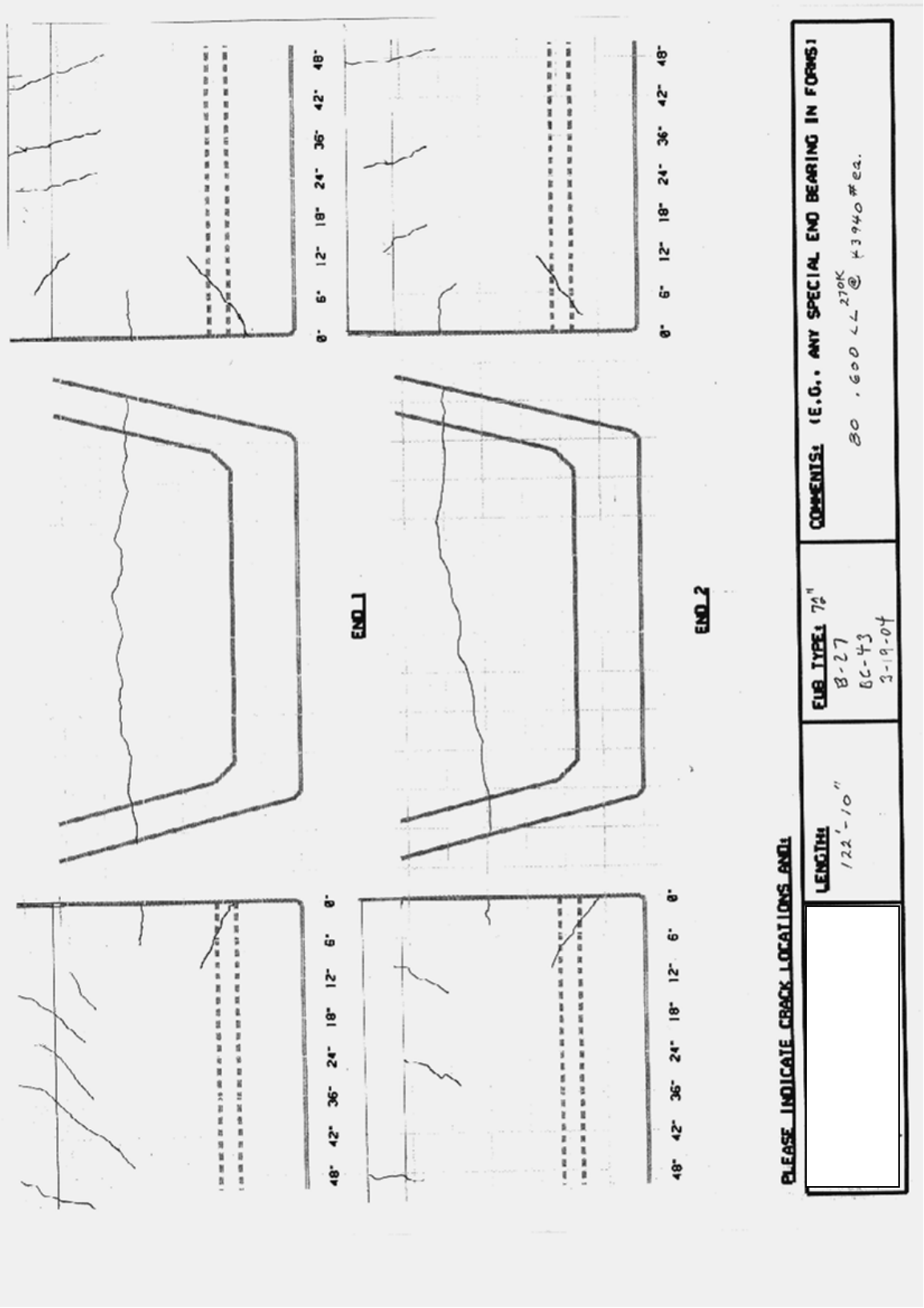
RT. SIDE

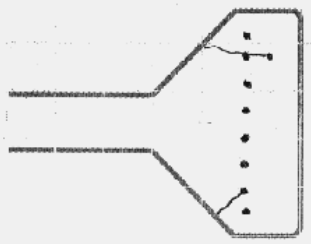
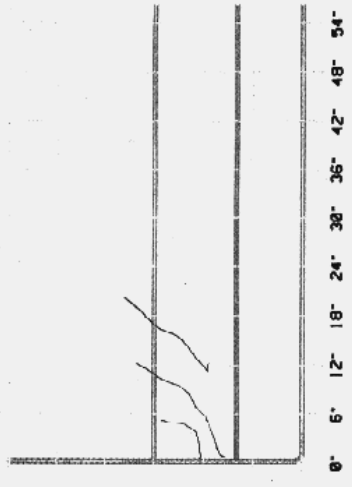
Length = 95' 4 1/2"  
39 #600 RTOK LL @ 4300# EQ.



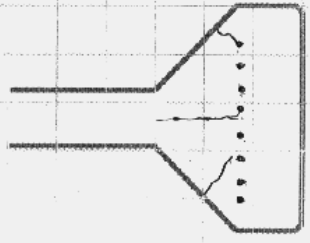
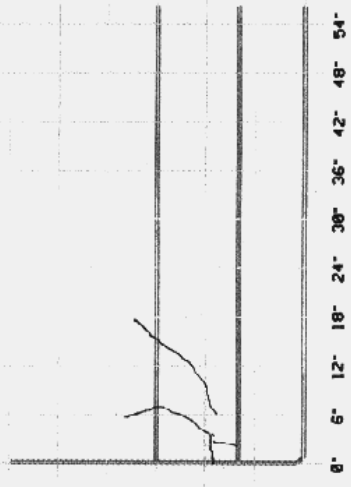
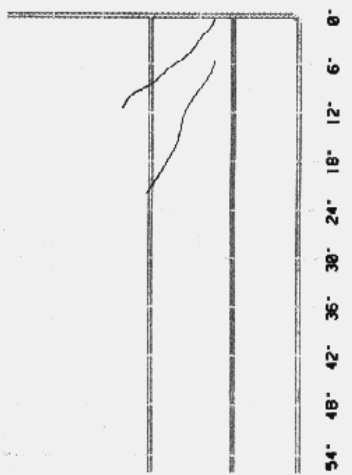
9-17-54



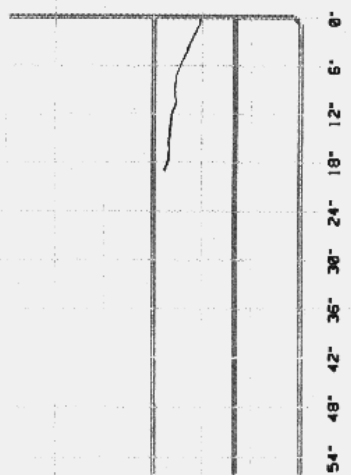




END 1

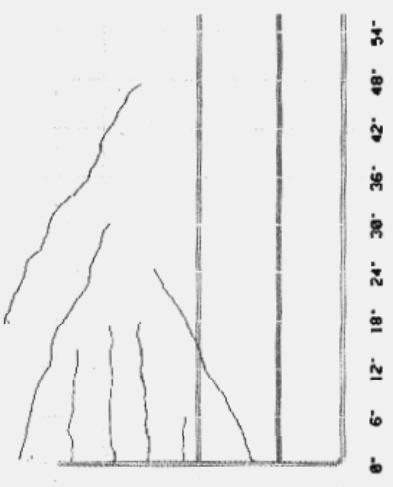


END 2

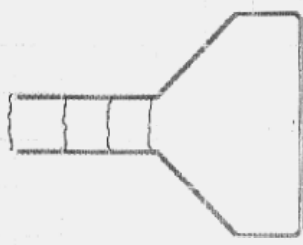


PLEASE INDICATE CRACK LOCATIONS AND:

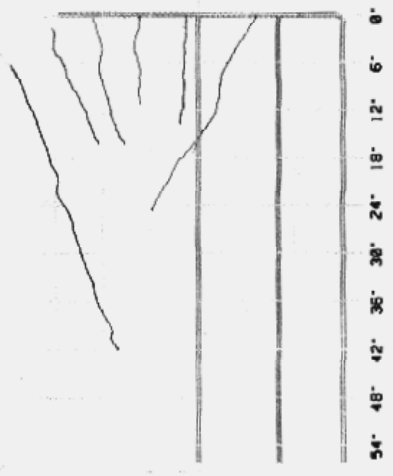
	LENGTH: 70' x 3" AV: 62 6 LB 1 11-4-04	BULB-T TYPE: ARCHIO TYPE: IV	COMMENTS: (E.O., ANY SPECIAL END BEARING IN FORMS) 49 1/2 270K LxLx @ 31000 #/ea.
--	---	---------------------------------	--



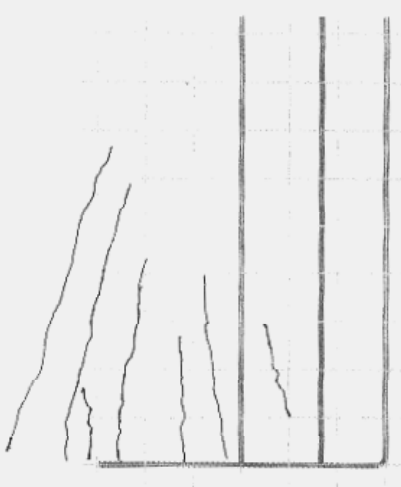
0' 6' 12' 18' 24' 30' 36' 42' 48' 54'



END 1



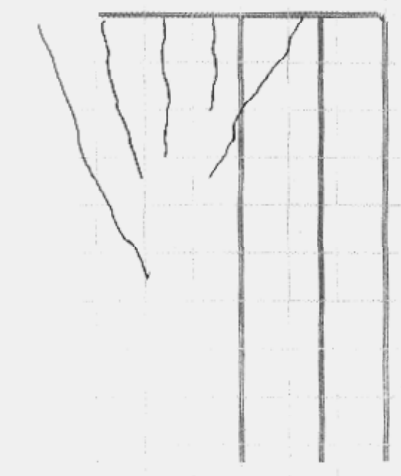
54' 48' 42' 36' 30' 24' 18' 12' 6' 0'



0' 6' 12' 18' 24' 30' 36' 42' 48' 54'



END 2



54' 48' 42' 36' 30' 24' 18' 12' 6' 0'

PLEASE INDICATE CRACK LOCATIONS AND

	LENGTH 117'-11" 84'-55" 12'-33'-03"	<input checked="" type="checkbox"/> GALB-I TYPE 72 GASHIO TYPE 4	COMMENTS: (E.G., ANY SPECIAL END BEARING IN FORM) 38 .600 LL 270K #3950 #22.
--	---	---	--

ORIGIN = 1

**APPENDIX B**  
**Vertical Crack Predictor**  
University of Florida  
2006

**INPUT:**

- 1) **Choice:** 1= 72" Florida Bulb T  
2= 78" Florida Bulb T  
3= AASHTO Type 6  
4= AASHTO Type 5  
5= AASHTO Type 4  
6= AASHTO Type 3  
7= AASHTO Type 2  
8= 48" Florida U Beam  
9= 54" Florida U Beam  
10= 63" Florida U Beam  
11= 72" Florida U Beam  
12= Custom

Choice := 2

If Choice = 12 then Specify the Cross Sectional Area of the Beam (Abeam) and the Area of the Bottom Flange (Abottomflange), the distance from the bottom of the beam to the centroid of the cross section "EcSpef", the distance from the centroid of the bottom flange to the bottom of the beam "FriceSpef", and the Moment of Inertia of the bottom flange "IBottomSpef":

Abeam := 225in<sup>2</sup>

Abottomflange := 225in<sup>2</sup>

EcSpef := 28.5in

FriceSpef := 7.5in

IBottomSpef := 9047in<sup>4</sup>

2) **Length of beams:**

Lbeams := (150 155 157 152)ft

3) **Expected Initial Camber**

InCamber := (3 3.1 3.2 3)in

4) **Number of Bottom Flange Prestressing Strands:**

NumberStrands := 49

5) **Average Concrete Release Strength at Time of Detensioning  
(Determined from Cylinder Breaks - NOT SPECIFIED VALUE):**

ReleaseStrength := 8000psi

6) **Concrete unit weight:**

$w_c := 150 \frac{\text{lbf}}{\text{ft}^3}$

**7) Bottom Flange Strand Jacking Force per strand:**

Fstrand := 44kip

**8) Distance Between Beams and Between Beams and the Bulkheads:**

FreeStrand := (50 3 3 3 45)ft

**9) Type of prestressing strand:**

1 = .500in Grade 270

2 = .500in Special Grade 270

3 = 9/16in Grade 270

4 = .600in Grade 270

StraChoice := 4

**10) Specify location of debonded strands (in the cutting order) and associated**

**debonded length (ft).** DO NOT INCLUDE UNITS. DEBONDED STRANDS SHOULD BE AT THE END OF THE CUTTING ORDER. DO NOT INCLUDE TOP STRANDS IN THE STRAND NUMBER COUNT.

Debond :=  $\begin{pmatrix} 42 & 43 & 44 & 45 & 46 & 47 & 48 & 49 \\ 5 & 5 & 5 & 5 & 10 & 10 & 15 & 15 \end{pmatrix}$       STRAND NUMBER  
DEBONDED LENGTH

**11) Specify Temperature Change (F) from Time of Beam Casting to Strand Detensioning.**

Positive = TEMP AT STRAND DETENSIONING IS COLDER THAN TEMP AT BEAM CASTING.

Negative = TEMP AT STRAND DETENSIONING IS WARMER THAN TEMP AT BEAM CASTING.

DO NOT INCLUDE THE UNITS

TempChange := -30

**12) Static Friction Coefficient Between Bottom of Prestressed Beam Ends and the Casting Bed ( $\mu$  is often between .3 and .45):**

$\mu$  := .3

**13) Dynamic Friction Coefficient Between Bottom of Prestressed Beam Ends and the Casting Bed ( $\mu_D$  is always less than  $\mu$ ):**

$\mu_D$  := .25

## Calculations :

NumbCalcs := 20      FullLength := 2

### Prestressing Strand Area

Aps :=  $\begin{cases} 0.153\text{in}^2 & \text{if StraChoice} = 1 \\ 0.167\text{in}^2 & \text{if StraChoice} = 2 \\ 0.192\text{in}^2 & \text{if StraChoice} = 3 \\ 0.2192\text{in}^2 & \text{if StraChoice} = 4 \\ \text{"error"} & \text{otherwise} \end{cases}$

Aps = 0.2192 in<sup>2</sup>

Modulus of elasticity of prestressing strands  
(can't be changed)

Eps := 28500ksi

Weight of the beam (kip/ft)

wt :=  $w_c \cdot A$

wt = 1.151042  $\frac{\text{kip}}{\text{ft}}$

## Beam Cross Sectional Area

$\frac{A}{w} := \begin{cases} 875\text{in}^2 & \text{if Choice} = 1 \\ 1105\text{in}^2 & \text{if Choice} = 2 \\ 1125\text{in}^2 & \text{if Choice} = 3 \\ 1053\text{in}^2 & \text{if Choice} = 4 \\ 789\text{in}^2 & \text{if Choice} = 5 \\ 559.5\text{in}^2 & \text{if Choice} = 6 \\ 369\text{in}^2 & \text{if Choice} = 7 \\ 1146\text{in}^2 & \text{if Choice} = 8 \\ 1212\text{in}^2 & \text{if Choice} = 9 \\ 1311\text{in}^2 & \text{if Choice} = 10 \\ 1410\text{in}^2 & \text{if Choice} = 11 \\ \text{Abeam} & \text{if Choice} = 12 \\ \text{"error"} & \text{otherwise} \end{cases}$

Eccent :=  $\begin{cases} 33.95\text{in} & \text{if Choice} = 1 \\ 40.39\text{in} & \text{if Choice} = 2 \\ 36.38\text{in} & \text{if Choice} = 3 \\ 31.96\text{in} & \text{if Choice} = 4 \\ 24.73\text{in} & \text{if Choice} = 5 \\ 20.27\text{in} & \text{if Choice} = 6 \\ 16.38\text{in} & \text{if Choice} = 7 \\ 19.67\text{in} & \text{if Choice} = 8 \\ 22.23\text{in} & \text{if Choice} = 9 \\ 27.04\text{in} & \text{if Choice} = 10 \\ 30.16\text{in} & \text{if Choice} = 11 \\ \text{EcSpef} & \text{if Choice} = 12 \\ \text{"error"} & \text{otherwise} \end{cases}$

A = 1105 in<sup>2</sup>

Eccentricities and Moments of Inertia of the bottom flange:

Frice :=	5.573in if Choice = 1	IBottom :=	3697in <sup>4</sup> if Choice = 1	
	7.509in if Choice = 2		8766in <sup>4</sup> if Choice = 2	
	7.597in if Choice = 3		9047in <sup>4</sup> if Choice = 3	
	7.597in if Choice = 4		9047in <sup>4</sup> if Choice = 4	
	7.266in if Choice = 5		7280in <sup>4</sup> if Choice = 5	
	6.233in if Choice = 6		3873in <sup>4</sup> if Choice = 6	Frice = 7.509 in
	5.2in if Choice = 7		1829in <sup>4</sup> if Choice = 7	
	5in if Choice = 8		4667in <sup>4</sup> if Choice = 8	IBottom = 8766 in <sup>4</sup>
	5in if Choice = 9		4667in <sup>4</sup> if Choice = 9	
	5in if Choice = 10		4667in <sup>4</sup> if Choice = 10	
	5in if Choice = 11		4667in <sup>4</sup> if Choice = 11	
	FriceSpef if Choice = 12		IBottomSpef if Choice = 12	
	"error" otherwise		"error" otherwise	

Area of Bottom Flange

$$\text{Abf} := \begin{cases} 325\text{in}^2 & \text{if Choice} = 1 \\ 399\text{in}^2 & \text{if Choice} = 2 \\ 404\text{in}^2 & \text{if Choice} = 3 \\ 404\text{in}^2 & \text{if Choice} = 4 \\ 361\text{in}^2 & \text{if Choice} = 5 \\ 262.75\text{in}^2 & \text{if Choice} = 6 \\ 180\text{in}^2 & \text{if Choice} = 7 \\ 560\text{in}^2 & \text{if Choice} = 8 \\ 560\text{in}^2 & \text{if Choice} = 9 \\ 560\text{in}^2 & \text{if Choice} = 10 \\ 560\text{in}^2 & \text{if Choice} = 11 \\ \text{Abottomflange} & \text{if Choice} = 12 \end{cases}$$

Direct Tension Strength of Concrete "TS"

$$\text{ConcTensStrength} := 5 \cdot \sqrt{\frac{\text{ReleaseStrength}}{\text{psi}}} \cdot \text{psi}$$

$$\text{ConcTensStrength} = 447.213595 \text{ psi}$$

$$\text{Abf} = 399 \text{ in}^2$$

$$\text{TensionArea} := \text{Abf}$$

$$\text{TensionArea} = 399 \text{ in}^2$$

$$\text{ConcAllowableTension} := \text{TensionArea} \cdot \text{ConcTensStrength}$$

$$\text{ConcAllowableTension} = 178.438225 \text{ kip}$$

Diameter of prestressing strand

$$\text{D} := \begin{cases} .5\text{in} & \text{if StraChoice} = 1 \\ .5\text{in} & \text{if StraChoice} = 2 \\ .5625\text{in} & \text{if StraChoice} = 3 \\ .6\text{in} & \text{if StraChoice} = 4 \\ \text{"error"} & \text{otherwise} \end{cases}$$

$$\text{D} = 0.6 \text{ in}$$



Assembles debonded lengths into a matrix

```

DebondLength :=
  for g ∈ 1 .. NumberStrands
    out_g ← 0
  for h ∈ 1 .. cols(Debond)
    out_Debond1,h ← Debond2,h
  out ← out·ft
  out
  
```

DebondLength =

	1
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0
16	0
17	0
18	0
19	0
20	0
21	0

ft

DebondLength =

	1
1	0
2	0
3	0
4	0
5	0
6	0
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0
16	0
17	0
18	0
19	0
20	0
21	0
22	0
23	0
24	0
25	0
26	0
27	0
28	0
29	0

ft

Initial conditions in the prestressing strands

TempStrain := .00000667·TempChange      TempStrain = -0.0002

TempStress := TempStrain·Eps      TempStress = -5.70285 ksi

OrigStressStrand :=  $\frac{F_{strand}}{A_{ps}}$       OrigStressStrand = 200.729927 ksi

OrigForceStrand := OrigStressStrand·Aps·NumberStrands      OrigForceStrand = 2156 kip

StressStrand := OrigStressStrand + TempStress      StressStrand = 195.027077 ksi

InitialStrainStrands :=  $\frac{StressStrand}{Eps}$       InitialStrainStrands = 0.00684

Adds debonding lengths to free strand lengths

```

AllStrandLengths := | for p ∈ 1 .. NumberStrands                if cols(FreeStrand) = 2
                    |   outp,1 ← FreeStrand1,1 + DebondLengthp
                    |   outp,2 ← FreeStrand1,2 + DebondLengthp
                    | if cols(FreeStrand) ≠ 2
                    |   for j ∈ 1 .. cols(FreeStrand)
                    |     for i ∈ 1 .. NumberStrands
                    |       outi,j ← FreeStrand1,j + 2·DebondLengthi
                    |     for k ∈ 1 .. NumberStrands
                    |       | outk,1 ← outk,1 - DebondLengthk
                    |       | outk,cols(Lbeams)+1 ← outk,cols(Lbeams)+1 - DebondLengthk
                    | out

```

Adds up all the strand lengths so that the averages for each strand cut can be determined

NS := NumberStrands - 1                      NS = 48

```

TotStrandLengths := | for w ∈ 1 .. cols(AllStrandLengths)
                    |   outNumberStrands,w ← AllStrandLengthsNumberStrands,w
                    |   for z ∈ NS .. 1
                    |     outz,w ← outz+1,w + AllStrandLengthsz,w
                    | out

```

AllStrandLengths =

	1	2	3	4	5
1	50	3	3	3	45
2	50	3	3	3	45
3	50	3	3	3	45
4	50	3	3	3	45
5	50	3	3	3	45
6	50	3	3	3	45
7	50	3	3	3	45
8	50	3	3	3	45
9	50	3	3	3	45
10	50	3	3	3	45
11	50	3	3	3	45
12	50	3	3	3	45
13	50	3	3	3	45
14	50	3	3	3	45
15	50	3	3	3	45
16	50	3	3	3	45
17	50	3	3	3	45
18	50	3	3	3	45
19	50	3	3	3	45
20	50	3	3	3	45
21	50	3	3	3	45
22	50	3	3	3	45
23	50	3	3	3	45
24	50	3	3	3	45
25	50	3	3	3	45
26	50	3	3	3	45
27	50	3	3	3	45
28	50	3	3	3	45

ft

AllStrandLengths =

	1	2	3	4	5
1	50	3	3	3	45
2	50	3	3	3	45
3	50	3	3	3	45
4	50	3	3	3	45
5	50	3	3	3	45
6	50	3	3	3	45
7	50	3	3	3	45
8	50	3	3	3	45
9	50	3	3	3	45
10	50	3	3	3	45
11	50	3	3	3	45
12	50	3	3	3	45
13	50	3	3	3	45
14	50	3	3	3	45
15	50	3	3	3	45
16	50	3	3	3	45
17	50	3	3	3	45
18	50	3	3	3	45
19	50	3	3	3	45
20	50	3	3	3	45
21	50	3	3	3	45
22	50	3	3	3	45

ft

	1	2	3	4	5
1	2520	287	287	287	2275
2	2470	284	284	284	2230
3	2420	281	281	281	2185
4	2370	278	278	278	2140
5	2320	275	275	275	2095
6	2270	272	272	272	2050
7	2220	269	269	269	2005
8	2170	266	266	266	1960
9	2120	263	263	263	1915
10	2070	260	260	260	1870
11	2020	257	257	257	1825
12	1970	254	254	254	1780
13	1920	251	251	251	1735
14	1870	248	248	248	1690
15	1820	245	245	245	1645
16	1770	242	242	242	1600
17	1720	239	239	239	1555
18	1670	236	236	236	1510
19	1620	233	233	233	1465
20	1570	230	230	230	1420
21	1520	227	227	227	1375
22	1470	224	224	224	1330
23	1420	221	221	221	1285
24	1370	218	218	218	1240
25	1320	215	215	215	1195
26	1270	212	212	212	1150
27	1220	209	209	209	1105
28	1170	206	206	206	1060
29	1120	203	203	203	1015

TotStrandLengths =

ft

TotStrandLengths =

	1	2	3	4	5
1	2520	287	287	287	2275
2	2470	284	284	284	2230
3	2420	281	281	281	2185
4	2370	278	278	278	2140
5	2320	275	275	275	2095
6	2270	272	272	272	2050
7	2220	269	269	269	2005
8	2170	266	266	266	1960
9	2120	263	263	263	1915
10	2070	260	260	260	1870
11	2020	257	257	257	1825
12	1970	254	254	254	1780
13	1920	251	251	251	1735
14	1870	248	248	248	1690
15	1820	245	245	245	1645
16	1770	242	242	242	1600
17	1720	239	239	239	1555
18	1670	236	236	236	1510
19	1620	233	233	233	1465
20	1570	230	230	230	1420
21	1520	227	227	227	1375
22	1470	224	224	224	1330
23	1420	221	221	221	1285
24	1370	218	218	218	1240
25	1320	215	215	215	1195
26	1270	212	212	212	1150

ft

Average Strand lengths in each free strand set after each strand is cut

```

AvgStrandLengths :=
  for w ∈ 1 .. cols(AllStrandLengths)
  for z ∈ 1 .. NumberStrands
    inter1_w ← TotStrandLengths_z,w
    inter2_w ← IndexS_z
    out_z,w ←  $\frac{\text{inter1}_w}{\text{inter2}_w}$ 
  for ww ∈ 1 .. cols(AllStrandLengths)
    out_NumberStrands+1,ww ← 0ft
  out

```

Concrete Modulus of Elasticity

$$E := \left( 40000 \cdot \sqrt{\frac{\text{ReleaseStrength}}{\text{psi}}} + 10^6 \right) \cdot \left( \frac{w_c}{145 \cdot \frac{\text{lbf}}{\text{ft}^3}} \right)^{1.5} \cdot \text{psi}$$

$$E = 4816.516412 \text{ ksi}$$

	1	2	3	4	5
1	51.429	5.857	5.857	5.857	46.429
2	51.458	5.917	5.917	5.917	46.458
3	51.489	5.979	5.979	5.979	46.489
4	51.522	6.043	6.043	6.043	46.522
5	51.556	6.111	6.111	6.111	46.556
6	51.591	6.182	6.182	6.182	46.591
7	51.628	6.256	6.256	6.256	46.628
8	51.667	6.333	6.333	6.333	46.667
9	51.707	6.415	6.415	6.415	46.707
10	51.75	6.5	6.5	6.5	46.75
11	51.795	6.59	6.59	6.59	46.795
12	51.842	6.684	6.684	6.684	46.842
13	51.892	6.784	6.784	6.784	46.892
14	51.944	6.889	6.889	6.889	46.944
15	52	7	7	7	47
16	52.059	7.118	7.118	7.118	47.059
17	52.121	7.242	7.242	7.242	47.121
18	52.188	7.375	7.375	7.375	47.188
19	52.258	7.516	7.516	7.516	47.258
20	52.333	7.667	7.667	7.667	47.333
21	52.414	7.828	7.828	7.828	47.414
22	52.5	8	8	8	47.5
23	52.593	8.185	8.185	8.185	47.593
24	52.692	8.385	8.385	8.385	47.692
25	52.8	8.6	8.6	8.6	47.8
26	52.917	8.833	8.833	8.833	47.917
27	53.043	9.087	9.087	9.087	48.043
28	53.182	9.364	9.364	9.364	48.182

AvgStrandLengths =

ft

	1	2	3	4	5
1	51.429	5.857	5.857	5.857	46.429
2	51.458	5.917	5.917	5.917	46.458
3	51.489	5.979	5.979	5.979	46.489
4	51.522	6.043	6.043	6.043	46.522
5	51.556	6.111	6.111	6.111	46.556
6	51.591	6.182	6.182	6.182	46.591
7	51.628	6.256	6.256	6.256	46.628
8	51.667	6.333	6.333	6.333	46.667
9	51.707	6.415	6.415	6.415	46.707
10	51.75	6.5	6.5	6.5	46.75
11	51.795	6.59	6.59	6.59	46.795
12	51.842	6.684	6.684	6.684	46.842
13	51.892	6.784	6.784	6.784	46.892
14	51.944	6.889	6.889	6.889	46.944
15	52	7	7	7	47
16	52.059	7.118	7.118	7.118	47.059
17	52.121	7.242	7.242	7.242	47.121
18	52.188	7.375	7.375	7.375	47.188
19	52.258	7.516	7.516	7.516	47.258
20	52.333	7.667	7.667	7.667	47.333
21	52.414	7.828	7.828	7.828	47.414
22	52.5	8	8	8	47.5
23	52.593	8.185	8.185	8.185	47.593
24	52.692	8.385	8.385	8.385	47.692
25	52.8	8.6	8.6	8.6	47.8
26	52.917	8.833	8.833	8.833	47.917
27	53.043	9.087	9.087	9.087	48.043
28	53.182	9.364	9.364	9.364	48.182
29	53.333	9.667	9.667	9.667	48.333

AvgStrandLengths =

ft

Creates matrix with jacking force for each strand

```
PrestressTransfer := | for g ∈ 1 .. NumberStrands
                      | out_g ← Fstrand
                      | out
```

Adds up total prestress transferred to the beam

```
TotPrestressTransfer := | for q ∈ 1 .. NumberStrands
                          | out_q ← Fstrand·q
                          | out
```

Adds up total prestress transferred to end of beam only  
prestress in debonded strands is not included

```
TotPrestressTransferEnd := | for q ∈ 1 .. 4
                            | out_q ← TotPrestressTransfer_q
                            | for r ∈ 5 .. NumberStrands
                            | | out_r ← out_{r-1} + Fstrand if DebondLength_r = 0ft
                            | | out_r ← out_{r-1} if DebondLength_r ≠ 0ft
                            | out
```



	1
1	44
2	44
3	44
4	44
5	44
6	44
7	44
8	44
9	44
10	44
11	44
12	44
13	44
14	44
15	44
16	44
17	44
18	44
19	44
20	44
21	44
22	44
23	44
24	44
25	44
26	44
27	44
28	44
29	44

PrestressTransfer = kip

	1
1	44
2	44
3	44
4	44
5	44
6	44
7	44
8	44
9	44
10	44
11	44
12	44
13	44
14	44
15	44
16	44
17	44
18	44
19	44
20	44
21	44
22	44
23	44
24	44
25	44
26	44
27	44
28	44
29	44

PrestressTransfer = kip

	1
1	44
2	88
3	132
4	176
5	220
6	264
7	308
8	352
9	396
10	440
11	484
12	528
13	572
14	616
15	660
16	704
17	748
18	792
19	836
20	880
21	924
22	968
23	1012
24	1056
25	1100
26	1144
27	1188
28	1232
29	1276

TotPrestressTransfer = kip

	1
1	44
2	88
3	132
4	176
5	220
6	264
7	308
8	352
9	396
10	440
11	484
12	528
13	572
14	616
15	660
16	704
17	748
18	792
19	836
20	880
21	924
22	968
23	1012
24	1056
25	1100
26	1144

TotPrestressTransfer = kip

	1
1	44
2	88
3	132
4	176
5	220
6	264
7	308
8	352
9	396
10	440
11	484
12	528
13	572
14	616
15	660
16	704
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18	792
19	836
20	880
21	924
22	968
23	1012
24	1056
25	1100
26	1144
27	1188
28	1232

TotPrestressTransferEnd = kip

	1
1	44
2	88
3	132
4	176
5	220
6	264
7	308
8	352
9	396
10	440
11	484
12	528
13	572
14	616
15	660
16	704
17	748
18	792
19	836
20	880
21	924
22	968
23	1012
24	1056
25	1100
26	1144
27	1188
28	1232

TotPrestressTransferEnd = kip

Calculates compression transfer length of the prestressing strands

$$\text{MPa} = 145.037738 \text{ psi}$$

$$\text{CompTransLength} := \frac{0.33 \cdot \text{OrigStressStrand}}{6.9 \cdot \text{MPa}} \cdot \frac{D}{\text{mm}} \cdot \sqrt{\frac{20.7}{\frac{\text{ReleaseStrength}}{\text{MPa}}}} \cdot \text{mm} \qquad \text{CompTransLength} = 24.329204 \text{ in}$$

Calculates friction forces on beam ends

$$\text{Bearing} := \frac{\text{wt} \cdot \text{Lbeams}}{2} \qquad \text{Bearing} = (86.328125 \ 89.205729 \ 90.356771 \ 87.479167) \text{ kip}$$

$$\text{FRf} := \mu \cdot \text{Bearing} \qquad \text{FRfDyn} := \mu_D \cdot \text{Bearing}$$

$$\text{FRf} = (25.898437 \ 26.761719 \ 27.107031 \ 26.24375) \text{ kip} \qquad \text{FRfDyn} = (21.582031 \ 22.301432 \ 22.589193 \ 21.869792) \text{ kip}$$

Converts static friction to a larger matrix

$$\text{FRfw} := \left| \begin{array}{l} \text{if cols}(\text{FreeStrand}) = 2 \\ \quad \left| \begin{array}{l} \text{out}_{1,1} \leftarrow \text{FRf} \\ \text{out}_{1,2} \leftarrow \text{FRf} \end{array} \right. \\ \text{for } g \in 1 \dots \text{cols}(\text{FRf}) \quad \text{if cols}(\text{FreeStrand}) \neq 2 \\ \quad \left| \begin{array}{l} \text{out}_{1,2:g-1} \leftarrow \text{FRf}_{1,g} \\ \text{out}_{1,2:g} \leftarrow \text{FRf}_{1,g} \end{array} \right. \\ \text{out} \end{array} \right. \qquad \text{Bearingw} := \left| \begin{array}{l} \text{for } g \in 1 \dots \text{cols}(\text{Bearing}) \\ \quad \left| \begin{array}{l} \text{out}_{1,2:g-1} \leftarrow \text{Bearing}_{1,g} \\ \text{out}_{1,2:g} \leftarrow \text{Bearing}_{1,g} \end{array} \right. \\ \text{out} \end{array} \right.$$

The total area of prestressing that has yet to be cut at each step in the cutting order

```

ApsUncut :=
  outNumberStrands ← 0 in2
  outNumberStrands-1 ← Aps
  for j ∈ NumberStrands - 2 .. 1
    outj ← (NumberStrands - j) · Aps
  out
  
```

ApsUncut =

	1
1	10.522
2	10.302
3	10.083
4	9.864
5	9.645
6	9.426
7	9.206
8	8.987
9	8.768
10	8.549
11	8.33
12	8.11
13	7.891
14	7.672
15	7.453
16	7.234
17	7.014
18	6.795
19	6.576
20	6.357
21	6.138
22	5.918
23	5.699
24	5.48
25	5.261
26	5.042

in<sup>2</sup>

ApsUncut =

	1
1	10.522
2	10.302
3	10.083
4	9.864
5	9.645
6	9.426
7	9.206
8	8.987
9	8.768
10	8.549
11	8.33
12	8.11
13	7.891
14	7.672
15	7.453
16	7.234
17	7.014
18	6.795
19	6.576
20	6.357
21	6.138
22	5.918
23	5.699
24	5.48
25	5.261
26	5.042

in<sup>2</sup>

The total area of prestressing that has yet to be cut at each step in the cutting order not including the debonded strands

$$\text{ApsUncutEnd} := \begin{cases} \text{out}_1 \leftarrow \text{ApsUncut}_1 - \text{cols}(\text{Debond}) \cdot \text{Aps} \\ \text{for } q \in 2 \dots \text{NumberStrands} \\ \quad \left| \begin{cases} \text{out}_q \leftarrow \text{out}_{q-1} - \text{Aps} & \text{if } \text{DebondLength}_q = 0\text{ft} \\ \text{out}_q \leftarrow \text{out}_{q-1} & \text{if } \text{DebondLength}_q \neq 0\text{ft} \end{cases} \\ \text{out} \end{cases}$$

ApsUncutEnd =

	1
1	8.768
2	8.549
3	8.33
4	8.11
5	7.891
6	7.672
7	7.453
8	7.234
9	7.014
10	6.795
11	6.576
12	6.357
13	6.138
14	5.918
15	5.699
16	5.48
17	5.261
18	5.042
19	4.822
20	4.603

in<sup>2</sup>

ApsUncutEnd =

	1
1	8.768
2	8.549
3	8.33
4	8.11
5	7.891
6	7.672
7	7.453
8	7.234
9	7.014
10	6.795
11	6.576
12	6.357
13	6.138
14	5.918
15	5.699
16	5.48
17	5.261
18	5.042
19	4.822
20	4.603
21	4.384
22	4.165
23	3.946
24	3.726
25	3.507
26	3.288
27	3.069
28	2.85
29	2.63

in<sup>2</sup>

Spring stiffnesses for prestressing strands

```

kSteel := | for q ∈ 1 .. NumberStrands - 1
           | for f ∈ 1 .. cols(FreeStrand)
           |   outq,f ←  $\frac{28500\text{ksi}}{\text{AvgStrandLengths}_{q+1,f}} \cdot \text{ApsUncut}_q$ 
           | out
    
```

Converts kSteel to a larger matrix

```

kSteelw := | for q ∈ 1 .. rows(kSteel)
            |   outq,1 ← kSteelq,1
            |   outq,2 ← kSteelq,2 if cols(FreeStrand) = 2
            |   if cols(FreeStrand) ≠ 2
            |     | outq,2·cols(Lbeams) ← kSteelq,cols(kSteel)
            |     | for c ∈ 2 .. cols(kSteel) - 1
            |     |   | outq,2·c-2 ← kSteelq,c
            |     |   | outq,2·c-1 ← kSteelq,c
            |     | out
    
```

Effective strand stiffness for each beam

```

keffSteel := | for q ∈ 1 .. NumberStrands - 1
              |   outq,1 ←  $\frac{1}{\frac{1}{kSteel_{q,1}} + \frac{1}{kSteel_{q,2}}}$  if cols(FreeStrand) = 2
              |   for f ∈ 1 .. cols(Lbeams) if cols(FreeStrand) ≠ 2
              |     | outq,f ←  $\frac{1}{\frac{1}{kSteel_{q,f}} + \frac{1}{kSteel_{q,f+1}}}$ 
              |   out
    
```

	1	2	3	4	5
1	485.612	4223.459	4223.459	4223.459	537.876
2	475.209	4092.546	4092.546	4092.546	526.318
3	464.806	3962.553	3962.553	3962.553	514.761
4	454.403	3833.509	3833.509	3833.509	503.205
5	444.001	3705.447	3705.447	3705.447	491.65
6	433.599	3578.399	3578.399	3578.399	480.094
7	423.197	3452.4	3452.4	3452.4	468.54
8	412.797	3327.485	3327.485	3327.485	456.986
9	402.396	3203.692	3203.692	3203.692	445.433
10	391.996	3081.061	3081.061	3081.061	433.881
11	381.597	2959.631	2959.631	2959.631	422.329
12	371.199	2839.448	2839.448	2839.448	410.779
13	360.801	2720.555	2720.555	2720.555	399.229
14	350.404	2603	2603	2603	387.681
15	340.008	2486.833	2486.833	2486.833	376.133
16	329.612	2372.106	2372.106	2372.106	364.587
17	319.218	2258.875	2258.875	2258.875	353.043
18	308.825	2147.196	2147.196	2147.196	341.499
19	298.433	2037.13	2037.13	2037.13	329.958
20	288.042	1928.743	1928.743	1928.743	318.418
21	277.653	1822.1	1822.1	1822.1	306.88
22	267.266	1717.273	1717.273	1717.273	295.344
23	256.88	1614.338	1614.338	1614.338	283.811
24	246.496	1513.372	1513.372	1513.372	272.28
25	236.115	1414.46	1414.46	1414.46	260.753
26	225.736	1317.691	1317.691	1317.691	249.228
27	215.359	1223.157	1223.157	1223.157	237.708
28	204.986	1130.959	1130.959	1130.959	226.192

kSteel =

$\frac{\text{kip}}{\text{in}}$



	1	2	3	4	5
1	485.612	4223.459	4223.459	4223.459	537.876
2	475.209	4092.546	4092.546	4092.546	526.318
3	464.806	3962.553	3962.553	3962.553	514.761
4	454.403	3833.509	3833.509	3833.509	503.205
5	444.001	3705.447	3705.447	3705.447	491.65
6	433.599	3578.399	3578.399	3578.399	480.094
7	423.197	3452.4	3452.4	3452.4	468.54
8	412.797	3327.485	3327.485	3327.485	456.986
9	402.396	3203.692	3203.692	3203.692	445.433
10	391.996	3081.061	3081.061	3081.061	433.881
11	381.597	2959.631	2959.631	2959.631	422.329
12	371.199	2839.448	2839.448	2839.448	410.779
13	360.801	2720.555	2720.555	2720.555	399.229
14	350.404	2603	2603	2603	387.681
15	340.008	2486.833	2486.833	2486.833	376.133
16	329.612	2372.106	2372.106	2372.106	364.587
17	319.218	2258.875	2258.875	2258.875	353.043
18	308.825	2147.196	2147.196	2147.196	341.499
19	298.433	2037.13	2037.13	2037.13	329.958
20	288.042	1928.743	1928.743	1928.743	318.418
21	277.653	1822.1	1822.1	1822.1	306.88
22	267.266	1717.273	1717.273	1717.273	295.344
23	256.88	1614.338	1614.338	1614.338	283.811
24	246.496	1513.372	1513.372	1513.372	272.28
25	236.115	1414.46	1414.46	1414.46	260.753
26	225.736	1317.691	1317.691	1317.691	249.228
27	215.359	1223.157	1223.157	1223.157	237.708
28	204.986	1130.959	1130.959	1130.959	226.192

kSteel =

$\frac{\text{kip}}{\text{in}}$

	1	2	3	4	5	6	7	8
1	485.612	4223.459	4223.459	4223.459	4223.459	4223.459	4223.459	537.876
2	475.209	4092.546	4092.546	4092.546	4092.546	4092.546	4092.546	526.318
3	464.806	3962.553	3962.553	3962.553	3962.553	3962.553	3962.553	514.761
4	454.403	3833.509	3833.509	3833.509	3833.509	3833.509	3833.509	503.205
5	444.001	3705.447	3705.447	3705.447	3705.447	3705.447	3705.447	491.65
6	433.599	3578.399	3578.399	3578.399	3578.399	3578.399	3578.399	480.094
7	423.197	3452.4	3452.4	3452.4	3452.4	3452.4	3452.4	468.54
8	412.797	3327.485	3327.485	3327.485	3327.485	3327.485	3327.485	456.986
9	402.396	3203.692	3203.692	3203.692	3203.692	3203.692	3203.692	445.433
10	391.996	3081.061	3081.061	3081.061	3081.061	3081.061	3081.061	433.881
11	381.597	2959.631	2959.631	2959.631	2959.631	2959.631	2959.631	422.329
12	371.199	2839.448	2839.448	2839.448	2839.448	2839.448	2839.448	410.779
13	360.801	2720.555	2720.555	2720.555	2720.555	2720.555	2720.555	399.229
14	350.404	2603	2603	2603	2603	2603	2603	387.681
15	340.008	2486.833	2486.833	2486.833	2486.833	2486.833	2486.833	376.133
16	329.612	2372.106	2372.106	2372.106	2372.106	2372.106	2372.106	364.587
17	319.218	2258.875	2258.875	2258.875	2258.875	2258.875	2258.875	353.043
18	308.825	2147.196	2147.196	2147.196	2147.196	2147.196	2147.196	341.499
19	298.433	2037.13	2037.13	2037.13	2037.13	2037.13	2037.13	329.958
20	288.042	1928.743	1928.743	1928.743	1928.743	1928.743	1928.743	318.418
21	277.653	1822.1	1822.1	1822.1	1822.1	1822.1	1822.1	306.88
22	267.266	1717.273	1717.273	1717.273	1717.273	1717.273	1717.273	295.344
23	256.88	1614.338	1614.338	1614.338	1614.338	1614.338	1614.338	283.811
24	246.496	1513.372	1513.372	1513.372	1513.372	1513.372	1513.372	272.28
25	236.115	1414.46	1414.46	1414.46	1414.46	1414.46	1414.46	260.753
26	225.736	1317.691	1317.691	1317.691	1317.691	1317.691	1317.691	249.228
27	215.359	1223.157	1223.157	1223.157	1223.157	1223.157	1223.157	237.708
28	204.986	1130.959	1130.959	1130.959	1130.959	1130.959	1130.959	226.192
29	194.617	1041.2	1041.2	1041.2	1041.2	1041.2	1041.2	214.68

kSteelw =

kip  
in

	1	2	3	4	5	6	7	8
1	485.612	4223.459	4223.459	4223.459	4223.459	4223.459	4223.459	537.876
2	475.209	4092.546	4092.546	4092.546	4092.546	4092.546	4092.546	526.318
3	464.806	3962.553	3962.553	3962.553	3962.553	3962.553	3962.553	514.761
4	454.403	3833.509	3833.509	3833.509	3833.509	3833.509	3833.509	503.205
5	444.001	3705.447	3705.447	3705.447	3705.447	3705.447	3705.447	491.65
6	433.599	3578.399	3578.399	3578.399	3578.399	3578.399	3578.399	480.094
7	423.197	3452.4	3452.4	3452.4	3452.4	3452.4	3452.4	468.54
8	412.797	3327.485	3327.485	3327.485	3327.485	3327.485	3327.485	456.986
9	402.396	3203.692	3203.692	3203.692	3203.692	3203.692	3203.692	445.433
10	391.996	3081.061	3081.061	3081.061	3081.061	3081.061	3081.061	433.881
11	381.597	2959.631	2959.631	2959.631	2959.631	2959.631	2959.631	422.329
12	371.199	2839.448	2839.448	2839.448	2839.448	2839.448	2839.448	410.779
13	360.801	2720.555	2720.555	2720.555	2720.555	2720.555	2720.555	399.229
14	350.404	2603	2603	2603	2603	2603	2603	387.681
15	340.008	2486.833	2486.833	2486.833	2486.833	2486.833	2486.833	376.133
16	329.612	2372.106	2372.106	2372.106	2372.106	2372.106	2372.106	364.587
17	319.218	2258.875	2258.875	2258.875	2258.875	2258.875	2258.875	353.043
18	308.825	2147.196	2147.196	2147.196	2147.196	2147.196	2147.196	341.499
19	298.433	2037.13	2037.13	2037.13	2037.13	2037.13	2037.13	329.958
20	288.042	1928.743	1928.743	1928.743	1928.743	1928.743	1928.743	318.418
21	277.653	1822.1	1822.1	1822.1	1822.1	1822.1	1822.1	306.88
22	267.266	1717.273	1717.273	1717.273	1717.273	1717.273	1717.273	295.344
23	256.88	1614.338	1614.338	1614.338	1614.338	1614.338	1614.338	283.811
24	246.496	1513.372	1513.372	1513.372	1513.372	1513.372	1513.372	272.28
25	236.115	1414.46	1414.46	1414.46	1414.46	1414.46	1414.46	260.753
26	225.736	1317.691	1317.691	1317.691	1317.691	1317.691	1317.691	249.228
27	215.359	1223.157	1223.157	1223.157	1223.157	1223.157	1223.157	237.708

kSteelw =

$\frac{\text{kip}}{\text{in}}$

	1	2	3	4
1	435.535	2111.73	2111.73	477.113
2	425.77	2046.273	2046.273	466.344
3	416.008	1981.276	1981.276	455.579
4	406.249	1916.755	1916.755	444.816
5	396.492	1852.724	1852.724	434.058
6	386.737	1789.2	1789.2	423.302
7	376.986	1726.2	1726.2	412.551
8	367.238	1663.743	1663.743	401.804
9	357.494	1601.846	1601.846	391.061
10	347.753	1540.53	1540.53	380.323
11	338.015	1479.816	1479.816	369.59
12	328.283	1419.724	1419.724	358.863
13	318.554	1360.277	1360.277	348.141
14	308.831	1301.5	1301.5	337.426
15	299.112	1243.417	1243.417	326.717
16	289.399	1186.053	1186.053	316.016
17	279.693	1129.437	1129.437	305.323
18	269.993	1073.598	1073.598	294.639
19	260.3	1018.565	1018.565	283.964
20	250.615	964.371	964.371	273.299
21	240.939	911.05	911.05	262.645
22	231.272	858.637	858.637	252.004
23	221.616	807.169	807.169	241.376
24	211.971	756.686	756.686	230.762
25	202.338	707.23	707.23	220.166
26	192.72	658.845	658.845	209.587
27	183.118	611.579	611.579	199.029
28	173.533	565.479	565.479	188.493
29	163.969	520.6	520.6	177.983

keffSteel =  $\frac{\text{kip}}{\text{in}}$

	1	2	3	4
1	435.535	2111.73	2111.73	477.113
2	425.77	2046.273	2046.273	466.344
3	416.008	1981.276	1981.276	455.579
4	406.249	1916.755	1916.755	444.816
5	396.492	1852.724	1852.724	434.058
6	386.737	1789.2	1789.2	423.302
7	376.986	1726.2	1726.2	412.551
8	367.238	1663.743	1663.743	401.804
9	357.494	1601.846	1601.846	391.061
10	347.753	1540.53	1540.53	380.323
11	338.015	1479.816	1479.816	369.59
12	328.283	1419.724	1419.724	358.863
13	318.554	1360.277	1360.277	348.141
14	308.831	1301.5	1301.5	337.426
15	299.112	1243.417	1243.417	326.717
16	289.399	1186.053	1186.053	316.016
17	279.693	1129.437	1129.437	305.323
18	269.993	1073.598	1073.598	294.639
19	260.3	1018.565	1018.565	283.964
20	250.615	964.371	964.371	273.299
21	240.939	911.05	911.05	262.645
22	231.272	858.637	858.637	252.004
23	221.616	807.169	807.169	241.376
24	211.971	756.686	756.686	230.762
25	202.338	707.23	707.23	220.166
26	192.72	658.845	658.845	209.587
27	183.118	611.579	611.579	199.029
28	173.533	565.479	565.479	188.493

keffSteel =  $\frac{\text{kip}}{\text{in}}$

Calculates the stiffness of only the nondebonded strands

```
kSteelEnd := | for q ∈ 1 .. NumberStrands - 1
               | for f ∈ 1 .. cols(FreeStrand)
               |   outq,f ←  $\frac{28500\text{ksi}}{\text{AvgStrandLengths}_{q+1,f}} \cdot \text{ApsUncutEnd}_q$ 
               | out
```

Makes kSteelEnd a larger matrix

```
kSteelEndw := | for q ∈ 1 .. rows(kSteelEnd)
               |   | outq,1 ← kSteelEndq,1
               |   | outq,2 ← kSteelEndq,2 if cols(FreeStrand) = 2
               |   | if cols(FreeStrand) ≠ 2
               |   |   | outq,2·cols(Lbeams) ← kSteelEndq,cols(kSteelEnd)
               |   |   | for c ∈ 2 .. cols(kSteelEnd) - 1
               |   |   |   | outq,2·c-2 ← kSteelEndq,c
               |   |   |   | outq,2·c-1 ← kSteelEndq,c
               |   | out
```

	1	2	3	4	5
1	404.677	3519.549	3519.549	3519.549	448.23
2	394.322	3395.942	3395.942	3395.942	436.732
3	383.97	3273.413	3273.413	3273.413	425.238
4	373.62	3151.996	3151.996	3151.996	413.747
5	363.273	3031.729	3031.729	3031.729	402.259
6	352.929	2912.651	2912.651	2912.651	390.775
7	342.588	2794.8	2794.8	2794.8	379.294
8	332.251	2678.22	2678.22	2678.22	367.818
9	321.917	2562.954	2562.954	2562.954	356.347
10	311.587	2449.048	2449.048	2449.048	344.88
11	301.261	2336.551	2336.551	2336.551	333.418
12	290.939	2225.513	2225.513	2225.513	321.962
13	280.623	2115.987	2115.987	2115.987	310.512
14	270.312	2008.029	2008.029	2008.029	299.068
15	260.006	1901.696	1901.696	1901.696	287.631
16	249.706	1797.05	1797.05	1797.05	276.203
17	239.414	1694.156	1694.156	1694.156	264.782
18	229.128	1593.081	1593.081	1593.081	253.371
19	218.851	1493.896	1493.896	1493.896	241.969
20	208.583	1396.676	1396.676	1396.676	230.578
21	198.324	1301.5	1301.5	1301.5	219.2
22	188.076	1208.452	1208.452	1208.452	207.835
23	177.84	1117.618	1117.618	1117.618	196.485
24	167.617	1029.093	1029.093	1029.093	185.151
25	157.41	942.974	942.974	942.974	173.835
26	147.219	859.364	859.364	859.364	162.54
27	137.047	778.373	778.373	778.373	151.269
28	126.896	700.117	700.117	700.117	140.023

kSteelEnd =  $\frac{\text{kip}}{\text{in}}$

	1	2	3	4	5
1	404.677	3519.549	3519.549	3519.549	448.23
2	394.322	3395.942	3395.942	3395.942	436.732
3	383.97	3273.413	3273.413	3273.413	425.238
4	373.62	3151.996	3151.996	3151.996	413.747
5	363.273	3031.729	3031.729	3031.729	402.259
6	352.929	2912.651	2912.651	2912.651	390.775
7	342.588	2794.8	2794.8	2794.8	379.294
8	332.251	2678.22	2678.22	2678.22	367.818
9	321.917	2562.954	2562.954	2562.954	356.347
10	311.587	2449.048	2449.048	2449.048	344.88
11	301.261	2336.551	2336.551	2336.551	333.418
12	290.939	2225.513	2225.513	2225.513	321.962
13	280.623	2115.987	2115.987	2115.987	310.512
14	270.312	2008.029	2008.029	2008.029	299.068
15	260.006	1901.696	1901.696	1901.696	287.631
16	249.706	1797.05	1797.05	1797.05	276.203
17	239.414	1694.156	1694.156	1694.156	264.782
18	229.128	1593.081	1593.081	1593.081	253.371
19	218.851	1493.896	1493.896	1493.896	241.969
20	208.583	1396.676	1396.676	1396.676	230.578
21	198.324	1301.5	1301.5	1301.5	219.2
22	188.076	1208.452	1208.452	1208.452	207.835
23	177.84	1117.618	1117.618	1117.618	196.485
24	167.617	1029.093	1029.093	1029.093	185.151
25	157.41	942.974	942.974	942.974	173.835
26	147.219	859.364	859.364	859.364	162.54

kSteelEnd =

$\frac{\text{kip}}{\text{in}}$



	1	2	3	4	5	6	7	8
1	404.68	3519.55	3519.55	3519.55	3519.55	3519.55	3519.55	448.23
2	394.32	3395.94	3395.94	3395.94	3395.94	3395.94	3395.94	436.73
3	383.97	3273.41	3273.41	3273.41	3273.41	3273.41	3273.41	425.24
4	373.62	3152	3152	3152	3152	3152	3152	413.75
5	363.27	3031.73	3031.73	3031.73	3031.73	3031.73	3031.73	402.26
6	352.93	2912.65	2912.65	2912.65	2912.65	2912.65	2912.65	390.77
7	342.59	2794.8	2794.8	2794.8	2794.8	2794.8	2794.8	379.29
8	332.25	2678.22	2678.22	2678.22	2678.22	2678.22	2678.22	367.82
9	321.92	2562.95	2562.95	2562.95	2562.95	2562.95	2562.95	356.35
10	311.59	2449.05	2449.05	2449.05	2449.05	2449.05	2449.05	344.88
11	301.26	2336.55	2336.55	2336.55	2336.55	2336.55	2336.55	333.42
12	290.94	2225.51	2225.51	2225.51	2225.51	2225.51	2225.51	321.96
13	280.62	2115.99	2115.99	2115.99	2115.99	2115.99	2115.99	310.51
14	270.31	2008.03	2008.03	2008.03	2008.03	2008.03	2008.03	299.07
15	260.01	1901.7	1901.7	1901.7	1901.7	1901.7	1901.7	287.63
16	249.71	1797.05	1797.05	1797.05	1797.05	1797.05	1797.05	276.2
17	239.41	1694.16	1694.16	1694.16	1694.16	1694.16	1694.16	264.78
18	229.13	1593.08	1593.08	1593.08	1593.08	1593.08	1593.08	253.37
19	218.85	1493.9	1493.9	1493.9	1493.9	1493.9	1493.9	241.97
20	208.58	1396.68	1396.68	1396.68	1396.68	1396.68	1396.68	230.58
21	198.32	1301.5	1301.5	1301.5	1301.5	1301.5	1301.5	219.2
22	188.08	1208.45	1208.45	1208.45	1208.45	1208.45	1208.45	207.83
23	177.84	1117.62	1117.62	1117.62	1117.62	1117.62	1117.62	196.48
24	167.62	1029.09	1029.09	1029.09	1029.09	1029.09	1029.09	185.15
25	157.41	942.97	942.97	942.97	942.97	942.97	942.97	173.84
26	147.22	859.36	859.36	859.36	859.36	859.36	859.36	162.54
27	137.05	778.37	778.37	778.37	778.37	778.37	778.37	151.27
28	126.9	700.12	700.12	700.12	700.12	700.12	700.12	140.02

kSteelEndw =

$\frac{\text{kip}}{\text{in}}$

	1	2	3	4	5	6	7	8	
1	404.68	3519.55	3519.55	3519.55	3519.55	3519.55	3519.55	448.23	
2	394.32	3395.94	3395.94	3395.94	3395.94	3395.94	3395.94	436.73	
3	383.97	3273.41	3273.41	3273.41	3273.41	3273.41	3273.41	425.24	
4	373.62	3152	3152	3152	3152	3152	3152	413.75	
5	363.27	3031.73	3031.73	3031.73	3031.73	3031.73	3031.73	402.26	
6	352.93	2912.65	2912.65	2912.65	2912.65	2912.65	2912.65	390.77	
7	342.59	2794.8	2794.8	2794.8	2794.8	2794.8	2794.8	379.29	
8	332.25	2678.22	2678.22	2678.22	2678.22	2678.22	2678.22	367.82	
9	321.92	2562.95	2562.95	2562.95	2562.95	2562.95	2562.95	356.35	
10	311.59	2449.05	2449.05	2449.05	2449.05	2449.05	2449.05	344.88	
11	301.26	2336.55	2336.55	2336.55	2336.55	2336.55	2336.55	333.42	
12	290.94	2225.51	2225.51	2225.51	2225.51	2225.51	2225.51	321.96	
13	280.62	2115.99	2115.99	2115.99	2115.99	2115.99	2115.99	310.51	
14	270.31	2008.03	2008.03	2008.03	2008.03	2008.03	2008.03	299.07	
15	260.01	1901.7	1901.7	1901.7	1901.7	1901.7	1901.7	287.63	
16	249.71	1797.05	1797.05	1797.05	1797.05	1797.05	1797.05	276.2	
17	239.41	1694.16	1694.16	1694.16	1694.16	1694.16	1694.16	264.78	
18	229.13	1593.08	1593.08	1593.08	1593.08	1593.08	1593.08	253.37	
19	218.85	1493.9	1493.9	1493.9	1493.9	1493.9	1493.9	241.97	
20	208.58	1396.68	1396.68	1396.68	1396.68	1396.68	1396.68	230.58	
21	198.32	1301.5	1301.5	1301.5	1301.5	1301.5	1301.5	219.2	
22	188.08	1208.45	1208.45	1208.45	1208.45	1208.45	1208.45	207.83	
23	177.84	1117.62	1117.62	1117.62	1117.62	1117.62	1117.62	196.48	
24	167.62	1029.09	1029.09	1029.09	1029.09	1029.09	1029.09	185.15	
25	157.41	942.97	942.97	942.97	942.97	942.97	942.97	173.84	
26	147.22	859.36	859.36	859.36	859.36	859.36	859.36	162.54	
27	137.05	778.37	778.37	778.37	778.37	778.37	778.37	151.27	
28	126.9	700.12	700.12	700.12	700.12	700.12	700.12	140.02	

kSteelEndw =

$\frac{\text{kip}}{\text{in}}$

Calculates the length of the concrete (for all strands) used for elastic shortening calculations

```

EffConcLength := | for h ∈ 1 .. NumberStrands                                if cols(FreeStrand) = 2
                  |   | outh,1 ← Lbeams -  $\frac{4}{3}$  · CompTransLength - 2 · DebondLengthh
                  |   | outh,1 ← Lbeams if FullLength = 1
                  |   | for w ∈ 1 .. cols(Lbeams)                                if cols(FreeStrand) ≠ 2
                  |   |   | for g ∈ 1 .. NumberStrands
                  |   |   |   | outg,w ← Lbeams1,w -  $\frac{4}{3}$  · CompTransLength - 2 · DebondLengthg
                  |   |   |   | outg,w ← Lbeams1,w if FullLength = 1
                  |   | out
  
```

Calculates the average concrete lengths (from above) at each strand cut

```

AvgEffConcLength := | for w ∈ 1 .. cols(EffConcLength)
                    |   | for i ∈ NumberStrands .. 1
                    |   |   |  $\sum_{n=1}^i \text{EffConcLength}_{n,w}$ 
                    |   |   | outi,w ←  $\frac{\quad}{i}$ 
                    |   | out
  
```

Absolute Value function

```

absVal(ξ) := | out ← ξ if ξ ≥ 0
              | out ← ξ · -1 if ξ < 0
              | out
  
```

	1	2	3	4
1	147.297	152.297	154.297	149.297
2	147.297	152.297	154.297	149.297
3	147.297	152.297	154.297	149.297
4	147.297	152.297	154.297	149.297
5	147.297	152.297	154.297	149.297
6	147.297	152.297	154.297	149.297
7	147.297	152.297	154.297	149.297
8	147.297	152.297	154.297	149.297
9	147.297	152.297	154.297	149.297
10	147.297	152.297	154.297	149.297
11	147.297	152.297	154.297	149.297
12	147.297	152.297	154.297	149.297
13	147.297	152.297	154.297	149.297
14	147.297	152.297	154.297	149.297
15	147.297	152.297	154.297	149.297
16	147.297	152.297	154.297	149.297
17	147.297	152.297	154.297	149.297
18	147.297	152.297	154.297	149.297
19	147.297	152.297	154.297	149.297
20	147.297	152.297	154.297	149.297
21	147.297	152.297	154.297	149.297
22	147.297	152.297	154.297	149.297
23	147.297	152.297	154.297	149.297
24	147.297	152.297	154.297	149.297
25	147.297	152.297	154.297	149.297
26	147.297	152.297	154.297	149.297
27	147.297	152.297	154.297	149.297

EffConcLength =

ft

	1	2	3	4
1	147.3	152.3	154.3	149.3
2	147.3	152.3	154.3	149.3
3	147.3	152.3	154.3	149.3
4	147.3	152.3	154.3	149.3
5	147.3	152.3	154.3	149.3
6	147.3	152.3	154.3	149.3
7	147.3	152.3	154.3	149.3
8	147.3	152.3	154.3	149.3
9	147.3	152.3	154.3	149.3
10	147.3	152.3	154.3	149.3
11	147.3	152.3	154.3	149.3
12	147.3	152.3	154.3	149.3
13	147.3	152.3	154.3	149.3
14	147.3	152.3	154.3	149.3
15	147.3	152.3	154.3	149.3
16	147.3	152.3	154.3	149.3
17	147.3	152.3	154.3	149.3
18	147.3	152.3	154.3	149.3
19	147.3	152.3	154.3	149.3
20	147.3	152.3	154.3	149.3
21	147.3	152.3	154.3	149.3
22	147.3	152.3	154.3	149.3
23	147.3	152.3	154.3	149.3
24	147.3	152.3	154.3	149.3
25	147.3	152.3	154.3	149.3
26	147.3	152.3	154.3	149.3

EffConcLength =

ft

AvgEffConcLength =

	1	2	3	4
1	147.297	152.297	154.297	149.297
2	147.297	152.297	154.297	149.297
3	147.297	152.297	154.297	149.297
4	147.297	152.297	154.297	149.297
5	147.297	152.297	154.297	149.297
6	147.297	152.297	154.297	149.297
7	147.297	152.297	154.297	149.297
8	147.297	152.297	154.297	149.297
9	147.297	152.297	154.297	149.297
10	147.297	152.297	154.297	149.297
11	147.297	152.297	154.297	149.297
12	147.297	152.297	154.297	149.297
13	147.297	152.297	154.297	149.297
14	147.297	152.297	154.297	149.297
15	147.297	152.297	154.297	149.297
16	147.297	152.297	154.297	149.297
17	147.297	152.297	154.297	149.297
18	147.297	152.297	154.297	149.297
19	147.297	152.297	154.297	149.297
20	147.297	152.297	154.297	149.297
21	147.297	152.297	154.297	149.297
22	147.297	152.297	154.297	149.297
23	147.297	152.297	154.297	149.297
24	147.297	152.297	154.297	149.297
25	147.297	152.297	154.297	149.297
26	147.297	152.297	154.297	149.297
27	147.297	152.297	154.297	149.297

ft

AvgEffConcLength =

	1	2	3	4
1	147.3	152.3	154.3	149.3
2	147.3	152.3	154.3	149.3
3	147.3	152.3	154.3	149.3
4	147.3	152.3	154.3	149.3
5	147.3	152.3	154.3	149.3
6	147.3	152.3	154.3	149.3
7	147.3	152.3	154.3	149.3
8	147.3	152.3	154.3	149.3
9	147.3	152.3	154.3	149.3
10	147.3	152.3	154.3	149.3
11	147.3	152.3	154.3	149.3
12	147.3	152.3	154.3	149.3
13	147.3	152.3	154.3	149.3
14	147.3	152.3	154.3	149.3
15	147.3	152.3	154.3	149.3
16	147.3	152.3	154.3	149.3
17	147.3	152.3	154.3	149.3
18	147.3	152.3	154.3	149.3
19	147.3	152.3	154.3	149.3
20	147.3	152.3	154.3	149.3
21	147.3	152.3	154.3	149.3
22	147.3	152.3	154.3	149.3
23	147.3	152.3	154.3	149.3
24	147.3	152.3	154.3	149.3
25	147.3	152.3	154.3	149.3
26	147.3	152.3	154.3	149.3

ft

Equivalent spring stiffness for beam

```

kConc := | for q ∈ 1 .. NumberStrands
          |   for z ∈ 1 .. cols(AvgEffConcLength)
          |     outq,z ←  $\frac{A \cdot E}{\text{AvgEffConcLength}_{q,z}}$ 
          |   out
    
```

	1	2	3	4
1	3011.07	2912.21	2874.47	2970.73
2	3011.07	2912.21	2874.47	2970.73
3	3011.07	2912.21	2874.47	2970.73
4	3011.07	2912.21	2874.47	2970.73
5	3011.07	2912.21	2874.47	2970.73
6	3011.07	2912.21	2874.47	2970.73
7	3011.07	2912.21	2874.47	2970.73
8	3011.07	2912.21	2874.47	2970.73
9	3011.07	2912.21	2874.47	2970.73
10	3011.07	2912.21	2874.47	2970.73
11	3011.07	2912.21	2874.47	2970.73
12	3011.07	2912.21	2874.47	2970.73
13	3011.07	2912.21	2874.47	2970.73
14	3011.07	2912.21	2874.47	2970.73
15	3011.07	2912.21	2874.47	2970.73
16	3011.07	2912.21	2874.47	2970.73
17	3011.07	2912.21	2874.47	2970.73
18	3011.07	2912.21	2874.47	2970.73
19	3011.07	2912.21	2874.47	2970.73
20	3011.07	2912.21	2874.47	2970.73

kConc =

$\frac{\text{kip}}{\text{in}}$

	1	2	3	4
1	3011.07	2912.21	2874.47	2970.73
2	3011.07	2912.21	2874.47	2970.73
3	3011.07	2912.21	2874.47	2970.73
4	3011.07	2912.21	2874.47	2970.73
5	3011.07	2912.21	2874.47	2970.73
6	3011.07	2912.21	2874.47	2970.73
7	3011.07	2912.21	2874.47	2970.73
8	3011.07	2912.21	2874.47	2970.73
9	3011.07	2912.21	2874.47	2970.73
10	3011.07	2912.21	2874.47	2970.73
11	3011.07	2912.21	2874.47	2970.73
12	3011.07	2912.21	2874.47	2970.73
13	3011.07	2912.21	2874.47	2970.73
14	3011.07	2912.21	2874.47	2970.73
15	3011.07	2912.21	2874.47	2970.73
16	3011.07	2912.21	2874.47	2970.73
17	3011.07	2912.21	2874.47	2970.73
18	3011.07	2912.21	2874.47	2970.73
19	3011.07	2912.21	2874.47	2970.73
20	3011.07	2912.21	2874.47	2970.73
21	3011.07	2912.21	2874.47	2970.73
22	3011.07	2912.21	2874.47	2970.73
23	3011.07	2912.21	2874.47	2970.73
24	3011.07	2912.21	2874.47	2970.73
25	3011.07	2912.21	2874.47	2970.73
26	3011.07	2912.21	2874.47	2970.73
27	3011.07	2912.21	2874.47	2970.73
28	3011.07	2912.21	2874.47	2970.73
29	3011.07	2912.21	2874.47	2970.73
30	3011.07	2912.21	2874.47	2970.73

kConc =

$\frac{\text{kip}}{\text{in}}$

Calculates the total prestress force left to shorten the beam and stretch the uncut strands after friction has been overcome

```

TotPTafterFric := for q ∈ 1 .. NumberStrands
  if cols(FreeStrand) = 2
    outq ← 0kip if TotPrestressTransferq ≤ FRf
    outq ← TotPrestressTransferq - FRfDyn if TotPrestressTransferq > FRf
  for b ∈ 1 .. cols(kConc)
    outq,b ← 0kip if TotPrestressTransferq ≤ FRf1,b
    outq,b ← TotPrestressTransferq - FRfDyn1,b if TotPrestressTransferq > FRf1,b
  out
  if cols(FreeStrand) ≠ 2

```

	1	2	3	4
1	22.42	21.7	21.41	22.13
2	66.42	65.7	65.41	66.13
3	110.42	109.7	109.41	110.13
4	154.42	153.7	153.41	154.13
5	198.42	197.7	197.41	198.13
6	242.42	241.7	241.41	242.13
7	286.42	285.7	285.41	286.13
8	330.42	329.7	329.41	330.13
9	374.42	373.7	373.41	374.13
10	418.42	417.7	417.41	418.13
11	462.42	461.7	461.41	462.13
12	506.42	505.7	505.41	506.13
13	550.42	549.7	549.41	550.13
14	594.42	593.7	593.41	594.13
15	638.42	637.7	637.41	638.13
16	682.42	681.7	681.41	682.13
17	726.42	725.7	725.41	726.13
18	770.42	769.7	769.41	770.13
19	814.42	813.7	813.41	814.13
20	858.42	857.7	857.41	858.13
21	902.42	901.7	901.41	902.13
22	946.42	945.7	945.41	946.13
23	990.42	989.7	989.41	990.13
24	1034.42	1033.7	1033.41	1034.13
25	1078.42	1077.7	1077.41	1078.13
26	1122.42	1121.7	1121.41	1122.13
27	1166.42	1165.7	1165.41	1166.13

TotPTafterFric =

	1	2	3	4
1	22.42	21.7	21.41	22.13
2	66.42	65.7	65.41	66.13
3	110.42	109.7	109.41	110.13
4	154.42	153.7	153.41	154.13
5	198.42	197.7	197.41	198.13
6	242.42	241.7	241.41	242.13
7	286.42	285.7	285.41	286.13
8	330.42	329.7	329.41	330.13
9	374.42	373.7	373.41	374.13
10	418.42	417.7	417.41	418.13
11	462.42	461.7	461.41	462.13
12	506.42	505.7	505.41	506.13
13	550.42	549.7	549.41	550.13
14	594.42	593.7	593.41	594.13
15	638.42	637.7	637.41	638.13
16	682.42	681.7	681.41	682.13
17	726.42	725.7	725.41	726.13
18	770.42	769.7	769.41	770.13
19	814.42	813.7	813.41	814.13
20	858.42	857.7	857.41	858.13
21	902.42	901.7	901.41	902.13
22	946.42	945.7	945.41	946.13
23	990.42	989.7	989.41	990.13
24	1034.42	1033.7	1033.41	1034.13
25	1078.42	1077.7	1077.41	1078.13
26	1122.42	1121.7	1121.41	1122.13

kip TotPTafterFric =

kip



Total axial shortening of the beams

```
Xtot :=
  for q ∈ 1 .. NumberStrands - 1
    for b ∈ 1 .. cols(TotPTafterFric)
      outq,b ←  $\frac{\text{TotPTafterFric}_{q,b}}{k\text{Conc}_{q,b} + \text{keffSteel}_{q,b}}$ 
    for v ∈ 1 .. cols(TotPTafterFric)
      outNumberStrands,v ←  $\frac{\text{TotPTafterFric}_{\text{NumberStrands},v}}{k\text{Conc}_{\text{NumberStrands},v}}$ 
  out
```

InCamber = (3 3.1 3.2 3) in

Lbeams = (150 155 157 152) ft

Eccent = 40.39 in

```
CambMov :=
  for j ∈ 1 .. cols(Lbeams)
    out1,j ←  $\frac{\text{InCamber}_{1,j} \cdot \text{Eccent}}{\frac{\text{Lbeams}_{1,j}}{2}}$ 
  out
```

CambMov = (0.134633 0.134633 0.137206 0.132862) in

Total axial shortening plus camber movement of the beams  
contributed by each strand cut

```
XtotInd :=
  for cc ∈ 1 .. cols(Xtot)
    out1,cc ← Xtot1,cc
  for q ∈ rows(Xtot) .. 2
    for c ∈ 1 .. cols(Xtot)
      outq,c ← Xtotq,c - Xtotq-1,c
  out
```

```
XtotIndividual :=
  for j ∈ 1 .. NumberStrands
    for k ∈ 1 .. cols(Lbeams)
      outj,k ←  $\frac{\text{XtotInd}_{j,k}}{\text{Xtot}_{\text{NumberStrands},k}} \cdot \text{CambMov}_{1,k} + \text{XtotInd}_{j,k}$ 
    out
```

X<sub>tot</sub> =

	1	2	3	4
1	0.007	0.004	0.004	0.006
2	0.019	0.013	0.013	0.019
3	0.032	0.022	0.023	0.032
4	0.045	0.032	0.032	0.045
5	0.058	0.041	0.042	0.058
6	0.071	0.051	0.052	0.071
7	0.085	0.062	0.062	0.085
8	0.098	0.072	0.073	0.098
9	0.111	0.083	0.083	0.111
10	0.125	0.094	0.095	0.125
11	0.138	0.105	0.106	0.138
12	0.152	0.117	0.118	0.152
13	0.165	0.129	0.13	0.166
14	0.179	0.141	0.142	0.18
15	0.193	0.153	0.155	0.194
16	0.207	0.166	0.168	0.208
17	0.221	0.18	0.181	0.222
18	0.235	0.193	0.195	0.236
19	0.249	0.207	0.209	0.25
20	0.263	0.221	0.223	0.265
21	0.277	0.236	0.238	0.279
22	0.292	0.251	0.253	0.294
23	0.306	0.266	0.269	0.308
24	0.321	0.282	0.285	0.323
25	0.336	0.298	0.301	0.338
26	0.35	0.314	0.317	0.353
27	0.365	0.331	0.334	0.368

in X<sub>tot</sub> =

	1	2	3	4
1	0.007	0.004	0.004	0.006
2	0.019	0.013	0.013	0.019
3	0.032	0.022	0.023	0.032
4	0.045	0.032	0.032	0.045
5	0.058	0.041	0.042	0.058
6	0.071	0.051	0.052	0.071
7	0.085	0.062	0.062	0.085
8	0.098	0.072	0.073	0.098
9	0.111	0.083	0.083	0.111
10	0.125	0.094	0.095	0.125
11	0.138	0.105	0.106	0.138
12	0.152	0.117	0.118	0.152
13	0.165	0.129	0.13	0.166
14	0.179	0.141	0.142	0.18
15	0.193	0.153	0.155	0.194
16	0.207	0.166	0.168	0.208
17	0.221	0.18	0.181	0.222
18	0.235	0.193	0.195	0.236
19	0.249	0.207	0.209	0.25
20	0.263	0.221	0.223	0.265
21	0.277	0.236	0.238	0.279
22	0.292	0.251	0.253	0.294
23	0.306	0.266	0.269	0.308

in

	1	2	3	4
1	0.007764	0.005128	0.005103	0.007629
2	0.015304	0.010603	0.010694	0.015239
3	0.015391	0.010884	0.01098	0.015335
4	0.015479	0.011174	0.011274	0.015432
5	0.015568	0.011471	0.011576	0.015529
6	0.015657	0.011777	0.011888	0.015627
7	0.015747	0.012091	0.012208	0.015727
8	0.015838	0.012414	0.012536	0.015827
9	0.01593	0.012746	0.012874	0.015928
10	0.016022	0.013086	0.01322	0.01603
11	0.016115	0.013434	0.013575	0.016133
12	0.016209	0.013791	0.013939	0.016236
13	0.016303	0.014155	0.014311	0.016341
14	0.016398	0.014528	0.014691	0.016446
15	0.016494	0.014909	0.01508	0.016553
16	0.016591	0.015297	0.015476	0.01666
17	0.016689	0.015692	0.015879	0.016768
18	0.016787	0.016093	0.016289	0.016877
19	0.016886	0.0165	0.016705	0.016987
20	0.016985	0.016912	0.017126	0.017098
21	0.017085	0.017327	0.017551	0.01721
22	0.017186	0.017746	0.017979	0.017322
23	0.017288	0.018166	0.018409	0.017436
24	0.01739	0.018586	0.01884	0.01755
25	0.017493	0.019004	0.019268	0.017664
26	0.017596	0.019418	0.019694	0.01778
27	0.0177	0.019827	0.020113	0.017895
28	0.017804	0.020228	0.020524	0.018012

XtotIndividual =

in

	1	2	3	4
1	0.007764	0.005128	0.005103	0.007629
2	0.015304	0.010603	0.010694	0.015239
3	0.015391	0.010884	0.01098	0.015335
4	0.015479	0.011174	0.011274	0.015432
5	0.015568	0.011471	0.011576	0.015529
6	0.015657	0.011777	0.011888	0.015627
7	0.015747	0.012091	0.012208	0.015727
8	0.015838	0.012414	0.012536	0.015827
9	0.01593	0.012746	0.012874	0.015928
10	0.016022	0.013086	0.01322	0.01603
11	0.016115	0.013434	0.013575	0.016133
12	0.016209	0.013791	0.013939	0.016236
13	0.016303	0.014155	0.014311	0.016341
14	0.016398	0.014528	0.014691	0.016446
15	0.016494	0.014909	0.01508	0.016553
16	0.016591	0.015297	0.015476	0.01666
17	0.016689	0.015692	0.015879	0.016768
18	0.016787	0.016093	0.016289	0.016877
19	0.016886	0.0165	0.016705	0.016987
20	0.016985	0.016912	0.017126	0.017098
21	0.017085	0.017327	0.017551	0.01721
22	0.017186	0.017746	0.017979	0.017322
23	0.017288	0.018166	0.018409	0.017436
24	0.01739	0.018586	0.01884	0.01755
25	0.017493	0.019004	0.019268	0.017664
26	0.017596	0.019418	0.019694	0.01778

XtotIndividual =

in

Calculates a reference total beam shortening for comparison to model results.  
 The model results for axial shortening should always be less than these reference numbers.

$$\text{Axialrr} := \left\{ \begin{array}{l} \text{out} \leftarrow \frac{(\text{NumberStrands} \cdot \text{Fstrand}) \cdot \text{Lbeams}}{A \cdot E} \quad \text{if } \text{cols}(\text{FreeStrand}) = 2 \\ \text{for } w \in 1 \dots \text{cols}(\text{Lbeams}) \quad \text{if } \text{cols}(\text{FreeStrand}) \neq 2 \\ \quad \text{out}_{1,w} \leftarrow \frac{(\text{NumberStrands} \cdot \text{Fstrand}) \cdot \text{Lbeams}_{1,w}}{A \cdot E} \\ \text{out} \end{array} \right. \quad \text{Axialref} := \left\{ \begin{array}{l} \text{for } j \in 1 \dots \text{cols}(\text{CambMov}) \\ \quad \text{out}_{1,j} \leftarrow \text{Axialrr}_{1,j} + \text{CambMov}_{1,j} \\ \text{out} \end{array} \right.$$

$$\text{Axialrr} = (0.729165 \quad 0.753471 \quad 0.763193 \quad 0.738887) \text{ in}$$

$$\text{Axialref} = (0.863799 \quad 0.888104 \quad 0.900399 \quad 0.871749) \text{ in}$$

$$\text{CambMov} = (0.134633 \quad 0.134633 \quad 0.137206 \quad 0.132862) \text{ in}$$

All the following functions with "Numb" at the end transform previously created matrices and make them much larger. If NumbCalcs = 20 then the matrices get 20 times larger

$$\text{kSteelwNumb} := \left\{ \begin{array}{l} \text{for } q \in 1 \dots \text{rows}(\text{kSteelw}) \\ \quad \text{for } c \in 1 \dots \text{cols}(\text{kSteelw}) \\ \quad \quad \text{for } \text{amp} \in \text{NumbCalcs} \dots 1 \\ \quad \quad \quad \text{out}_{\text{NumbCalcs} \cdot q - \text{amp} + 1, c} \leftarrow \text{kSteelw}_{q, c} \\ \text{out} \end{array} \right.$$

$$\text{kSteelNumb} := \left\{ \begin{array}{l} \text{for } q \in 1 \dots \text{rows}(\text{kSteel}) \\ \quad \text{for } c \in 1 \dots \text{cols}(\text{kSteel}) \\ \quad \quad \text{for } \text{amp} \in \text{NumbCalcs} \dots 1 \\ \quad \quad \quad \text{out}_{\text{NumbCalcs} \cdot q - \text{amp} + 1, c} \leftarrow \text{kSteel}_{q, c} \\ \text{out} \end{array} \right.$$

```

XtotIndividualNumb := | for q ∈ 1 .. rows(XtotIndividual)
                       | for c ∈ 1 .. cols(XtotIndividual)
                       | for amp ∈ NumbCalcs.. 1
                       |
                       |   outNumbCalcs·q-amp+1,c ←  $\frac{XtotIndividual_{q,c}}{NumbCalcs}$ 
                       |
                       | out

```

```

RefSlideNumb := | for q ∈ 1 .. rows(RefSlide)
                  | for c ∈ 1 .. cols(RefSlide)
                  | for amp ∈ NumbCalcs.. 1
                  |
                  |   outNumbCalcs·q-amp+1,c ←  $\frac{RefSlide_{q,c}}{NumbCalcs}$ 
                  |
                  | out

```

```

kSteelEndNumb := | for q ∈ 1 .. rows(kSteelEnd)
                   | for c ∈ 1 .. cols(kSteelEnd)
                   | for amp ∈ NumbCalcs.. 1
                   |
                   |   outNumbCalcs·q-amp+1,c ← kSteelEndq,c
                   |
                   | out

```

```

AvgStrandLengthsNumb := | for q ∈ 1 .. rows(AvgStrandLengths)
                          | for c ∈ 1 .. cols(AvgStrandLengths)
                          | for amp ∈ NumbCalcs.. 1
                          |
                          |   outNumbCalcs·q-amp+1,c ← AvgStrandLengthsq,c
                          |
                          | out

```

```

ApsUncutNumb := | for q ∈ 1 .. rows(ApsUncut)
                  for c ∈ 1 .. cols(ApsUncut)
                    for amp ∈ NumbCalcs.. 1
                      outNumbCalcs·q-amp+1,c ← ApsUncutc
                  | out

```

```

DebondLengthNumb := | for q ∈ 1 .. rows(DebondLength)
                       for c ∈ 1 .. cols(DebondLength)
                         for amp ∈ NumbCalcs.. 1
                           outNumbCalcs·q-amp+1,c ← DebondLengthq,c
                       | out

```

```

ApsUncutEndNumb := | for q ∈ 1 .. rows(ApsUncutEnd)
                      for c ∈ 1 .. cols(ApsUncutEnd)
                        for amp ∈ NumbCalcs.. 1
                          outNumbCalcs·q-amp+1,c ← ApsUncutEndq,c
                      | out

```

```

TotPrestressTransferEndNumb := | for q ∈ 1 .. rows(TotPrestressTransferEnd)
                                  for c ∈ 1 .. cols(TotPrestressTransferEnd)
                                    for amp ∈ NumbCalcs.. 1
                                      outNumbCalcs·q-amp+1,c ← TotPrestressTransferEndq,c
                                  | out

```

Creates a matrix of zeros

```
CreateZeros := | for z ∈ 1..3  
                | for g ∈ 1..cols(FreeStrand)  
                |   outz,g ← 0  
                | out
```

$$\text{CreateZeros} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Takes three variables and places them into a matrix so they can be transferred around as one variable

```
ConstPack(qq, XXX, XXXp) := | out ← CreateZeros  
                             | out1,1 ← qq  
                             | for g ∈ 1..cols(CreateZeros)  
                             |   | out2,g ← XXX1,g  
                             |   | out3,g ← XXXp1,g  
                             | out
```



Determines global motion and tension pull

```

Global(PackIn) :=
  for ss ∈ 1 .. cols(Lbeams)
    SlideIndicatorss ← 1
    SlideValuess ← 0in
    SlideTotperq1,ss ← 0in
  qG ← PackIn1,1
  for yy ∈ 1 .. cols(FreeStrand)
    XXGyy ← PackIn2,yyin
    XXprevGyy ← PackIn3,yyin
  for iterate ∈ 1 .. 1000
    for kk ∈ 1 .. cols(Lbeams)
      if SlideIndicatorkk = 2
        if ΔUSFGkk ≥ 0kip
          XXGkk ← XXGkk + SlideValuekk
          XXGkk+1 ← XXGkk+1 - SlideValuekk
          SlideTotperqiterate,kk ← SlideTotperqiterate-1,kk + SlideValuekk
        if ΔUSFGkk < 0kip
          XXGkk ← XXGkk - SlideValuekk
          XXGkk+1 ← XXGkk+1 + SlideValuekk
          SlideTotperqiterate,kk ← SlideTotperqiterate-1,kk - SlideValuekk
    if iterate > 1

```

```

for h ∈ 1 .. cols(Lbeams) + 1
  TensionPullGh ← XXGh · kSteelNumbqG,h + TempStress · ApsUncutNumbqG
TensionPullwG1 ← TensionPullG1
TensionPullwG2.cols(Lbeams) ← TensionPullGcols(Lbeams)+1
for aa ∈ 2 .. cols(Lbeams)
  TensionPullwG2.aa-2 ← TensionPullGaa
  TensionPullwG2.aa-1 ← TensionPullGaa
for cc ∈ 1 .. cols(Lbeams)
  ΔUSFGcc ← TensionPullwG2.cc - TensionPullwG2.cc-1
  ΔUSFGItiterate,cc ← ΔUSFGcc
  SlideIndicatorcc ← 2 if absVal $\left(\frac{\Delta USFG_{cc}}{kip}\right) \cdot kip \geq 2 \cdot FRf_{1,cc}$ 
  SlideValuecc ← RefSlideNumbqG,cc
  SlideIndicatorcc ← 1 if absVal $\left(\frac{\Delta USFG_{cc}}{kip}\right) \cdot kip < 2 \cdot FRfDyn_{1,cc}$  if SlideIndicatorcc = 2
  SlideTotperqGcc ← SlideTotperqiterate,cc
for uu ∈ 1 .. cols(Lbeams) + 1
  XXoutuu ← XXGuu
break if max(SlideIndicator) = 1
out ←  $\left(\frac{XXout}{in} \frac{\Delta USFGIt}{kip} \frac{SlideTotperqG}{in}\right)$ 
out

```

Determines movement at each of the beam ends  
 Determines CT values for all beam ends

```

Equation := for q ∈ 1 .. (NumberStrands - 1) · NumbCalcs
  if cols(FreeStrand) = 2
    Slideqq ← 0in
    TotalSlideqq ← 0in
    if q = 1
      Δq,2 ←  $\frac{k_{\text{SteelwNumb}}_{q,1} \cdot X_{\text{totIndividualNumb}}_q}{k_{\text{SteelwNumb}}_{q,2} + k_{\text{SteelwNumb}}_{q,1}}$ 
      Δq,1 ←  $X_{\text{totIndividualNumb}}_q - \frac{k_{\text{SteelwNumb}}_{q,1} \cdot X_{\text{totIndividualNumb}}_q}{k_{\text{SteelwNumb}}_{q,2} + k_{\text{SteelwNumb}}_{q,1}}$ 
    if q > 1
      Δq,2 ←  $\frac{k_{\text{SteelwNumb}}_{q,1} \cdot X_{\text{totIndividualNumb}}_q - \Delta_{\text{USF}}_{q-1}}{k_{\text{SteelwNumb}}_{q,2} + k_{\text{SteelwNumb}}_{q,1}}$ 
      Δq,1 ←  $X_{\text{totIndividualNumb}}_q - \Delta_{q,2}$ 
      if Δq,2 < 0·in
        Δq,2 ← 0·in
        Δq,1 ←  $X_{\text{totIndividualNumb}}_q$ 
      if Δq,1 < 0·in
        Δq,1 ← 0·in
        Δq,2 ←  $X_{\text{totIndividualNumb}}_q$ 
  
```

for  $g \in 1 \dots \text{cols}(\text{Lbeams})$

if  $\text{cols}(\text{FreeStrand}) \neq 2$

if  $q = 1$

Slide $_{q,g} \leftarrow 0 \cdot \text{in}$

TotalSlide $_{q,g} \leftarrow 0 \cdot \text{in}$

$$\Delta_{q,2 \cdot g} \leftarrow \frac{k_{\text{SteelwNumb}}_{q,2 \cdot g-1} \cdot X_{\text{totIndividualNumb}}_{q,g}}{k_{\text{SteelwNumb}}_{q,2 \cdot g} + k_{\text{SteelwNumb}}_{q,2 \cdot g-1}}$$

$$\Delta_{q,2 \cdot g-1} \leftarrow X_{\text{totIndividualNumb}}_{q,g} - \frac{k_{\text{SteelwNumb}}_{q,2 \cdot g-1} \cdot X_{\text{totIndividualNumb}}_{q,g}}{k_{\text{SteelwNumb}}_{q,2 \cdot g} + k_{\text{SteelwNumb}}_{q,2 \cdot g-1}}$$

if  $\Delta_{q,2 \cdot g} < 0 \cdot \text{in}$

$\Delta_{q,2 \cdot g} \leftarrow 0 \cdot \text{in}$

$\Delta_{q,2 \cdot g-1} \leftarrow X_{\text{totIndividualNumb}}_{q,g}$

if  $\Delta_{q,2 \cdot g-1} < 0 \cdot \text{in}$

$\Delta_{q,2 \cdot g-1} \leftarrow 0 \cdot \text{in}$

$\Delta_{q,2 \cdot g} \leftarrow X_{\text{totIndividualNumb}}_{q,g}$

if  $q > 1$

$$\Delta_{q,2 \cdot g} \leftarrow \frac{k_{\text{SteelwNumb}}_{q,2 \cdot g-1} \cdot X_{\text{totIndividualNumb}}_{q,g} - \Delta_{\text{USF}}_{q-1,g}}{k_{\text{SteelwNumb}}_{q,2 \cdot g} + k_{\text{SteelwNumb}}_{q,2 \cdot g-1}}$$

$$\Delta_{q,2 \cdot g-1} \leftarrow X_{\text{totIndividualNumb}}_{q,g} - \Delta_{q,2 \cdot g}$$

if  $\Delta_{q,2 \cdot g} < 0 \cdot \text{in}$

$\Delta_{q,2 \cdot g} \leftarrow 0 \cdot \text{in}$

$\Delta_{q,2 \cdot g-1} \leftarrow X_{\text{totIndividualNumb}}_{q,g}$

if  $\Delta_{q,2 \cdot g-1} < 0 \cdot \text{in}$

```

| | | | |  $\Delta_{q,2 \cdot g-1} \leftarrow 0 \cdot \text{in}$ 
| | | | |  $\Delta_{q,2 \cdot g} \leftarrow \text{XtotIndividualNumb}_{q,g}$ 
|
| if q = 1
| |  $\text{XX}_{q,1} \leftarrow \Delta_{q,1}$ 
| |  $\text{XX}_{q,2} \leftarrow \Delta_{q,2}$  if cols(FreeStrand) = 2
| | if cols(FreeStrand) ≠ 2
| | |  $\text{XX}_{q,\text{cols(Lbeams)+1}} \leftarrow \Delta_{q,\text{cols(Lbeams)} \cdot 2}$ 
| | | for w ∈ 2 .. cols(Lbeams)
| | | |  $\text{XX}_{q,w} \leftarrow \Delta_{q,2 \cdot w-2} + \Delta_{q,2 \cdot w-1}$ 
|
| if q > 1
| |  $\text{XX}_{q,1} \leftarrow \text{XX}_{q-1,1} + \Delta_{q,1}$ 
| |  $\text{XX}_{q,2} \leftarrow \text{XX}_{q-1,2} + \Delta_{q,2}$  if cols(FreeStrand) = 2
| | if cols(FreeStrand) ≠ 2
| | |  $\text{XX}_{q,\text{cols(Lbeams)+1}} \leftarrow \text{XX}_{q-1,\text{cols(Lbeams)+1}} + \Delta_{q,\text{cols(Lbeams)} \cdot 2}$ 
| | | for w ∈ 2 .. cols(Lbeams)
| | | |  $\text{XX}_{q,w} \leftarrow \text{XX}_{q-1,w} + \Delta_{q,2 \cdot w-2} + \Delta_{q,2 \cdot w-1}$ 
|
| if cols(FreeStrand) ≠ 2 if q > 1
| |  $\text{SendGlobal} \leftarrow \text{ConstPack} \left( q, \frac{(\text{XX}^T)^{\langle q \rangle T}}{\text{in}}, \frac{(\text{XX}^T)^{\langle q-1 \rangle T}}{\text{in}} \right)$ 
| |  $\text{ReceiveGlobal} \leftarrow \text{Global}(\text{SendGlobal})_{1,1}^T \cdot \text{in}$ 
| | for we ∈ 1 .. cols(FreeStrand)
| | |  $\text{XX}_{q,we} \leftarrow \text{ReceiveGlobal}_{we}$ 

```

```

XXq,we ← receiveGlobal1,we
ReceiveSlide ← Global(SendGlobal)1,3T·in
for ss ∈ 1 .. cols(Lbeams)
    Slideq,ss ← ReceiveSlide1,ss
    TotalSlideq,ss ← TotalSlideq-1,ss + Slideq,ss
for h ∈ 1 .. cols(Lbeams) + 1
    TensionPullq,h ← kSteelNumbq,h·XXq,h + TempStress·ApsUncutNumbq
    TensionPullEndq,h ← kSteelEndNumbq,h·XXq,h + TempStress·ApsUncutEndNumbq
    RTLq,h ←  $\frac{0.33}{6.9} \cdot \frac{\text{TensionPull}_{q,h}}{\text{MPa}} \cdot \frac{D}{\text{mm}} \cdot \sqrt{\frac{20.7}{\text{ReleaseStrength}}}$  ·mm if DebondLengthNumbq = 0ft
    RTLq,h ← 0mm if DebondLengthNumbq ≠ 0ft
    TensionPullwq,1 ← TensionPullq,1
    TensionPullEndwq,1 ← TensionPullEndq,1
    RTLwq,1 ← RTLq,1
    if cols(FreeStrand) = 2
        TensionPullwq,2 ← TensionPullq,2
        TensionPullEndwq,2 ← TensionPullEndq,2
        RTLwq,2 ← RTLq,2
    if cols(FreeStrand) ≠ 2
        TensionPullwq,2:cols(Lbeams) ← TensionPullq,cols(Lbeams)+1

```

```

TensionPullEndwq, 2·cols(Lbeams) ← TensionPullEndq, cols(Lbeams)+1
RTLwq, 2·cols(Lbeams) ← RTLq, cols(Lbeams)+1
for aa ∈ 2 .. cols(Lbeams)
    TensionPullEndwq, 2·aa-2 ← TensionPullEndq, aa
    TensionPullEndwq, 2·aa-1 ← TensionPullEndq, aa
    TensionPullwq, 2·aa-2 ← TensionPullq, aa
    TensionPullwq, 2·aa-1 ← TensionPullq, aa
    RTLwq, 2·aa-2 ← RTLq, aa
    RTLwq, 2·aa-1 ← RTLq, aa
if cols(FreeStrand) = 2
    for bb ∈ 1 .. 2
        if DebondLengthNumbq = 0ft
            CRTLwq, bb ←  $\frac{RTLw_{q, bb} \cdot TotPrestressTransferEndNumb_q}{CompTransLength}$ 
            VCTq, bb ← CRTLwq, bb + ConcAllowableTension - TensionPullEndwq, bb - FRfw1, bb
        if DebondLengthNumbq ≠ 0ft
            CRTLwq, bb ← CRTLwq-1, bb
            VCTq, bb ← VCTq-1, bb
            FrictionForceq, bb ← FRfw1, bb
    ΔUSFq ← TensionPullwq, 2 - TensionPullwq, 1
if cols(FreeStrand) ≠ 2
    for bb ∈ 1 .. 2·cols(Lbeams)

```

$$\text{CRTLw}_{q,bb} \leftarrow \frac{\text{RTLw}_{q,bb} \cdot \text{TotPrestressTransferEndNumb}_q}{\text{CompTransLength}} \quad \text{if } \text{DebondLengthNumb}_q = 0\text{ft}$$

$$\text{CRTLw}_{q,bb} \leftarrow \text{CRTLw}_{q-1,bb} \quad \text{if } \text{DebondLengthNumb}_q \neq 0\text{ft}$$

for  $cc \in 1 \dots \text{cols}(\text{Lbeams})$

$$\Delta\text{USF}_{q,cc} \leftarrow \text{TensionPullw}_{q,2:cc} - \text{TensionPullw}_{q,2:cc-1}$$

for  $i \in 1 \dots \text{cols}(\text{Lbeams})$

if  $\Delta\text{USF}_{q,i} \geq 0\text{kip}$

if  $\text{DebondLengthNumb}_q = 0\text{ft}$

$$\text{VCT}_{q,2:i-1} \leftarrow \text{CRTLw}_{q,2:i-1} + \text{ConcAllowableTension} - \text{TensionPullEndw}_{q,2:i-1} - \text{FRfw}_{1,2:i-1}$$

$$\text{VCT}_{q,2:i} \leftarrow \text{CRTLw}_{q,2:i} + \text{ConcAllowableTension} - \text{TensionPullEndw}_{q,2:i} - \text{FRfw}_{1,2:i} + \Delta\text{USF}_{q,i}$$

if  $\text{DebondLengthNumb}_q \neq 0\text{ft}$

$$\text{VCT}_{q,2:i-1} \leftarrow \text{VCT}_{q-1,2:i-1}$$

$$\text{VCT}_{q,2:i} \leftarrow \text{VCT}_{q-1,2:i}$$

$$\text{FrictionForce}_{q,2:i-1} \leftarrow \text{FRfw}_{1,2:i-1}$$

$$\text{FrictionForce}_{q,2:i} \leftarrow \text{FRfw}_{1,2:i} - \Delta\text{USF}_{q,i}$$

if  $\Delta\text{USF}_{q,i} < 0\text{kip}$

if  $\text{DebondLengthNumb}_q = 0\text{ft}$

$$\text{VCT}_{q,2:i-1} \leftarrow \text{CRTLw}_{q,2:i-1} + \text{ConcAllowableTension} - \text{TensionPullEndw}_{q,2:i-1} - \text{FRfw}_{1,2:i-1} + \text{absVal}\left(\frac{\Delta\text{USF}_{q,i}}{\text{kip}}\right)$$

$$\text{VCT}_{q,2:i} \leftarrow \text{CRTLw}_{q,2:i} + \text{ConcAllowableTension} - \text{TensionPullEndw}_{q,2:i} - \text{FRfw}_{1,2:i}$$

if  $\text{DebondLengthNumb}_q \neq 0\text{ft}$

$$\text{VCT}_{q,2:i-1} \leftarrow \text{VCT}_{q-1,2:i-1}$$





All the following functions take extremely large matrices  
and make them small matrices (the ones shown in the results)

```
TotEndMovx :=
  for c ∈ 1 .. cols(EndMovx)
    out1,c ← EndMovx1,c
  for q ∈ 2 .. rows(EndMovx)
    for cc ∈ 1 .. cols(EndMovx)
      outq,cc ← outq-1,cc + EndMovxq,cc
  out
```

```
EndMovxPerStrand :=
  for q ∈ 1 ..  $\frac{\text{rows}(\text{TotEndMovx})}{\text{NumbCalcs}}$ 
    for ce ∈ 1 .. cols(TotEndMovx)
      outq,ce ← TotEndMovxq·NumbCalcs,ce
    for c ∈ 1 .. cols(XtotIndividual)
      outNumberStrands, 2·c-1 ←  $\frac{\text{XtotIndividual}_{\text{NumberStrands}, c}}{2}$  + outNumberStrands-1, 2·c-1
      outNumberStrands, 2·c ←  $\frac{\text{XtotIndividual}_{\text{NumberStrands}, c}}{2}$  + outNumberStrands-1, 2·c
  out
```

$$\text{TensPullPerStrand} := \left| \begin{array}{l} \text{for } q \in 1 \dots \frac{\text{rows}(\text{TensPull})}{\text{NumbCalcs}} \\ \quad \text{for } ce \in 1 \dots \text{cols}(\text{TensPull}) \\ \quad \quad \text{out}_{q,ce} \leftarrow \text{TensPull}_{q \cdot \text{NumbCalcs}, ce} \\ \text{out} \end{array} \right.$$

$$\text{TensPullEndPerStrand} := \left| \begin{array}{l} \text{for } q \in 1 \dots \frac{\text{rows}(\text{TensPullEnd})}{\text{NumbCalcs}} \\ \quad \text{for } ce \in 1 \dots \text{cols}(\text{TensPullEnd}) \\ \quad \quad \text{out}_{q,ce} \leftarrow \text{TensPullEnd}_{q \cdot \text{NumbCalcs}, ce} \\ \text{out} \end{array} \right.$$

$$\text{ReverseTransLengthPerStrand} := \left| \begin{array}{l} \text{for } q \in 1 \dots \frac{\text{rows}(\text{ReverseTransLength})}{\text{NumbCalcs}} \\ \quad \text{for } ce \in 1 \dots \text{cols}(\text{ReverseTransLength}) \\ \quad \quad \text{out}_{q,ce} \leftarrow \text{ReverseTransLength}_{q \cdot \text{NumbCalcs}, ce} \\ \text{out} \end{array} \right.$$

$$\text{CompAtRTLPerStrand} := \left| \begin{array}{l} \text{for } q \in 1 \dots \frac{\text{rows}(\text{CompAtRTL})}{\text{NumbCalcs}} \\ \quad \text{for } ce \in 1 \dots \text{cols}(\text{CompAtRTL}) \\ \quad \quad \text{out}_{q,ce} \leftarrow \text{CompAtRTL}_{q \cdot \text{NumbCalcs}, ce} \\ \text{out} \end{array} \right.$$

$$\text{StrandMovXXPerStrand} := \left| \begin{array}{l} \text{for } q \in 1 \dots \frac{\text{rows}(\text{StrandMovXX})}{\text{NumbCalcs}} \\ \quad \text{for } ce \in 1 \dots \text{cols}(\text{StrandMovXX}) \\ \quad \quad \text{out}_{q,ce} \leftarrow \text{StrandMovXX}_{q \cdot \text{NumbCalcs}, ce} \\ \text{out} \end{array} \right.$$

$$\text{UnbalanceForcePerStrand} := \left| \begin{array}{l} \text{for } q \in 1 \dots \frac{\text{rows}(\text{UnbalanceForce})}{\text{NumbCalcs}} \\ \quad \text{for } ce \in 1 \dots \text{cols}(\text{UnbalanceForce}) \\ \quad \quad \text{out}_{q,ce} \leftarrow \text{UnbalanceForce}_{q \cdot \text{NumbCalcs}, ce} \\ \text{out} \end{array} \right.$$

```

FrictionValuePerStrand :=
  for q ∈ 1 ..  $\frac{\text{rows}(\text{FrictionValue})}{\text{NumbCalcs}}$ 
    for ce ∈ 1 .. cols(FrictionValue)
      outq,ce ← FrictionValueq·NumbCalcs,ce
  out

```

```

BeamSlidePerStrand :=
  for q ∈ 1 ..  $\frac{\text{rows}(\text{BeamSlide})}{\text{NumbCalcs}}$ 
    for ce ∈ 1 .. cols(BeamSlide)
      outq,ce ← BeamSlideq·NumbCalcs,ce
  out

```

```

CrackPredictorPerStrand :=
  for q ∈ 1 ..  $\frac{\text{rows}(\text{CrackPredictor})}{\text{NumbCalcs}}$ 
    for ce ∈ 1 .. cols(CrackPredictor)
      outq,ce ← CrackPredictorq·NumbCalcs,ce
  out

```

```

NSN := NumberStrands·NumbCalcs

```

## END MOVEMENT RESULTS:

Total End Movement for Each Beam End after each strand cut (this does NOT include global motion)  
 In order to determine the total motion of each beam end this value must be added to the sliding value

	1	2	3	4	5	6	7	8
1	0.00772	0.00004	0.00337	0.00176	0.00174	0.00337	0.00004	0.00759
2	0.02303	0.00004	0.01044	0.0053	0.00536	0.01044	0.00004	0.02282
3	0.03842	0.00004	0.01772	0.00889	0.00905	0.01772	0.00004	0.03816
4	0.0539	0.00004	0.02867	0.00912	0.01161	0.02644	0.00004	0.05359
5	0.06947	0.00004	0.03953	0.00973	0.01464	0.03499	0.00004	0.06912
6	0.08512	0.00004	0.05067	0.01037	0.01748	0.04404	0.00004	0.08475
7	0.10087	0.00004	0.06184	0.01129	0.02057	0.05315	0.00004	0.10047
8	0.11671	0.00004	0.07339	0.01215	0.02319	0.06307	0.00004	0.1163
9	0.13264	0.00004	0.08586	0.01243	0.0262	0.07293	0.00004	0.13223
10	0.14866	0.00004	0.09774	0.01364	0.02943	0.08292	0.00004	0.14826
11	0.16478	0.00004	0.11048	0.01432	0.03336	0.09257	0.00004	0.16439
12	0.18098	0.00004	0.12311	0.01549	0.03614	0.10373	0.00004	0.18063
13	0.19729	0.00004	0.13674	0.01601	0.04007	0.1141	0.00004	0.19697
14	0.21369	0.00004	0.14972	0.01756	0.04389	0.12498	0.00004	0.21342
15	0.23018	0.00004	0.16281	0.01939	0.04657	0.13738	0.00004	0.22997
16	0.24677	0.00004	0.17738	0.02011	0.05148	0.14794	0.00004	0.24663
17	0.26346	0.00004	0.19106	0.02213	0.05457	0.16073	0.00004	0.2634
18	0.28025	0.00004	0.2063	0.02298	0.06006	0.17153	0.00004	0.28027
19	0.29713	0.00004	0.222	0.02378	0.06529	0.18301	0.00004	0.29726
20	0.31412	0.00004	0.23655	0.02613	0.06864	0.19678	0.00004	0.31436
21	0.3312	0.00004	0.25124	0.02877	0.07334	0.20964	0.00004	0.33157
22	0.34839	0.00004	0.26748	0.03027	0.07908	0.22187	0.00004	0.34889
23	0.36568	0.00004	0.28514	0.03078	0.08528	0.23408	0.00004	0.36633
24	0.38307	0.00004	0.30009	0.03442	0.08998	0.24822	0.00004	0.38388
25	0.40056	0.00004	0.31538	0.03813	0.09533	0.26213	0.00004	0.40154

EndMovxPerStrand =

in

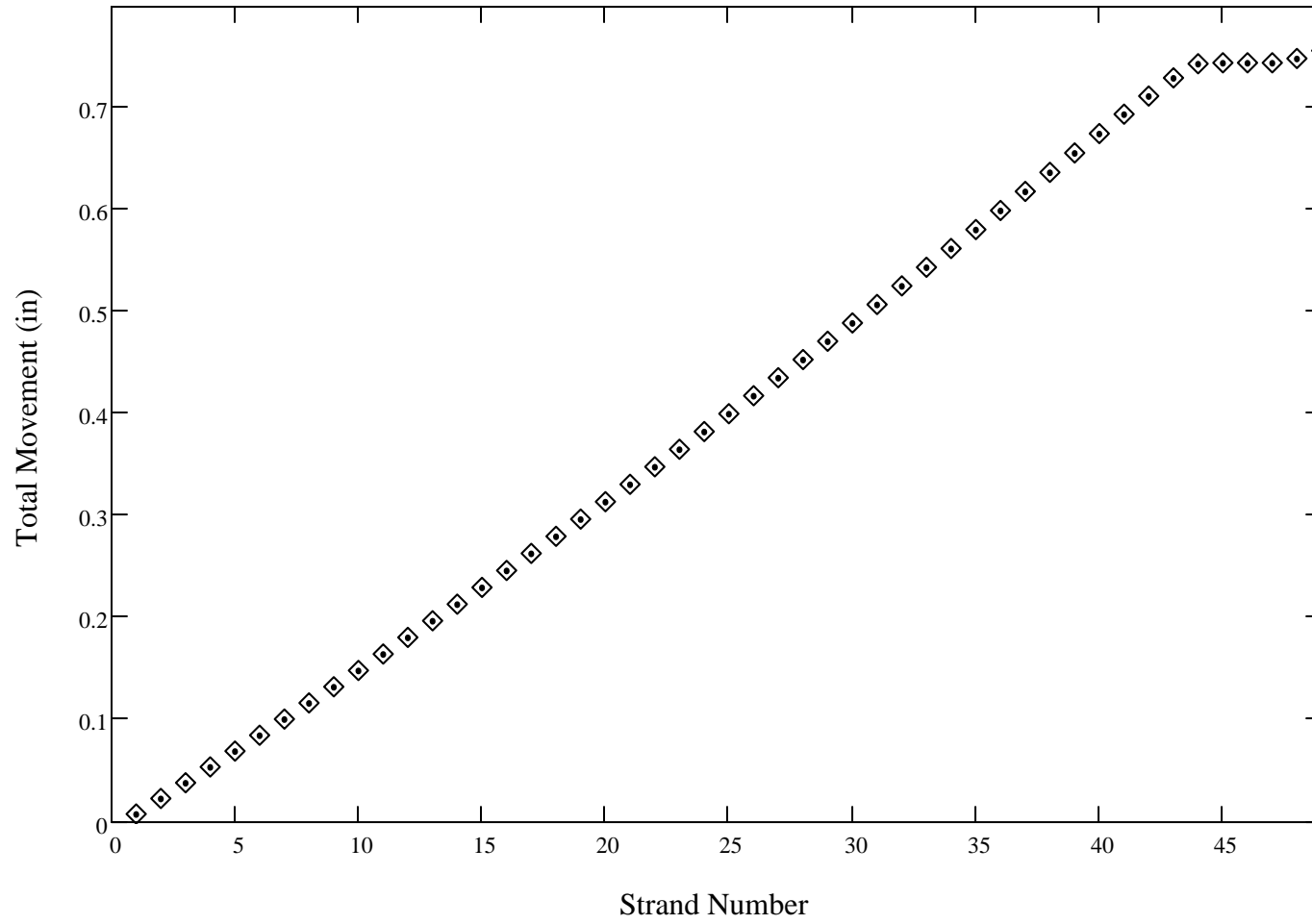
	1	2	3	4	5	6	7	8
1	0.00772	0.00004	0.00337	0.00176	0.00174	0.00337	0.00004	0.00759
2	0.02303	0.00004	0.01044	0.0053	0.00536	0.01044	0.00004	0.02282
3	0.03842	0.00004	0.01772	0.00889	0.00905	0.01772	0.00004	0.03816
4	0.0539	0.00004	0.02867	0.00912	0.01161	0.02644	0.00004	0.05359
5	0.06947	0.00004	0.03953	0.00973	0.01464	0.03499	0.00004	0.06912
6	0.08512	0.00004	0.05067	0.01037	0.01748	0.04404	0.00004	0.08475
7	0.10087	0.00004	0.06184	0.01129	0.02057	0.05315	0.00004	0.10047
8	0.11671	0.00004	0.07339	0.01215	0.02319	0.06307	0.00004	0.1163
9	0.13264	0.00004	0.08586	0.01243	0.0262	0.07293	0.00004	0.13223
10	0.14866	0.00004	0.09774	0.01364	0.02943	0.08292	0.00004	0.14826
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25	0.40056	0.00004	0.31538	0.03813	0.09533	0.26213	0.00004	0.40154

EndMovxPerStrand =

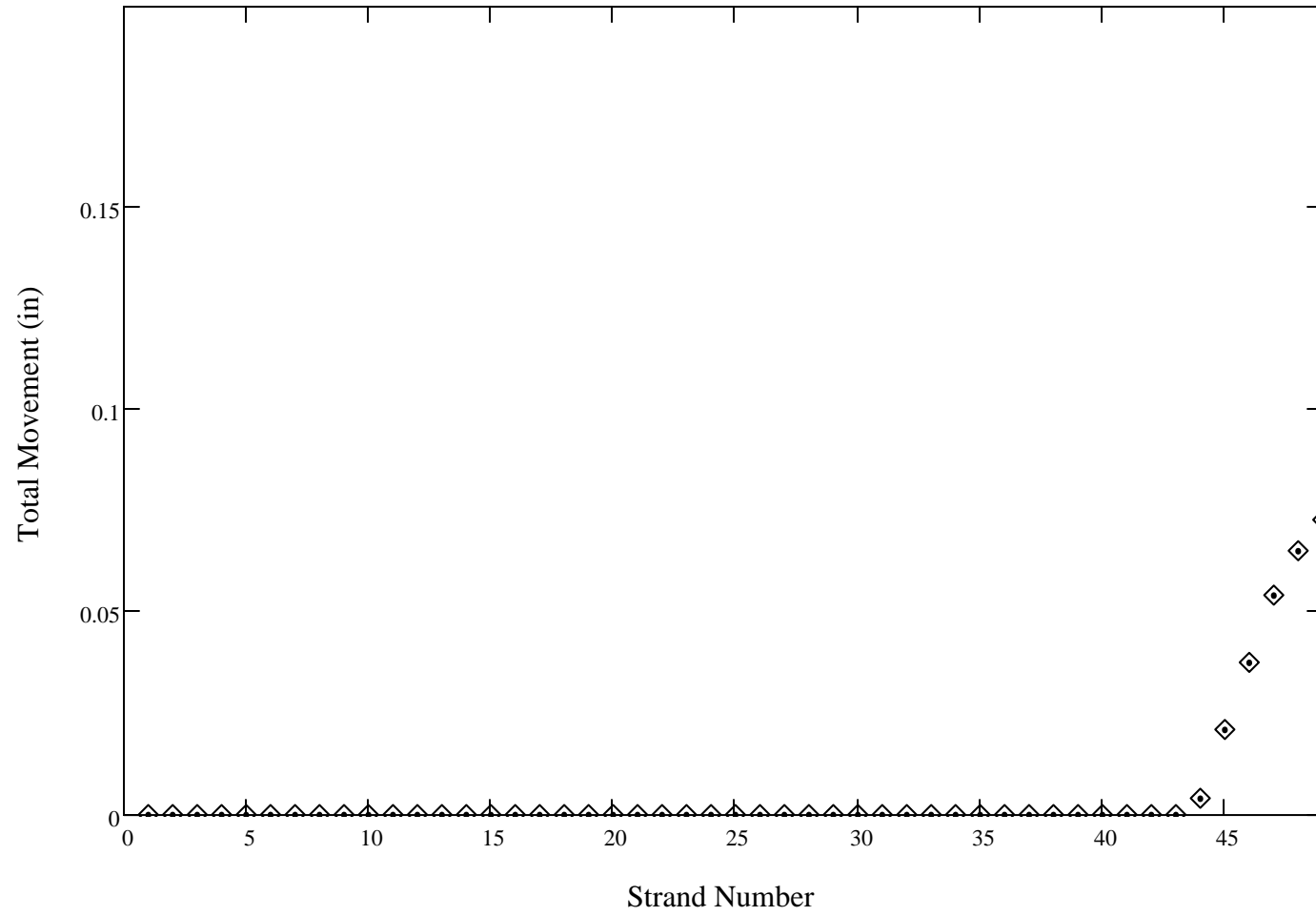
in

Axialref = (0.863799 0.888104 0.900399 0.871749) in

Total End Movement B1 End1

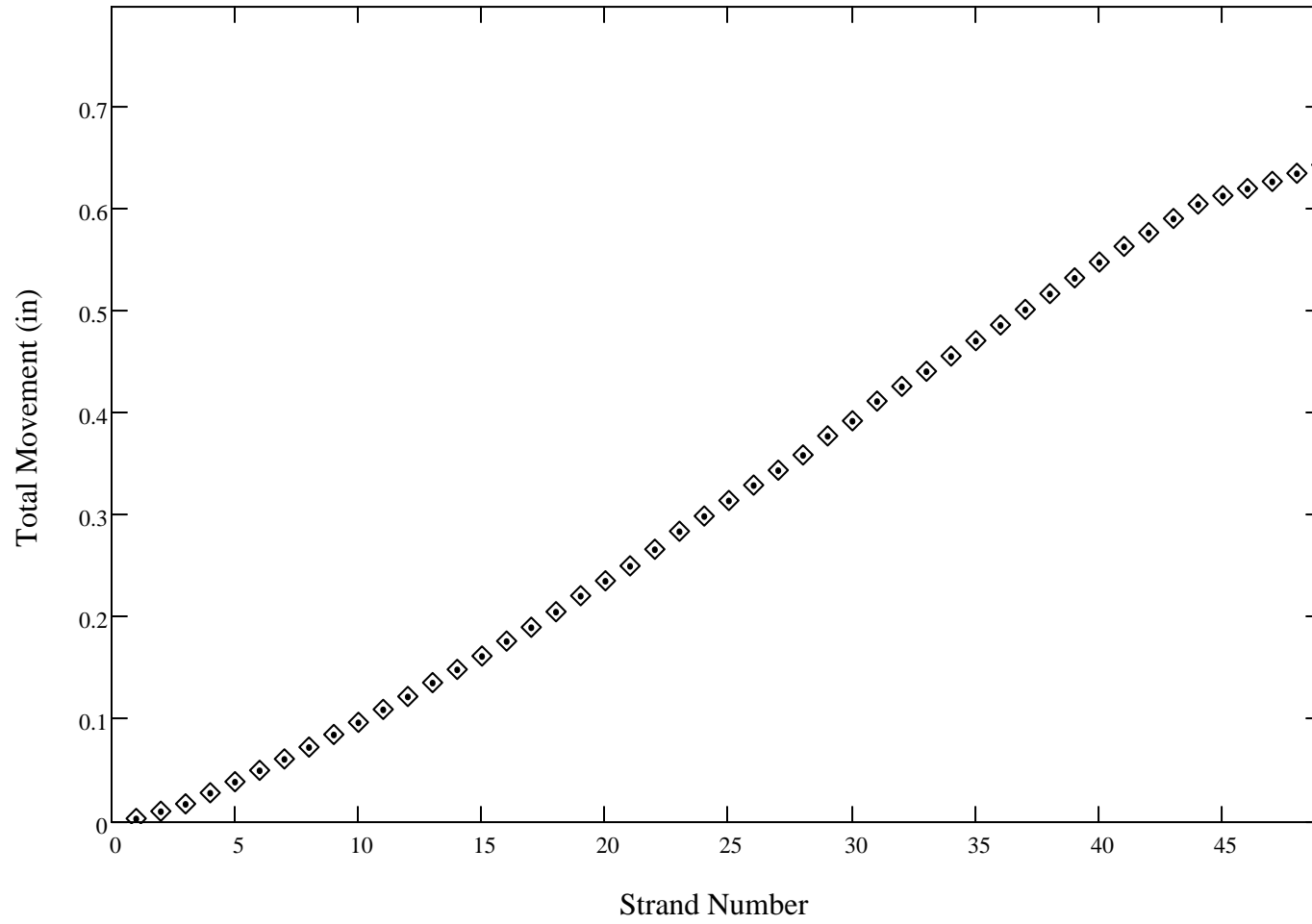


Total End Movement B1 End2

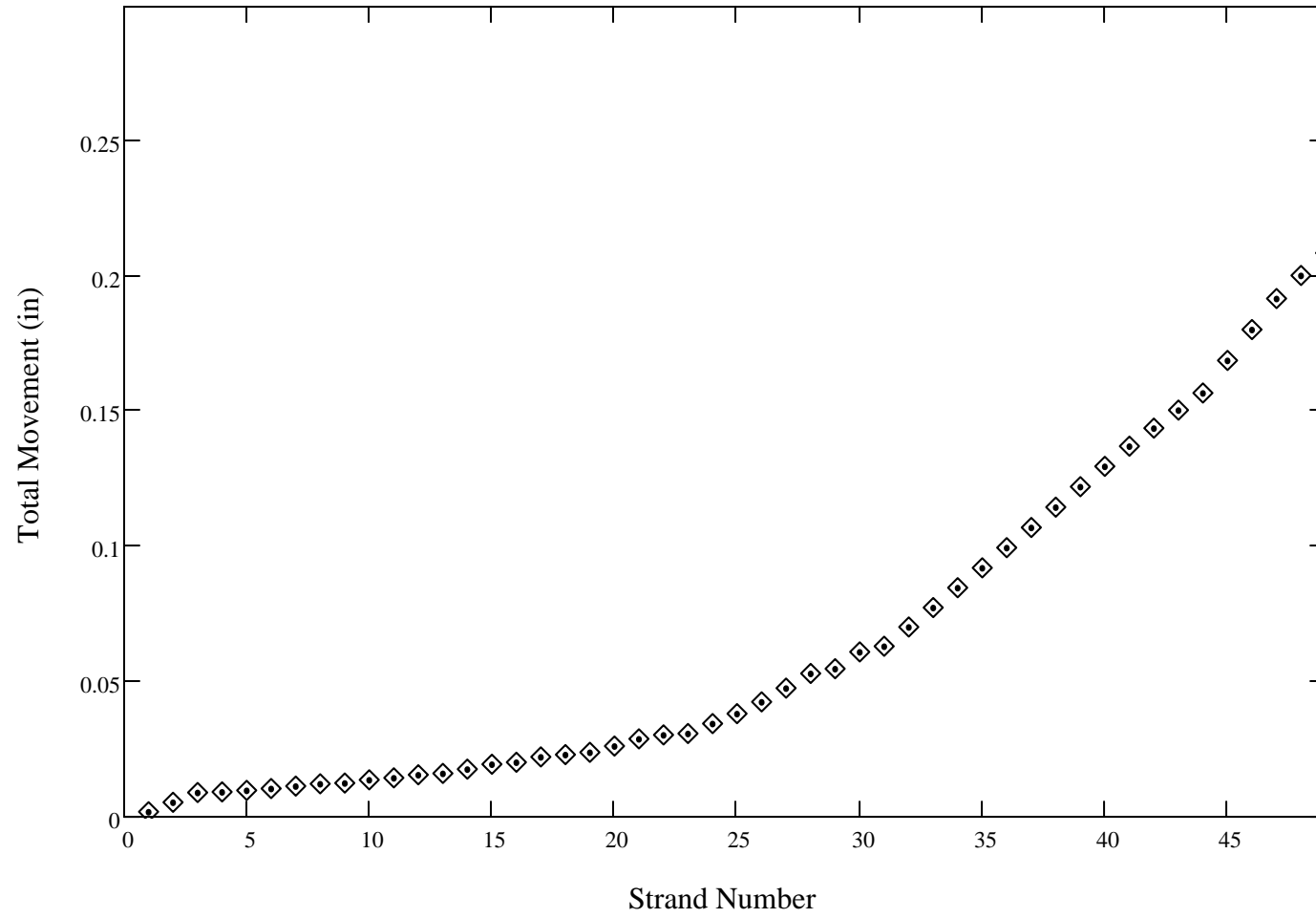




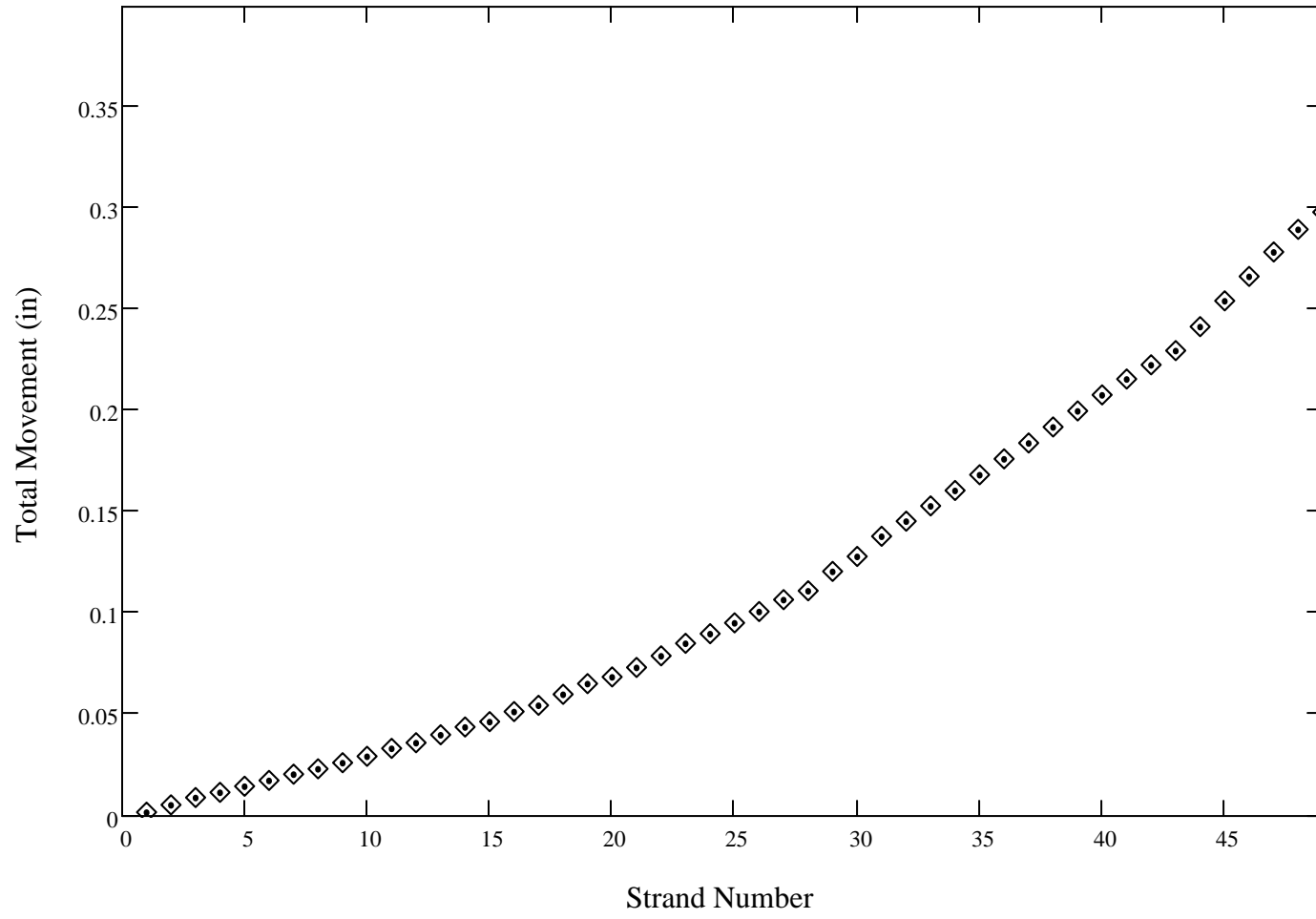
Total End Movement B2 End1



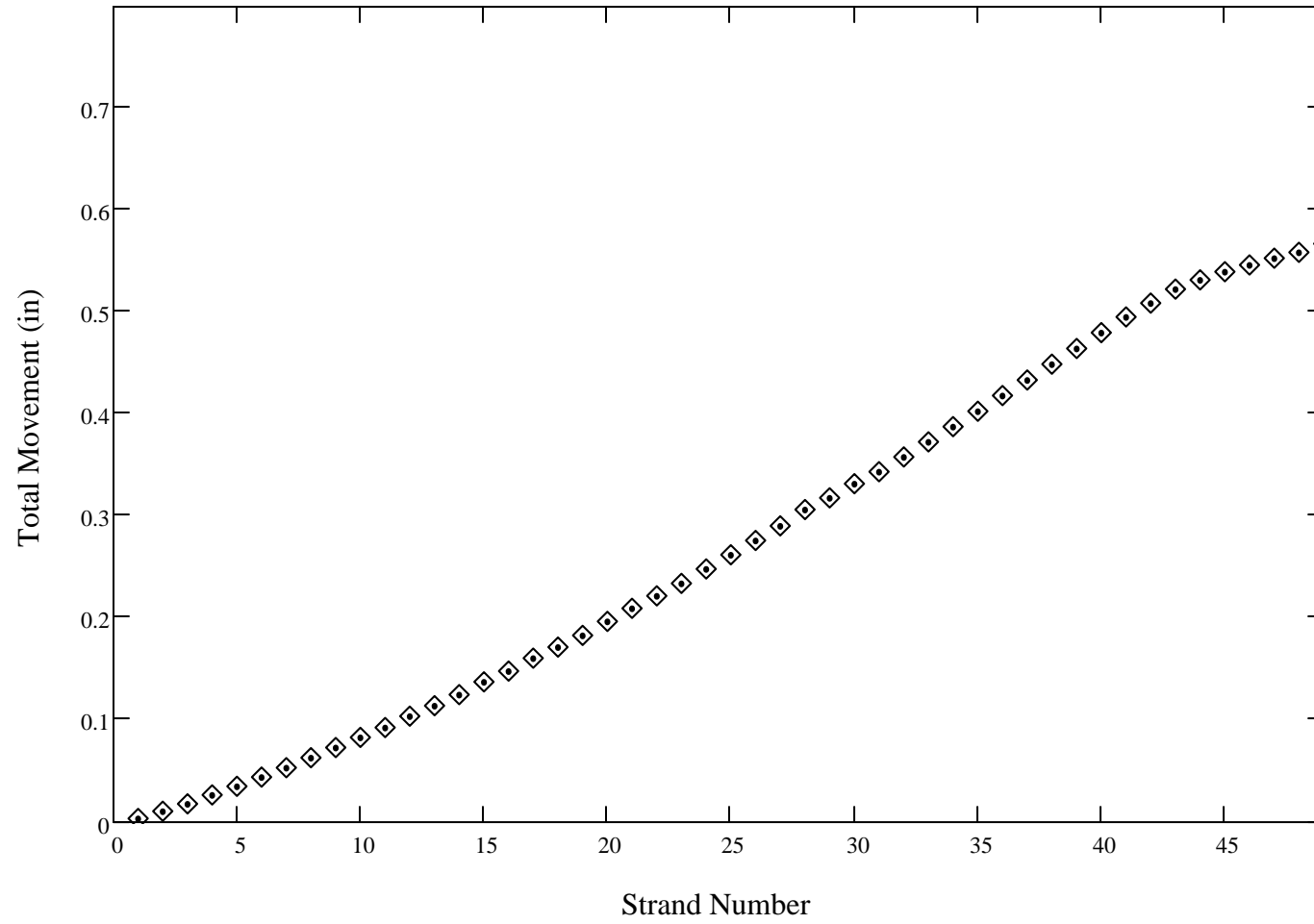
Total End Movement B2 End2



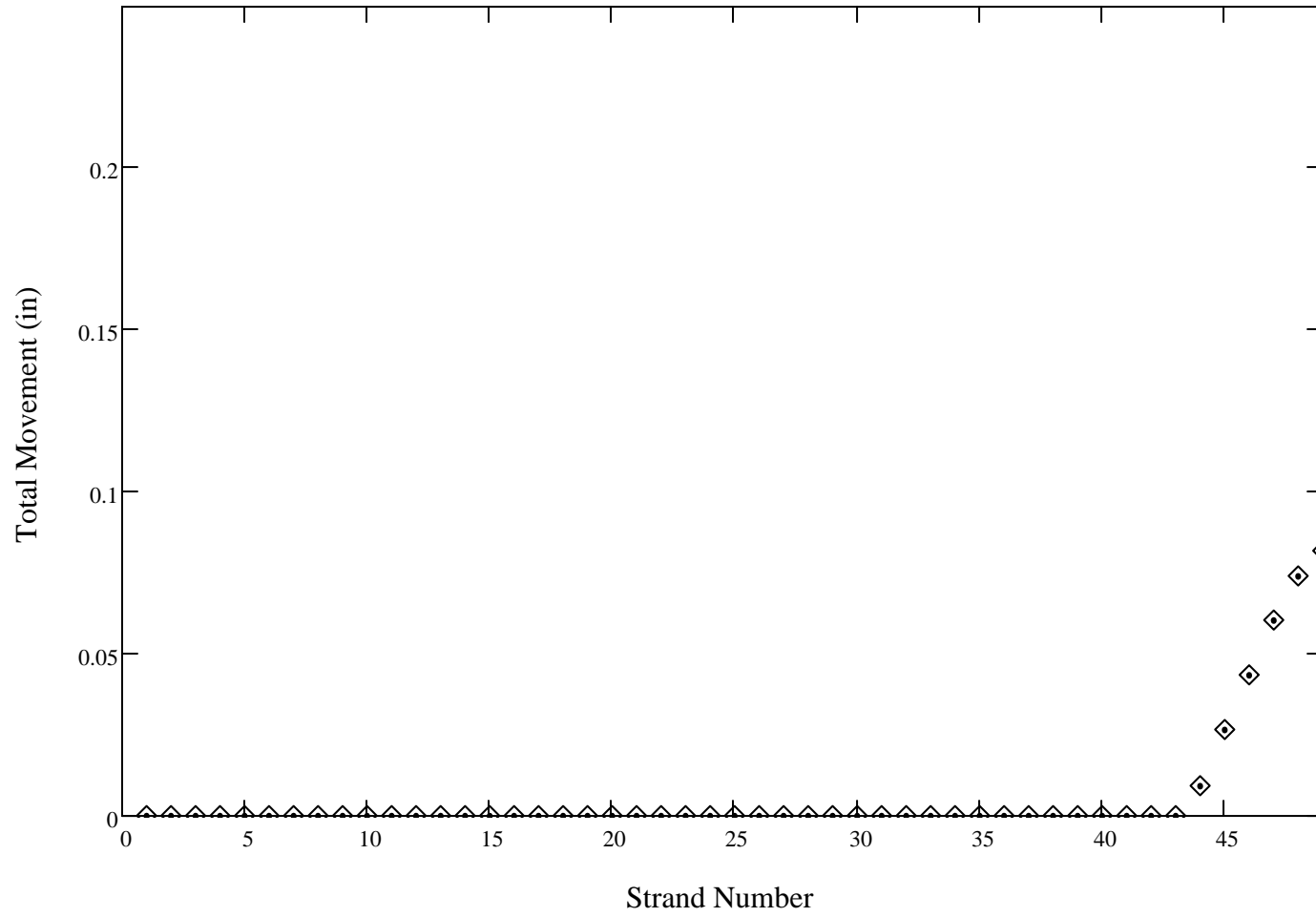
Total End Movement B3 End1



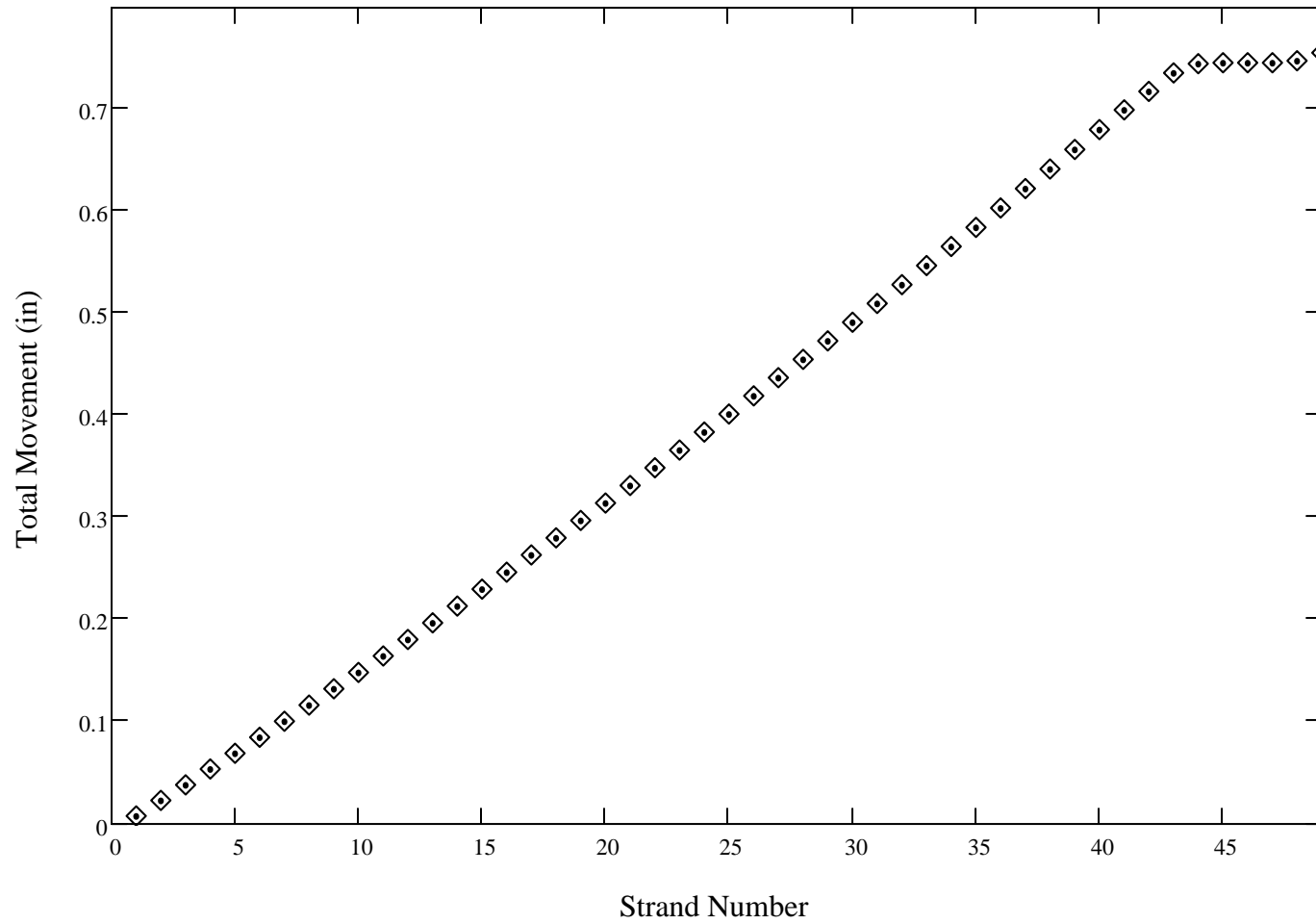
Total End Movement B3 End2



Total End Movement B4 End1



Total End Movement B4 End2



End Movement Each Strand Contributes  
Does NOT include global motion

```

EndMov :=
  for c ∈ 1 .. cols(EndMovxPerStrand)
    out1,c ← EndMovxPerStrand1,c
  for q ∈ 2 .. NumberStrands
    for cc ∈ 1 .. cols(EndMovxPerStrand)
      outq,cc ← EndMovxPerStrandq,cc - EndMovxPerStrandq-1,cc
  out

```

	1	2	3	4	5	6	7	8
1	0.007724	0.00004	0.00337	0.001758	0.001737	0.003366	0.000043	0.007586
2	0.015304	0	0.007065	0.003538	0.003623	0.007071	0	0.015239
3	0.015391	0	0.007286	0.003598	0.003693	0.007287	0	0.015335
4	0.015479	0	0.010947	0.000226	0.00256	0.008714	0	0.015432
5	0.015568	0	0.010862	0.000609	0.003028	0.008548	0	0.015529
6	0.015657	0	0.011134	0.000643	0.002835	0.009053	0	0.015627
7	0.015747	0	0.011171	0.00092	0.003098	0.00911	0	0.015727
8	0.015838	0	0.011556	0.000858	0.002616	0.00992	0	0.015827
9	0.01593	0	0.012464	0.000282	0.003012	0.009862	0	0.015928
10	0.016022	0	0.011881	0.001205	0.003228	0.009992	0	0.01603
11	0.016115	0	0.012748	0.000686	0.003927	0.009648	0	0.016133
12	0.016209	0	0.01262	0.00117	0.002781	0.011158	0	0.016236
13	0.016303	0	0.013634	0.000521	0.003937	0.010374	0	0.016341
14	0.016398	0	0.012981	0.001547	0.003816	0.010875	0	0.016446
15	0.016494	0	0.013085	0.001824	0.002675	0.012404	0	0.016553
16	0.016591	0	0.014574	0.000722	0.004914	0.010562	0	0.01666
17	0.016689	0	0.013675	0.002017	0.003093	0.012786	0	0.016768
18	0.016787	0	0.015242	0.000851	0.005492	0.010797	0	0.016877
19	0.016886	0	0.015699	0.000801	0.005224	0.011481	0	0.016987

EndMov =

in

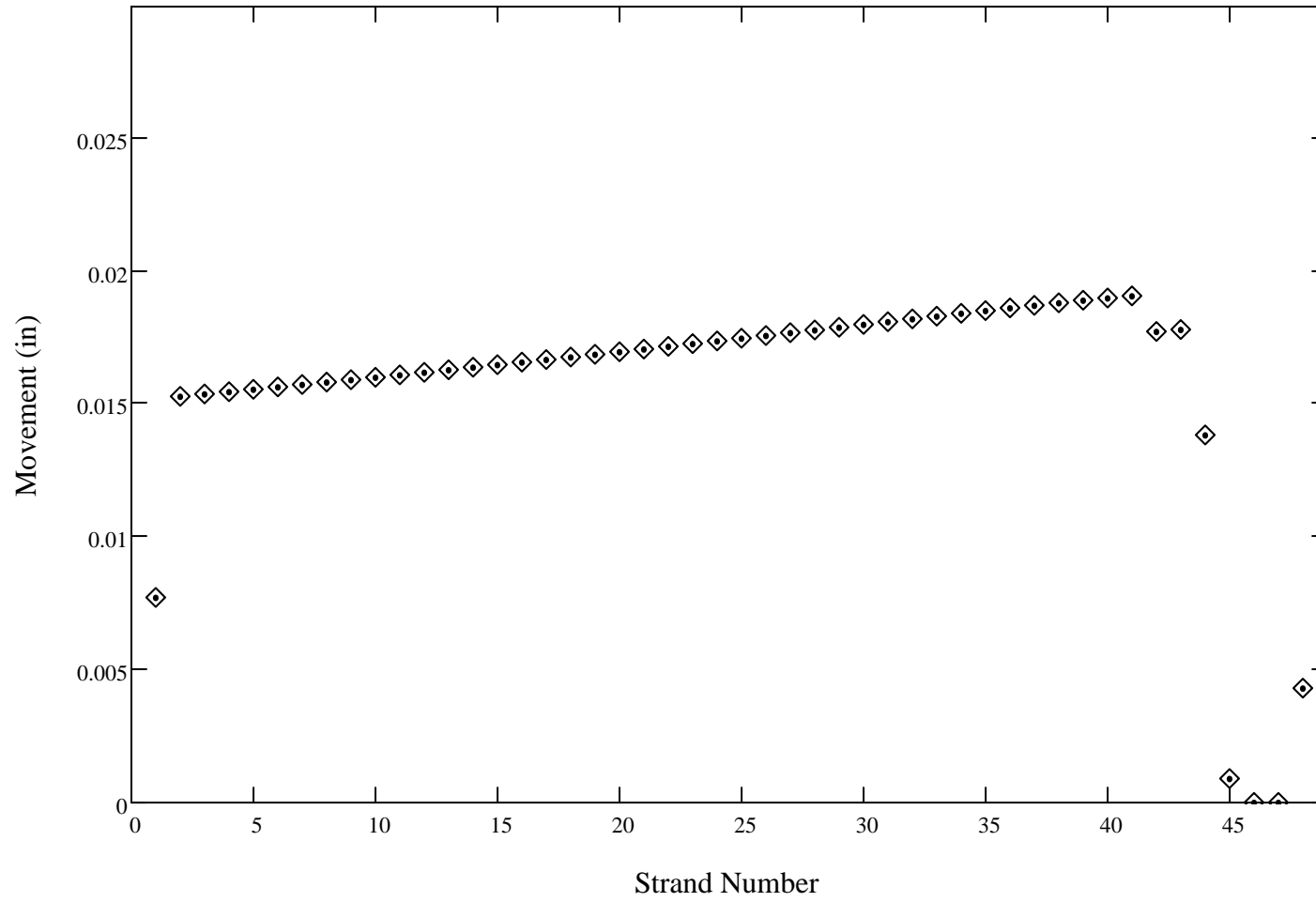
	1	2	3	4	5	6	7	8
1	0.007724	0.00004	0.00337	0.001758	0.001737	0.003366	0.000043	0.007586
2	0.015304	0	0.007065	0.003538	0.003623	0.007071	0	0.015239
3	0.015391	0	0.007286	0.003598	0.003693	0.007287	0	0.015335
4	0.015479	0	0.010947	0.000226	0.00256	0.008714	0	0.015432
5	0.015568	0	0.010862	0.000609	0.003028	0.008548	0	0.015529
6	0.015657	0	0.011134	0.000643	0.002835	0.009053	0	0.015627
7	0.015747	0	0.011171	0.00092	0.003098	0.00911	0	0.015727
8	0.015838	0	0.011556	0.000858	0.002616	0.00992	0	0.015827
9	0.01593	0	0.012464	0.000282	0.003012	0.009862	0	0.015928
10	0.016022	0	0.011881	0.001205	0.003228	0.009992	0	0.01603
11	0.016115	0	0.012748	0.000686	0.003927	0.009648	0	0.016133
12	0.016209	0	0.01262	0.00117	0.002781	0.011158	0	0.016236
13	0.016303	0	0.013634	0.000521	0.003937	0.010374	0	0.016341
14	0.016398	0	0.012981	0.001547	0.003816	0.010875	0	0.016446
15	0.016494	0	0.013085	0.001824	0.002675	0.012404	0	0.016553
16	0.016591	0	0.014574	0.000722	0.004914	0.010562	0	0.01666
17	0.016689	0	0.013675	0.002017	0.003093	0.012786	0	0.016768
18	0.016787	0	0.015242	0.000851	0.005492	0.010797	0	0.016877
19	0.016886	0	0.015699	0.000801	0.005224	0.011481	0	0.016987
20	0.016985	0	0.014557	0.002354	0.003352	0.013774	0	0.017098
21	0.017085	0	0.014684	0.002644	0.004696	0.012855	0	0.01721
22	0.017186	0	0.016247	0.001499	0.005746	0.012233	0	0.017322
23	0.017288	0	0.017661	0.000505	0.006199	0.01221	0	0.017436
24	0.01739	0	0.014948	0.003637	0.004701	0.014139	0	0.01755
25	0.017493	0	0.015292	0.003712	0.005352	0.013916	0	0.017664
26	0.017596	0	0.01505	0.004369	0.005614	0.01408	0	0.01778
27	0.0177	0	0.014773	0.005055	0.005876	0.014237	0	0.017895
28	0.017804	0	0.014795	0.005432	0.004415	0.016109	0	0.018012
29	0.017909	0	0.018816	0.001801	0.009562	0.011362	0	0.018128
30	0.018014	0	0.014791	0.006201	0.007387	0.013922	0	0.018245

EndMov =

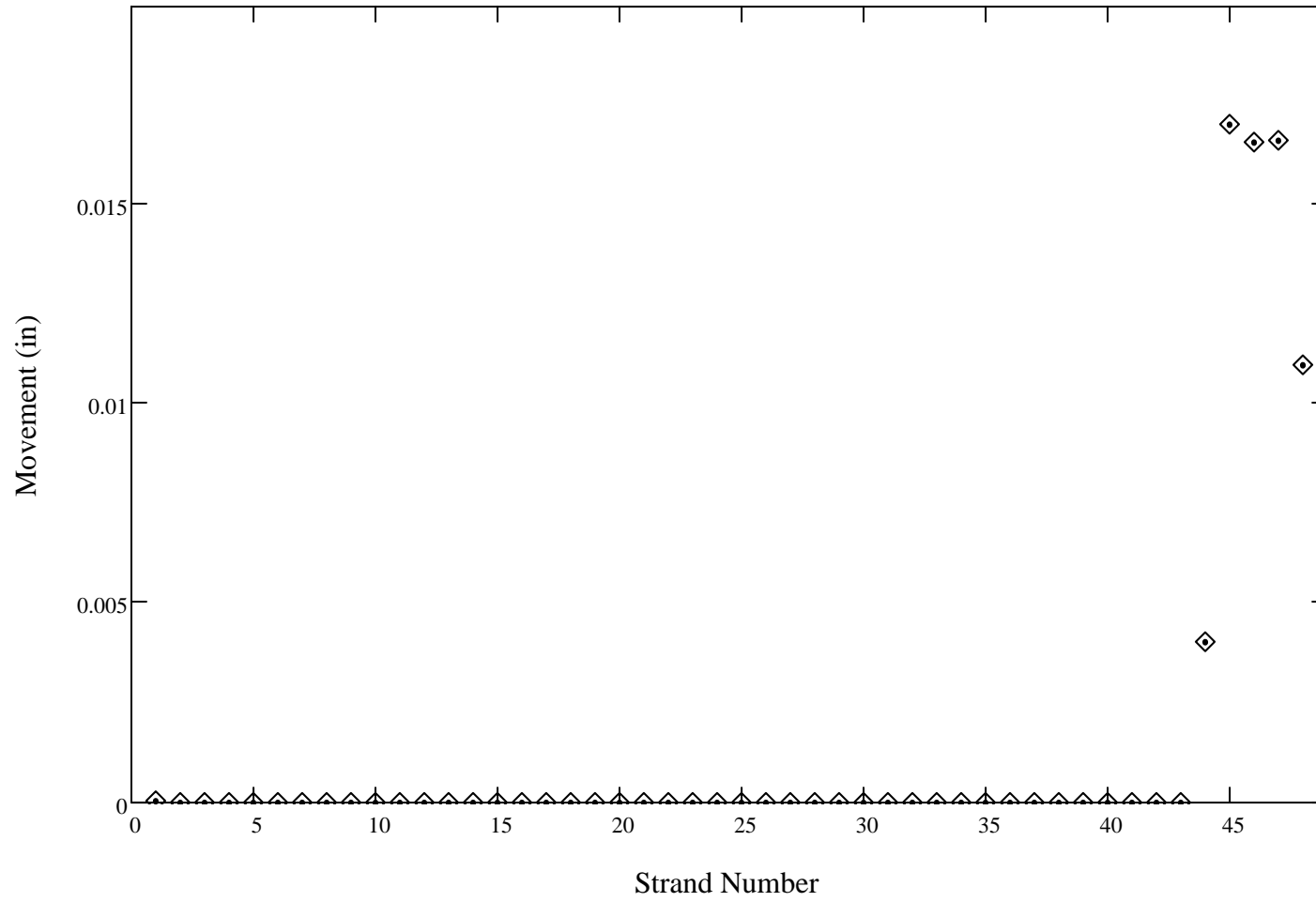
in

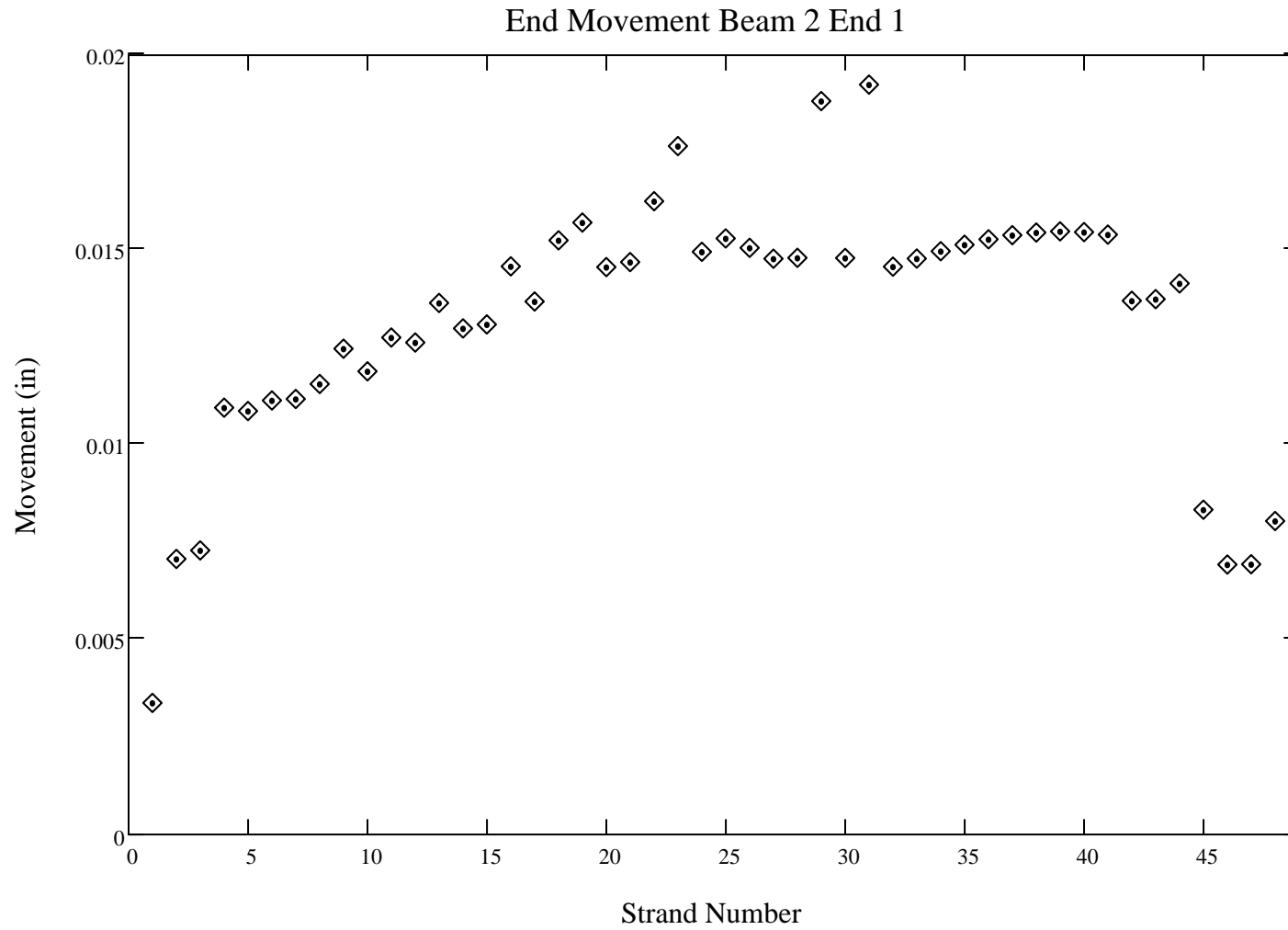


End Movement Beam 1 End 1

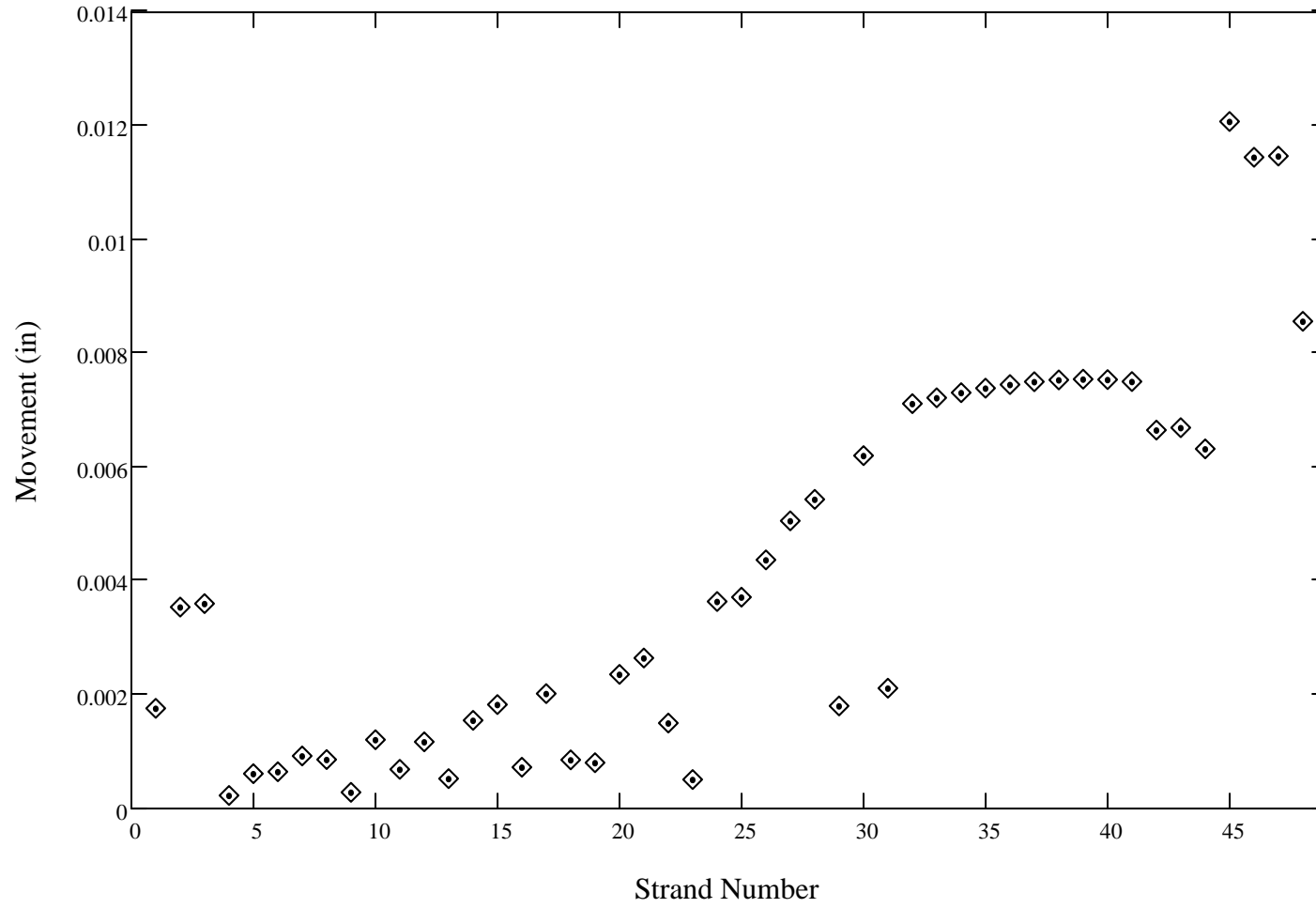


End Movement Beam 1 End 2

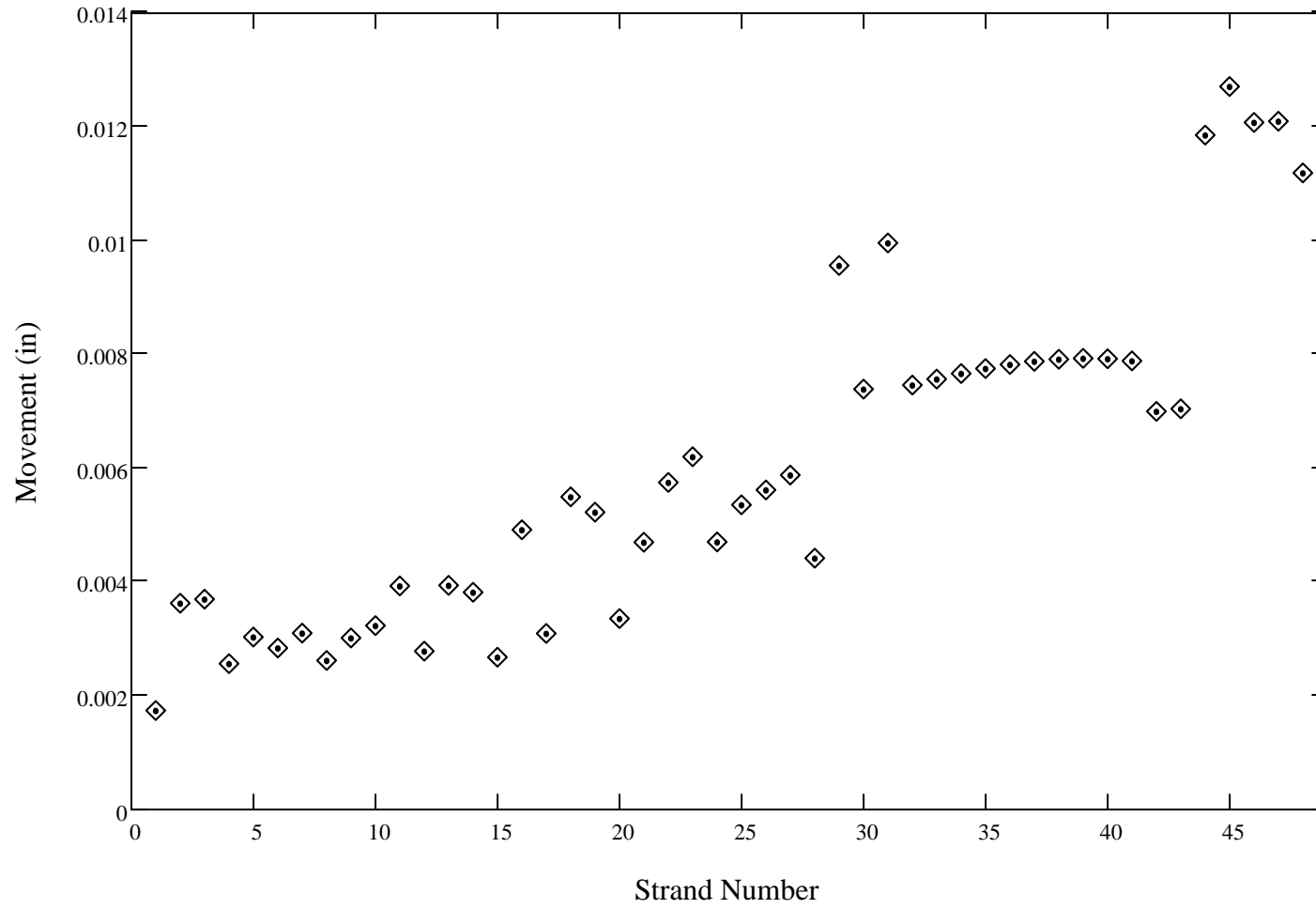




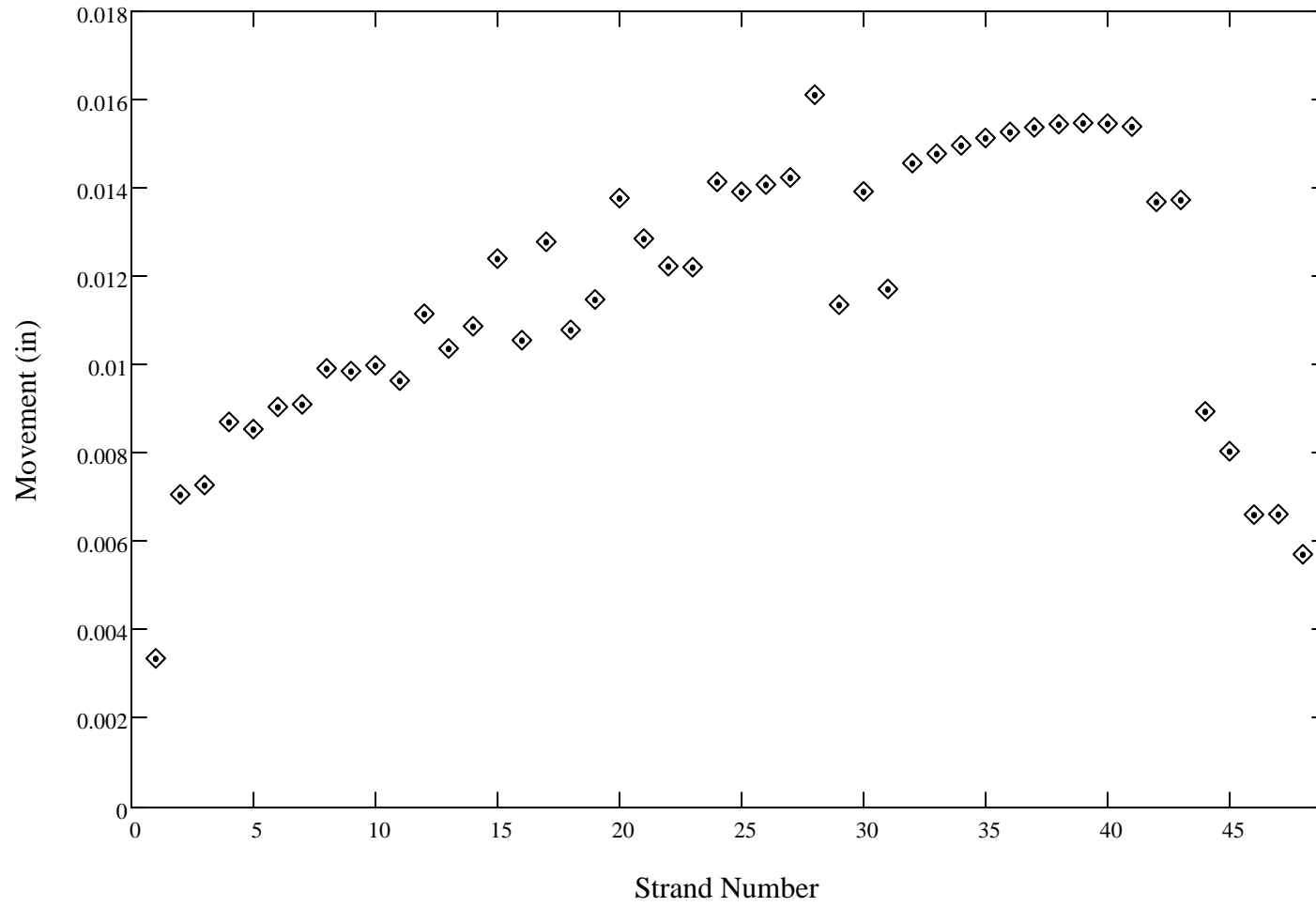
End Movement Beam 2 End 2



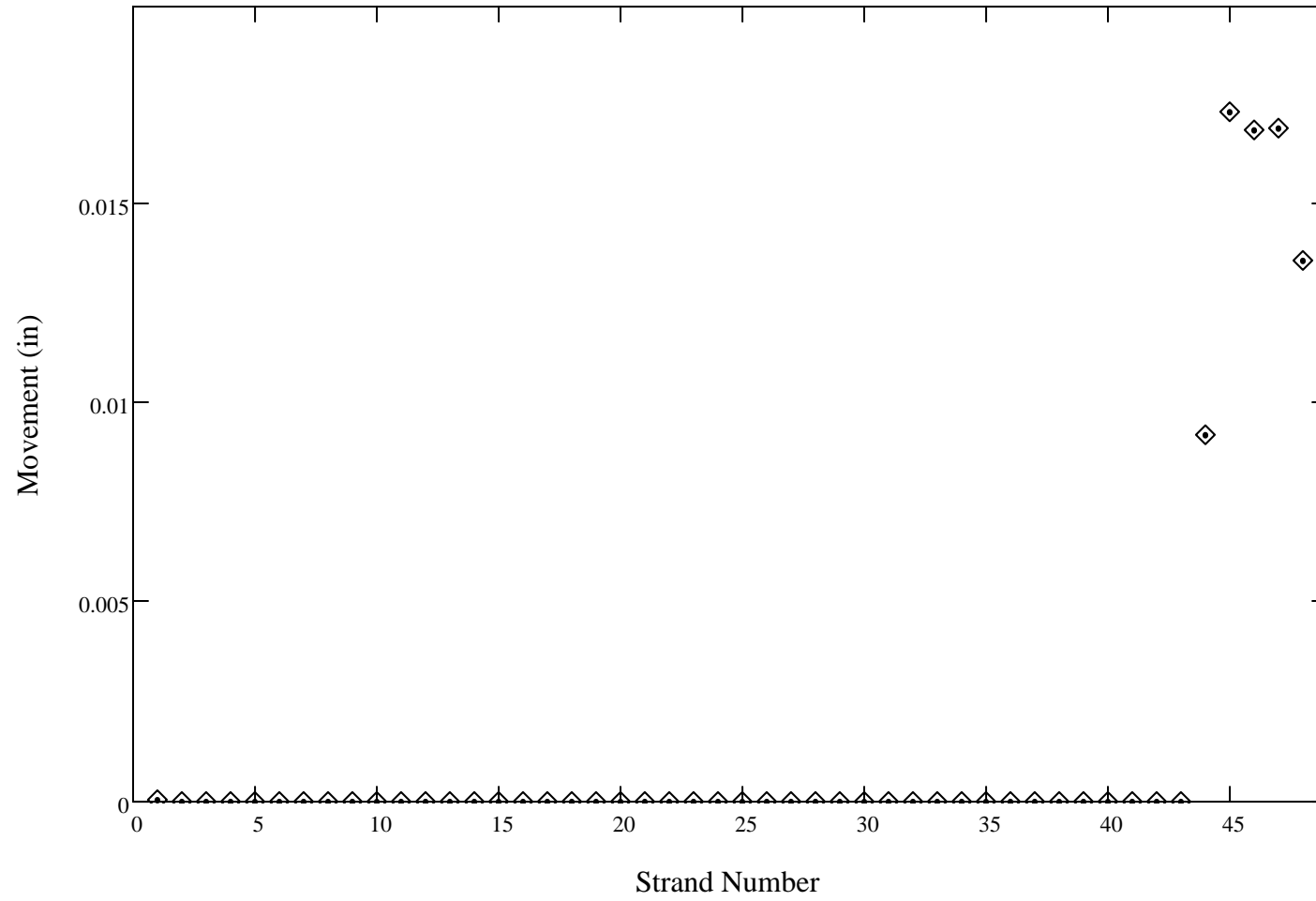
End Movement Beam 3 End 1



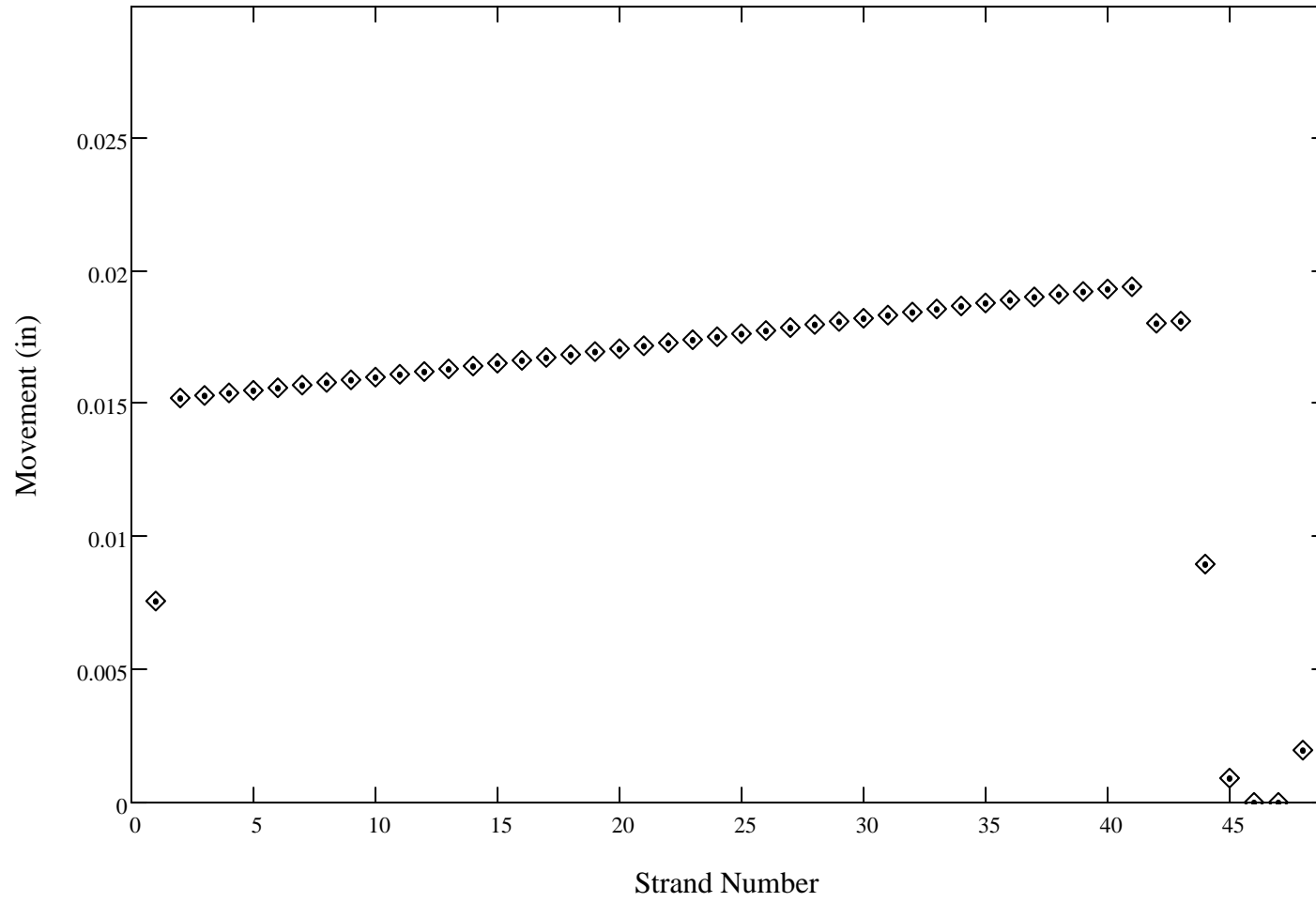
End Movement Beam 3 End 2



End Movement Beam 4 End 1



End Movement Beam 4 End 2





Global motion of beams- reference frame = initial beam positions and location of bulkheads  
 Positive number indicates beam slides right, negative number indicates beam slides left

	1	2	3	4
1	0	0	0	0
2	0	0	0	0
3	0.003113	0	0	0
4	0.009488	0	0	-0.0059
5	0.017108	0	0	-0.011946
6	0.024921	0	0	-0.018146
7	0.031791	0	0	-0.024507
8	0.040019	0	0	-0.027773
9	0.047263	0	0	-0.034484
10	0.058434	0	0	-0.041384
11	0.066096	0	0	-0.048485
12	0.073985	0	0	-0.052141
13	0.082115	0	0	-0.05968
14	0.090501	0	0	-0.067457
15	0.099159	0	0	-0.071474
16	0.108108	0	0	-0.079778
17	0.117368	0	0	-0.084076
18	0.12696	0	0	-0.088529
19	0.13691	0	0	-0.097771
20	0.142077	0	0	-0.102573
21	0.152826	0	0	-0.107569
22	0.164024	0	0	-0.112776
23	0.169867	0	0	-0.118212
24	0.175976	0	0	-0.123898
25	0.182376	0	0	-0.129858
26	0.189095	0	0	-0.136119

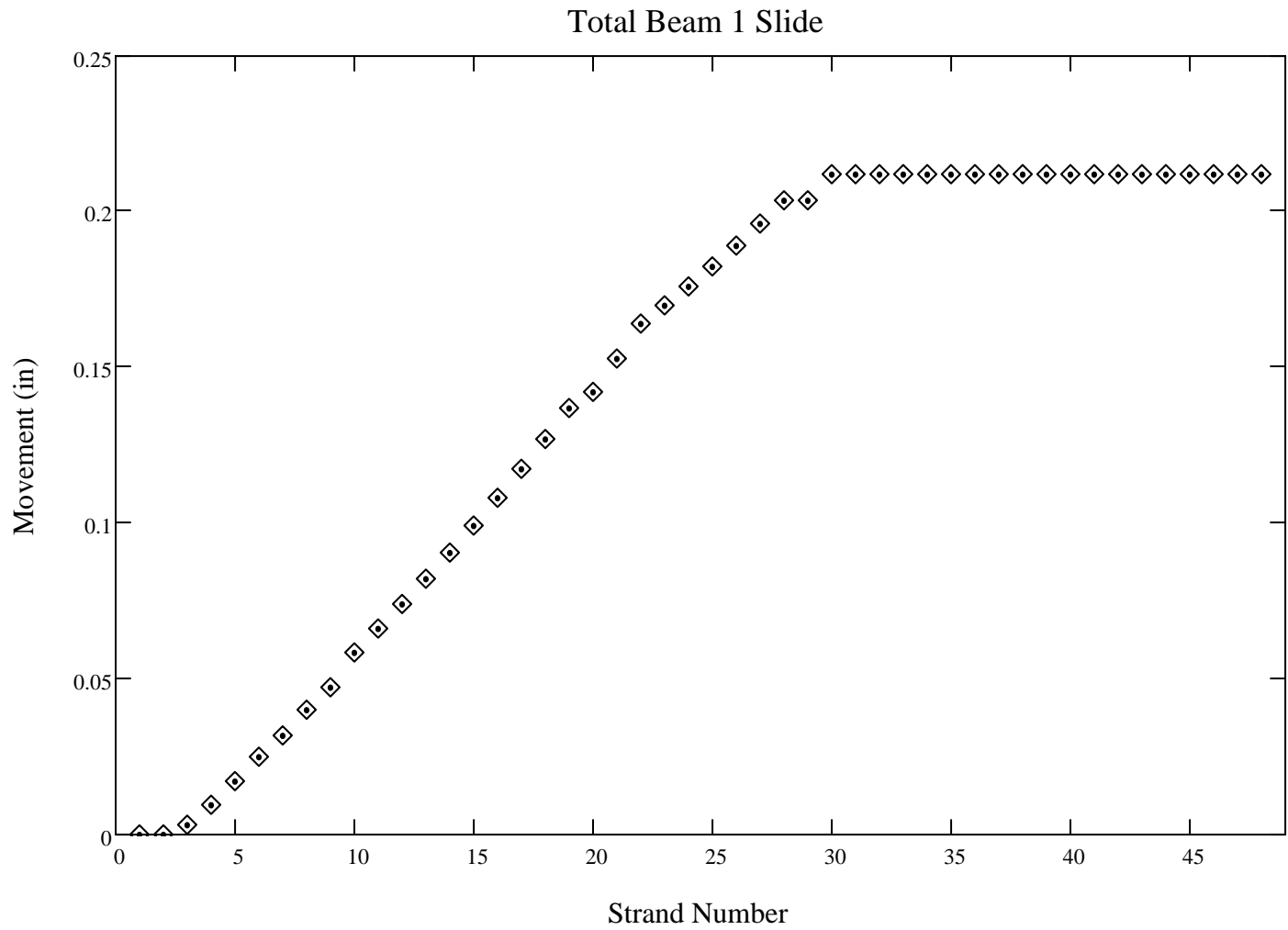
BeamSlidePerStrand =

in

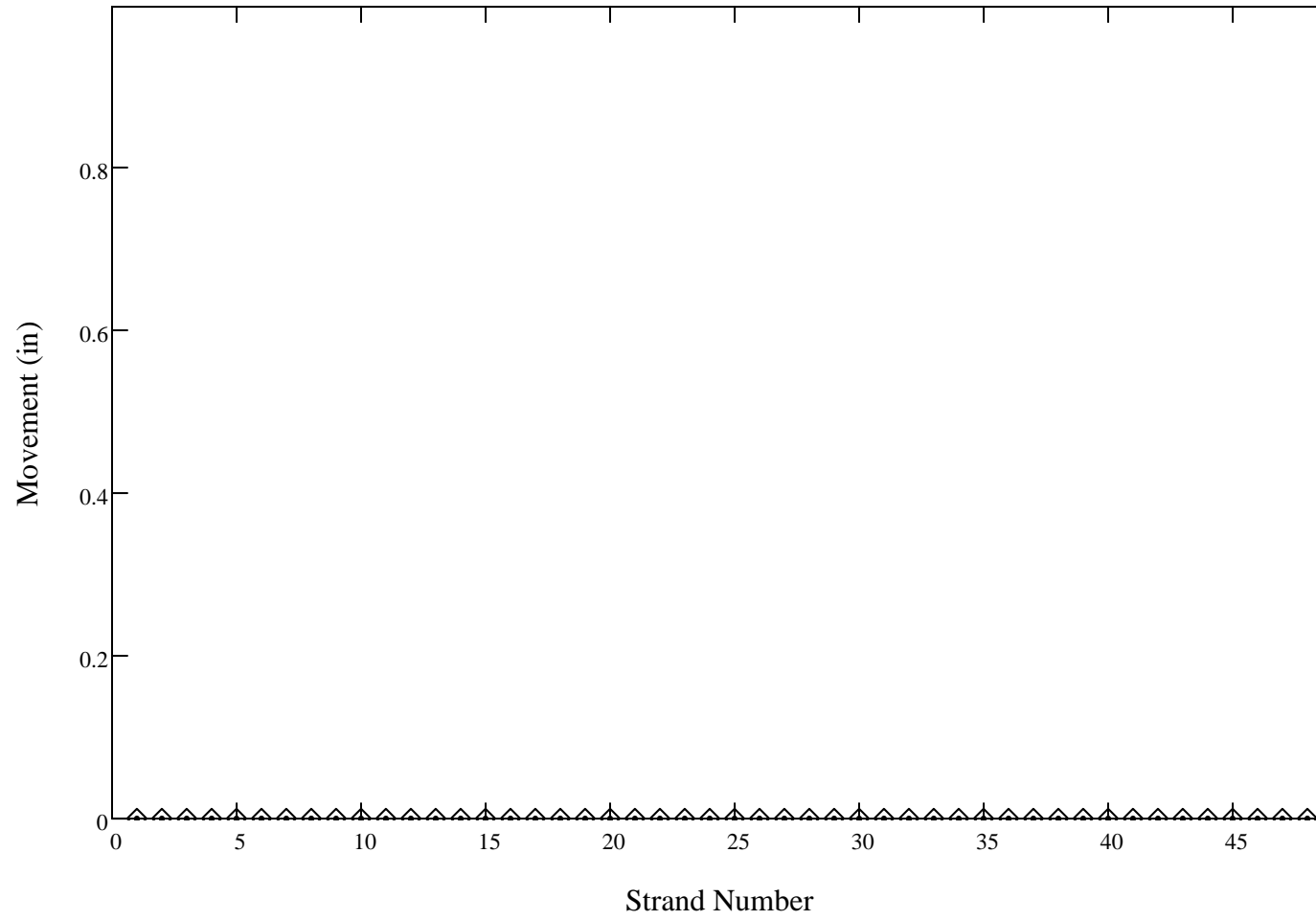
	1	2	3	4
1	0	0	0	0
2	0	0	0	0
3	0.003113	0	0	0
4	0.009488	0	0	-0.0059
5	0.017108	0	0	-0.011946
6	0.024921	0	0	-0.018146
7	0.031791	0	0	-0.024507
8	0.040019	0	0	-0.027773
9	0.047263	0	0	-0.034484
10	0.058434	0	0	-0.041384
11	0.066096	0	0	-0.048485
12	0.073985	0	0	-0.052141
13	0.082115	0	0	-0.05968
14	0.090501	0	0	-0.067457
15	0.099159	0	0	-0.071474
16	0.108108	0	0	-0.079778
17	0.117368	0	0	-0.084076
18	0.12696	0	0	-0.088529
19	0.13691	0	0	-0.097771
20	0.142077	0	0	-0.102573
21	0.152826	0	0	-0.107569
22	0.164024	0	0	-0.112776
23	0.169867	0	0	-0.118212
24	0.175976	0	0	-0.123898
25	0.182376	0	0	-0.129858
26	0.189095	0	0	-0.136119
27	0.196167	0	0	-0.142712
28	0.203629	0	0	-0.142712

BeamSlidePerStrand =

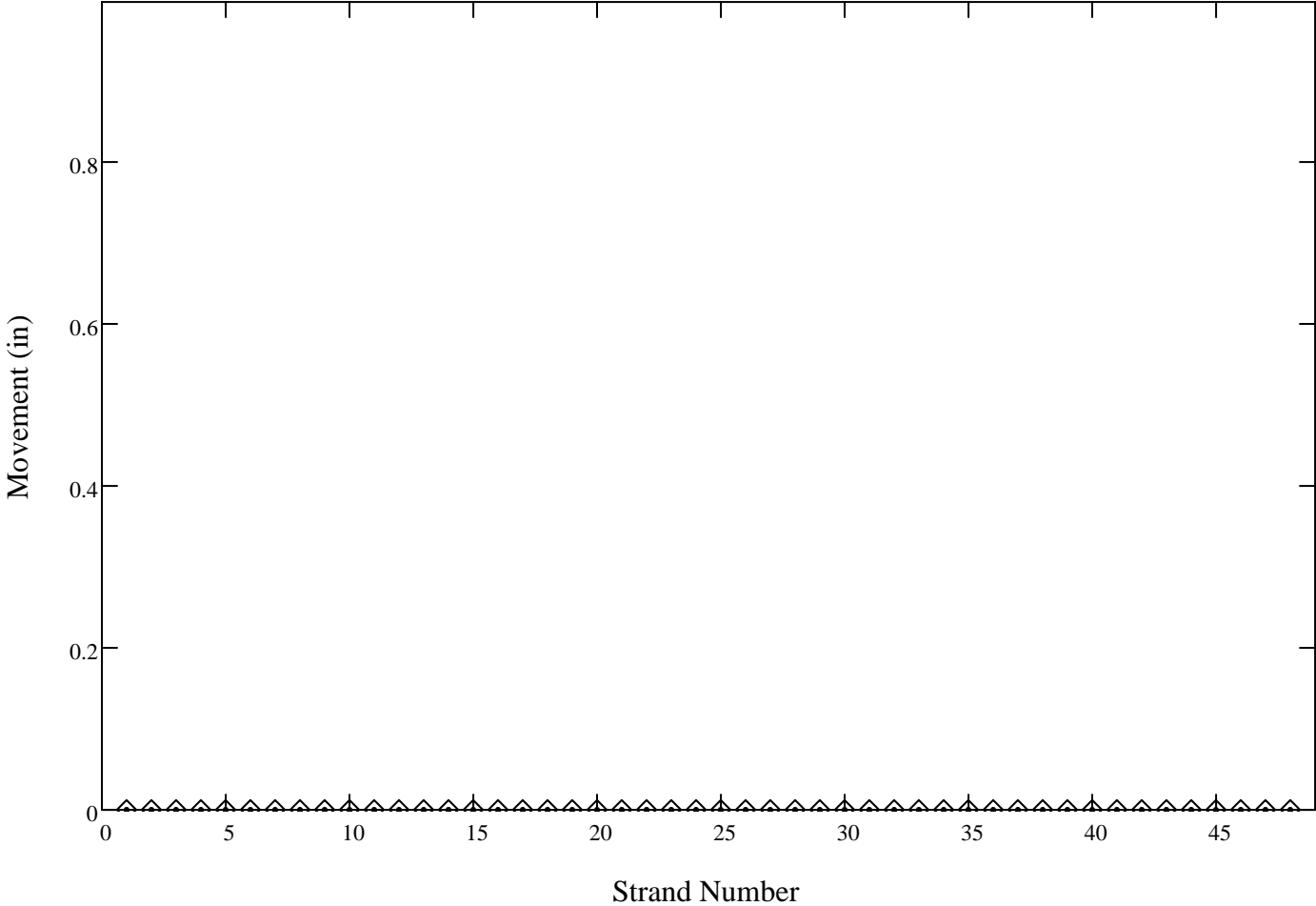
in

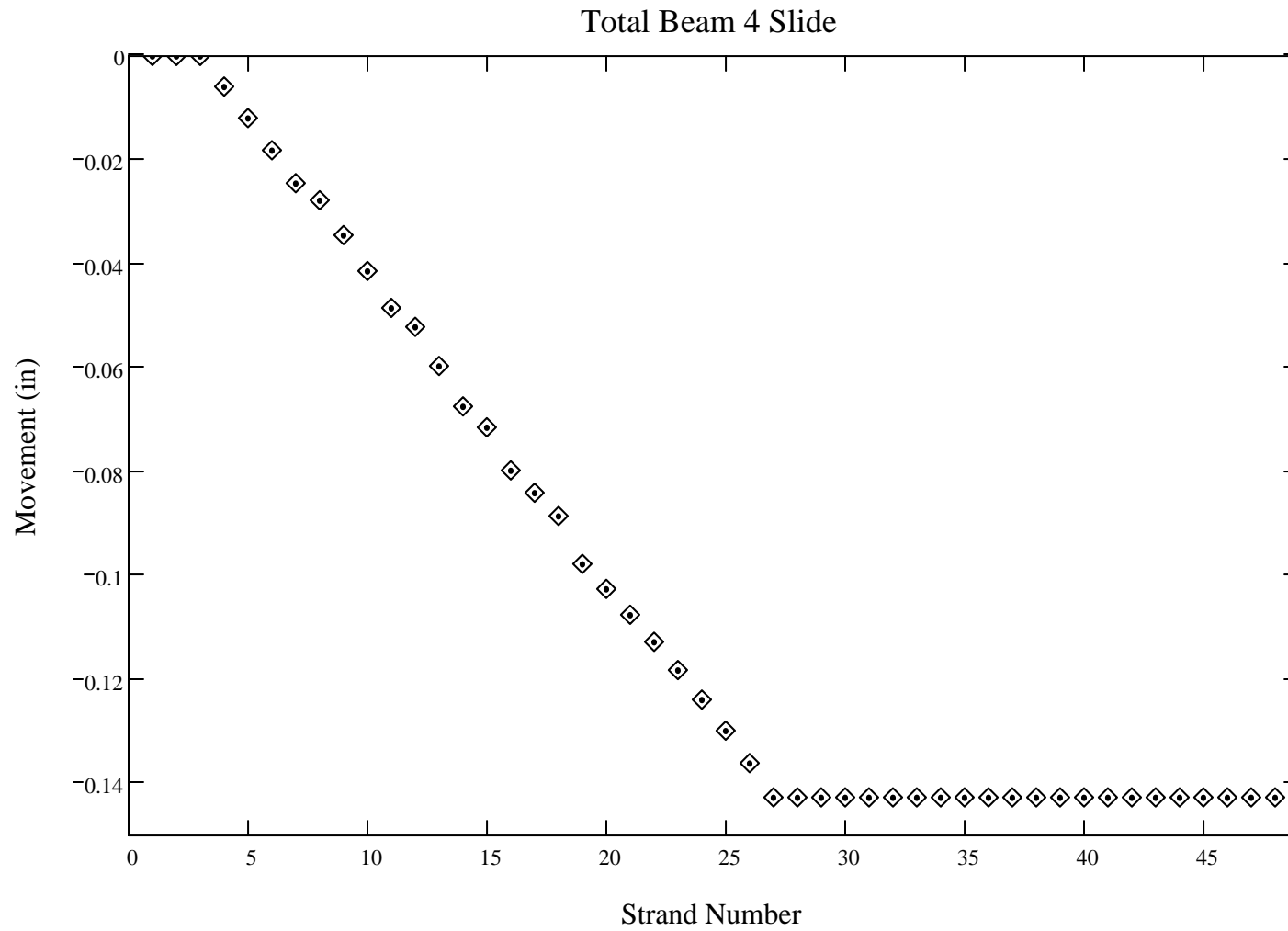


### Total Beam 2 Slide



Total Beam 3 Slide





## BEAM RESULTS:

Total Axial Shortening of Beams after each strand cut

```

BeamShorten := | for q ∈ 1 .. rows(EndMovxPerStrand)
                 for c ∈ 1 ..  $\frac{\text{cols(EndMovxPerStrand)}}{2}$ 
                 outq,c ← EndMovxPerStrandq,2·c-1 + EndMovxPerStrandq,2·c
                 | out
    
```

	1	2	3	4
1	0.008	0.005	0.005	0.008
2	0.023	0.016	0.016	0.023
3	0.038	0.027	0.027	0.038
4	0.054	0.038	0.038	0.054
5	0.07	0.049	0.05	0.069
6	0.085	0.061	0.062	0.085
7	0.101	0.073	0.074	0.101
8	0.117	0.086	0.086	0.116
BeamShorten = 9	0.133	0.098	0.099	0.132
10	0.149	0.111	0.112	0.148
11	0.165	0.125	0.126	0.164
12	0.181	0.139	0.14	0.181
13	0.197	0.153	0.154	0.197
14	0.214	0.167	0.169	0.213
15	0.23	0.182	0.184	0.23
16	0.247	0.197	0.199	0.247
17	0.263	0.213	0.215	0.263
18	0.28	0.229	0.232	0.28

in

	1	2	3	4
1	0.008	0.005	0.005	0.008
2	0.023	0.016	0.016	0.023
3	0.038	0.027	0.027	0.038
4	0.054	0.038	0.038	0.054
5	0.07	0.049	0.05	0.069
6	0.085	0.061	0.062	0.085
7	0.101	0.073	0.074	0.101
8	0.117	0.086	0.086	0.116
9	0.133	0.098	0.099	0.132
10	0.149	0.111	0.112	0.148
11	0.165	0.125	0.126	0.164
12	0.181	0.139	0.14	0.181
13	0.197	0.153	0.154	0.197
14	0.214	0.167	0.169	0.213
15	0.23	0.182	0.184	0.23
16	0.247	0.197	0.199	0.247
17	0.263	0.213	0.215	0.263
18	0.28	0.229	0.232	0.28
19	0.297	0.246	0.248	0.297
20	0.314	0.263	0.265	0.314
21	0.331	0.28	0.283	0.332
22	0.348	0.298	0.301	0.349
23	0.366	0.316	0.319	0.366
24	0.383	0.335	0.338	0.384
25	0.401	0.354	0.357	0.402
26	0.418	0.373	0.377	0.419
27	0.436	0.393	0.397	0.437
28	0.454	0.413	0.418	0.455
29	0.472	0.434	0.439	0.473
30	0.49	0.455	0.46	0.492

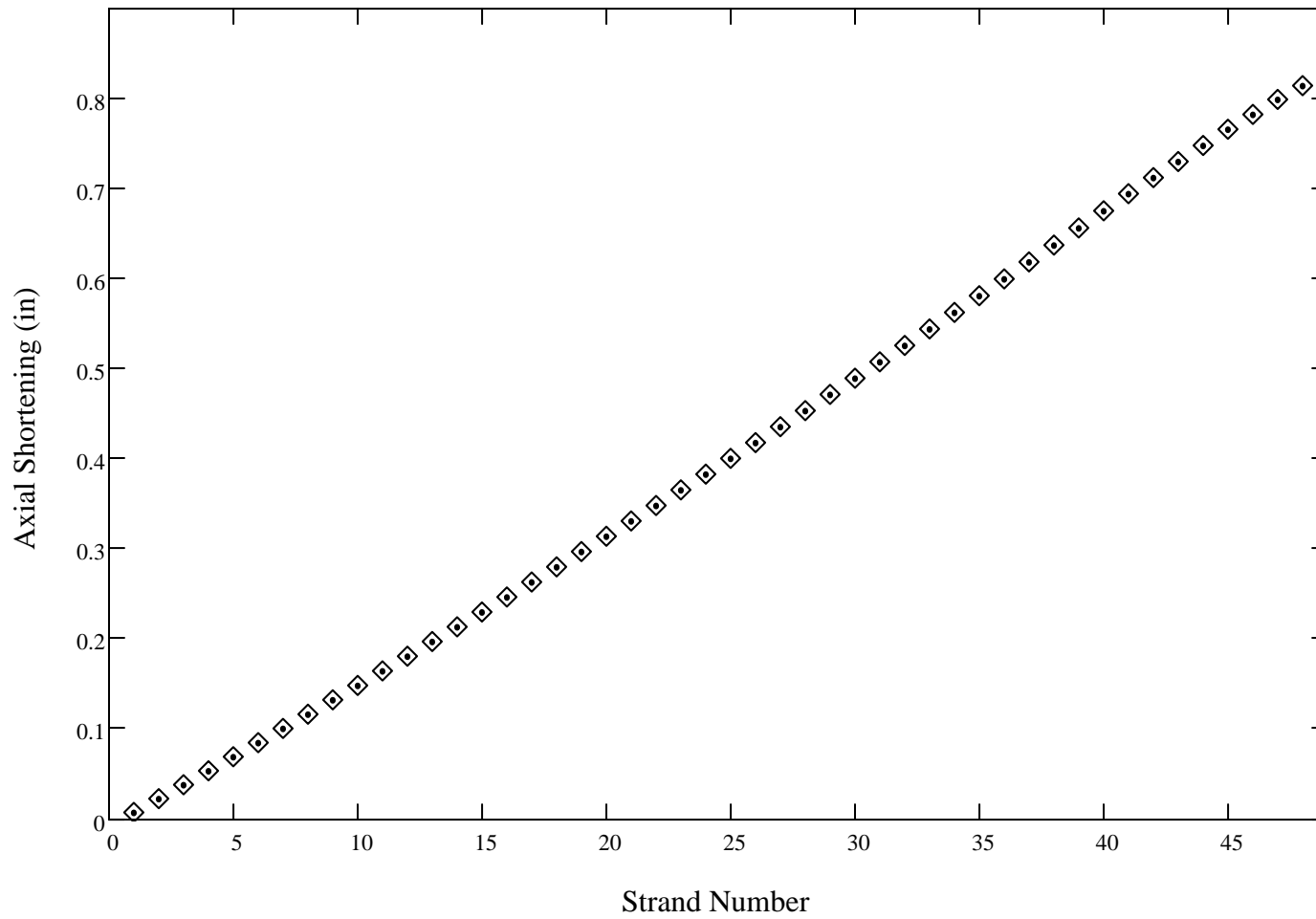
BeamShorten =

in

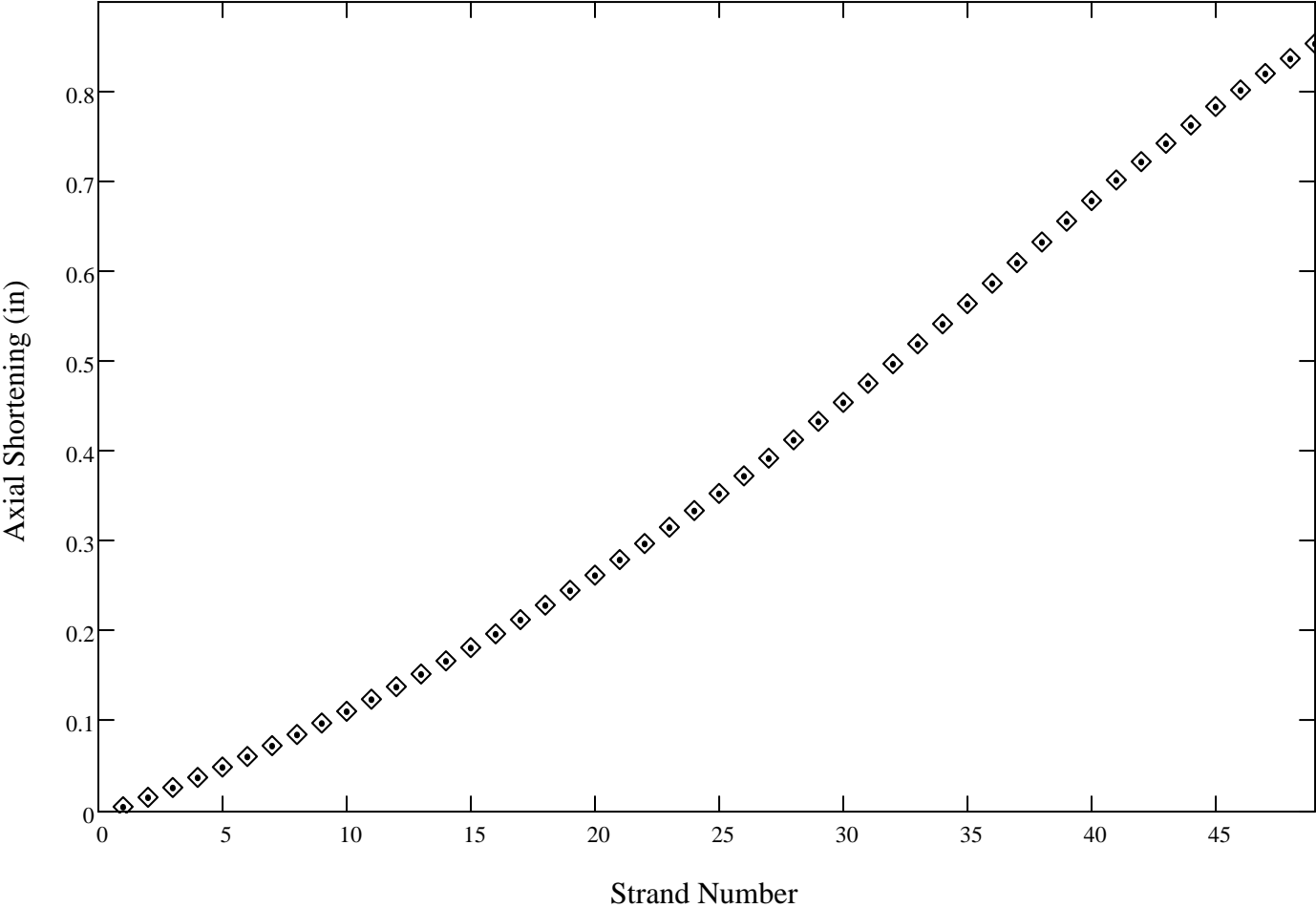
Axialref = (0.863799 0.888104 0.900399 0.871749) in



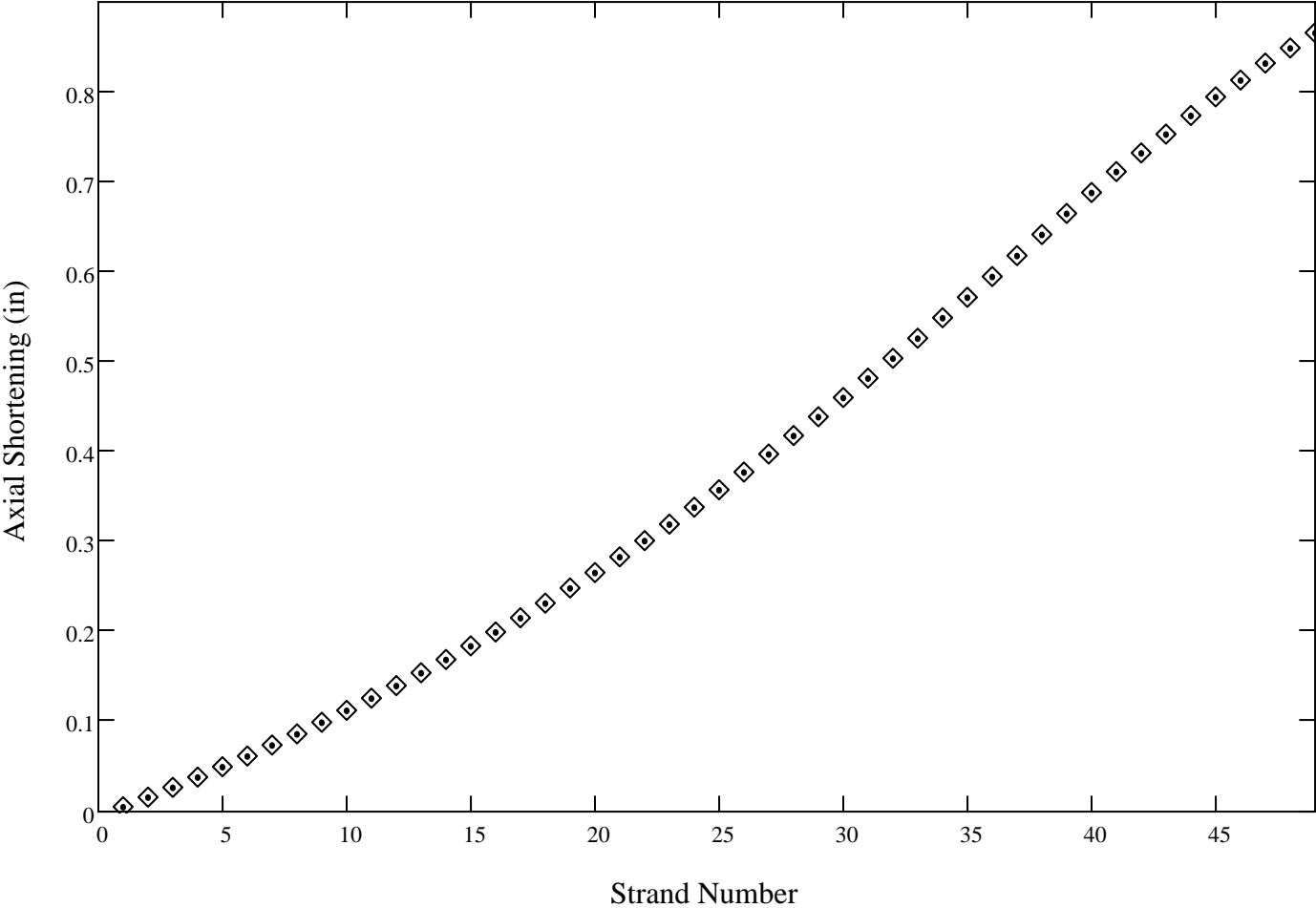
Axial Shortening of Beam 1



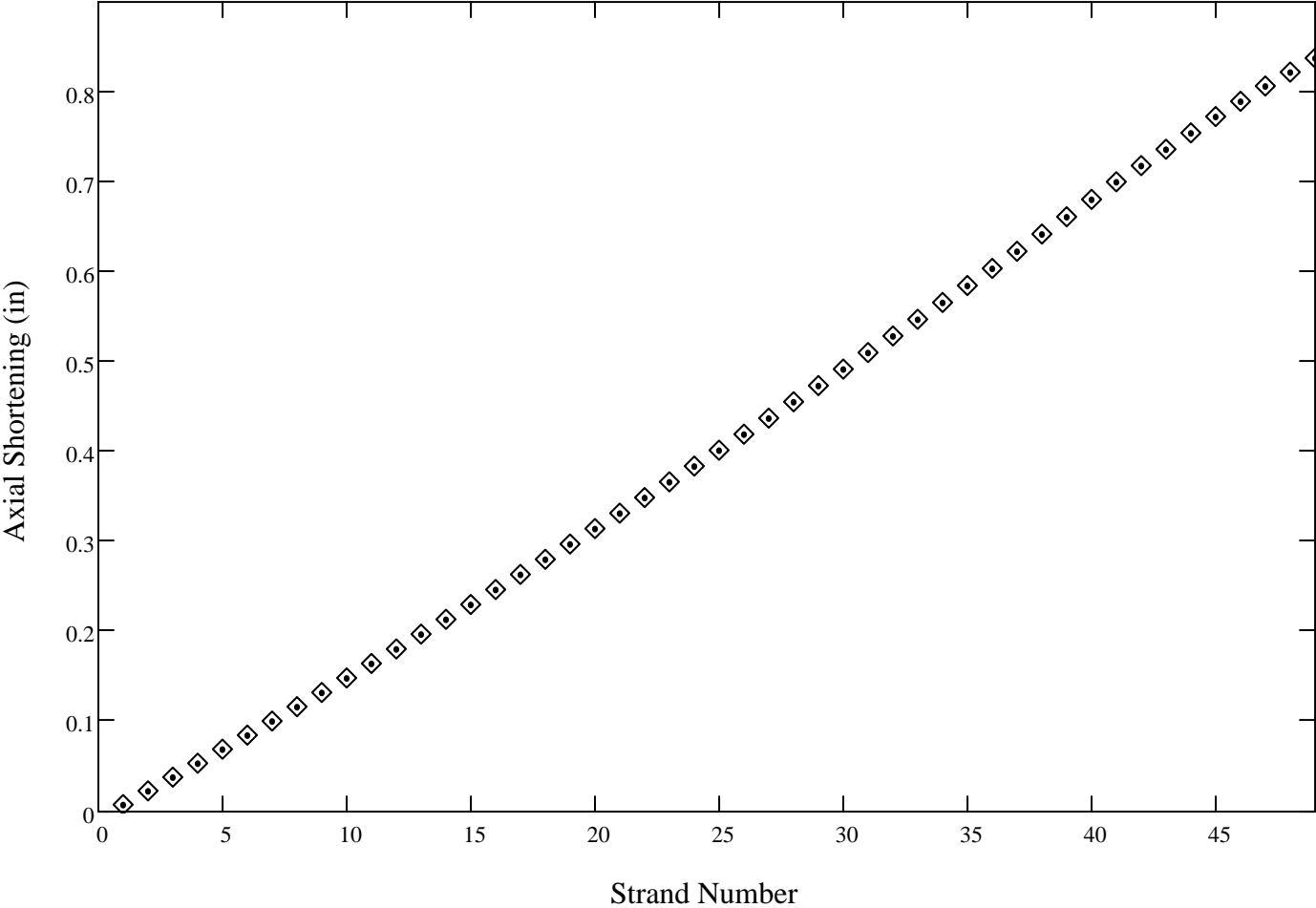
Axial Shortening of Beam 2



Axial Shortening of Beam 3



Axial Shortening of Beam 4



Beam Length - accounts for beam axial shortening

```

BeamLength :=
  for q ∈ 1 .. rows(BeamShorten)
  |
  |   outq ← Lbeams - BeamShortenq if cols(FreeStrand) = 2
  |   for c ∈ 1 .. cols(BeamShorten)           if cols(FreeStrand) ≠ 2
  |   |   outq,c ← Lbeams1,c - BeamShortenq,c
  |   |
  |   out

```

	1	2	3	4
1	149.999	155	157	151.999
2	149.998	154.999	156.999	151.998
3	149.997	154.998	156.998	151.997
4	149.996	154.997	156.997	151.996
5	149.994	154.996	156.996	151.994
6	149.993	154.995	156.995	151.993
7	149.992	154.994	156.994	151.992
8	149.99	154.993	156.993	151.99
9	149.989	154.992	156.992	151.989
10	149.988	154.991	156.991	151.988
11	149.986	154.99	156.99	151.986
12	149.985	154.988	156.988	151.985
13	149.984	154.987	156.987	151.984
14	149.982	154.986	156.986	151.982
15	149.981	154.985	156.985	151.981
16	149.979	154.984	156.983	151.979
17	149.978	154.982	156.982	151.978
18	149.977	154.981	156.981	151.977
19	149.975	154.98	156.979	151.975
20	149.974	154.978	156.978	151.974

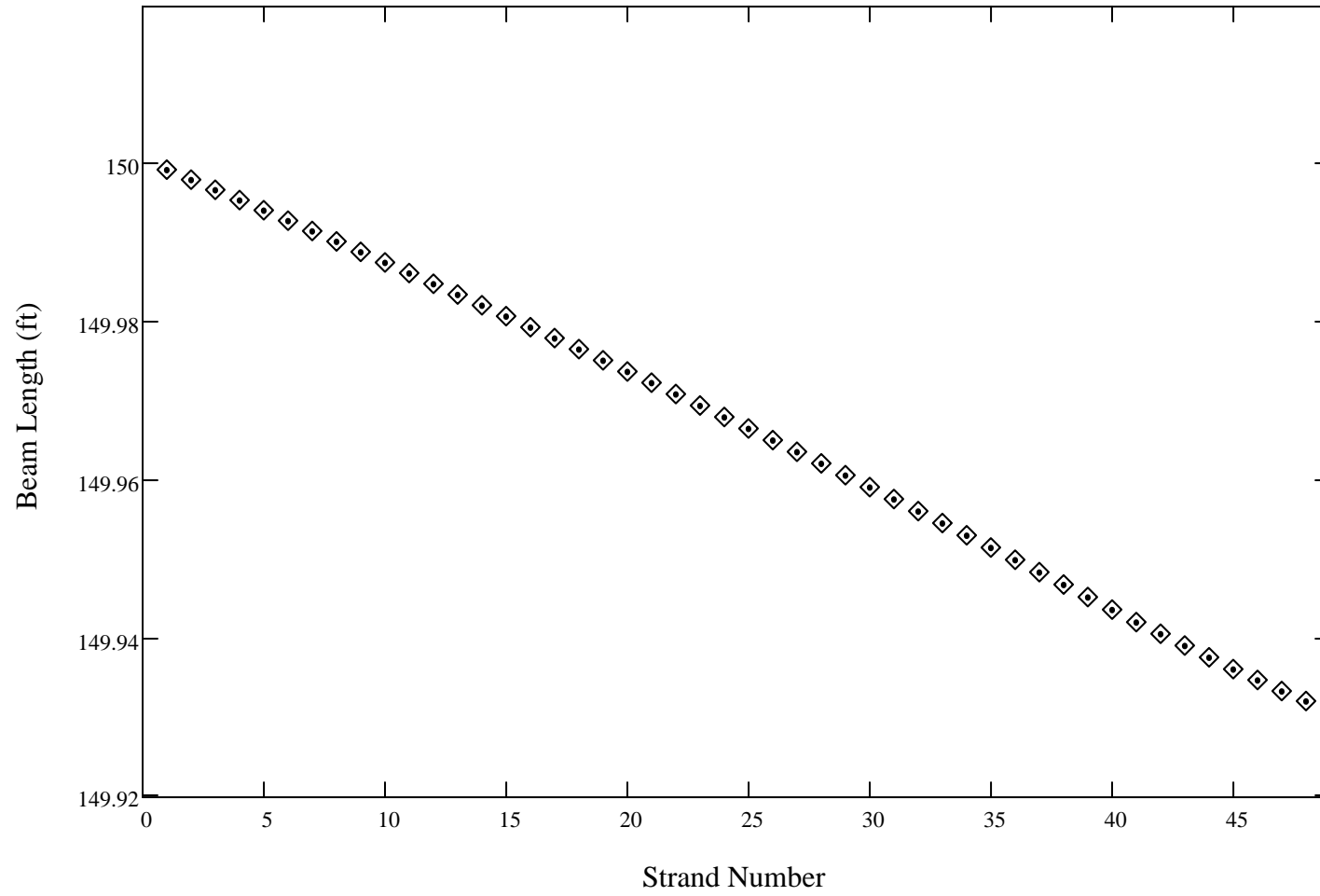
BeamLength = ft

	1	2	3	4
1	149.999	155	157	151.999
2	149.998	154.999	156.999	151.998
3	149.997	154.998	156.998	151.997
4	149.996	154.997	156.997	151.996
5	149.994	154.996	156.996	151.994
6	149.993	154.995	156.995	151.993
7	149.992	154.994	156.994	151.992
8	149.99	154.993	156.993	151.99
9	149.989	154.992	156.992	151.989
10	149.988	154.991	156.991	151.988
11	149.986	154.99	156.99	151.986
12	149.985	154.988	156.988	151.985
13	149.984	154.987	156.987	151.984
14	149.982	154.986	156.986	151.982
15	149.981	154.985	156.985	151.981
16	149.979	154.984	156.983	151.979
17	149.978	154.982	156.982	151.978
18	149.977	154.981	156.981	151.977
19	149.975	154.98	156.979	151.975
20	149.974	154.978	156.978	151.974
21	149.972	154.977	156.976	151.972
22	149.971	154.975	156.975	151.971
23	149.97	154.974	156.973	151.969
24	149.968	154.972	156.972	151.968
25	149.967	154.971	156.97	151.967
26	149.965	154.969	156.969	151.965
27	149.964	154.967	156.967	151.964
28	149.962	154.966	156.965	151.962
29	149.961	154.964	156.963	151.961
30	149.959	154.962	156.962	151.959

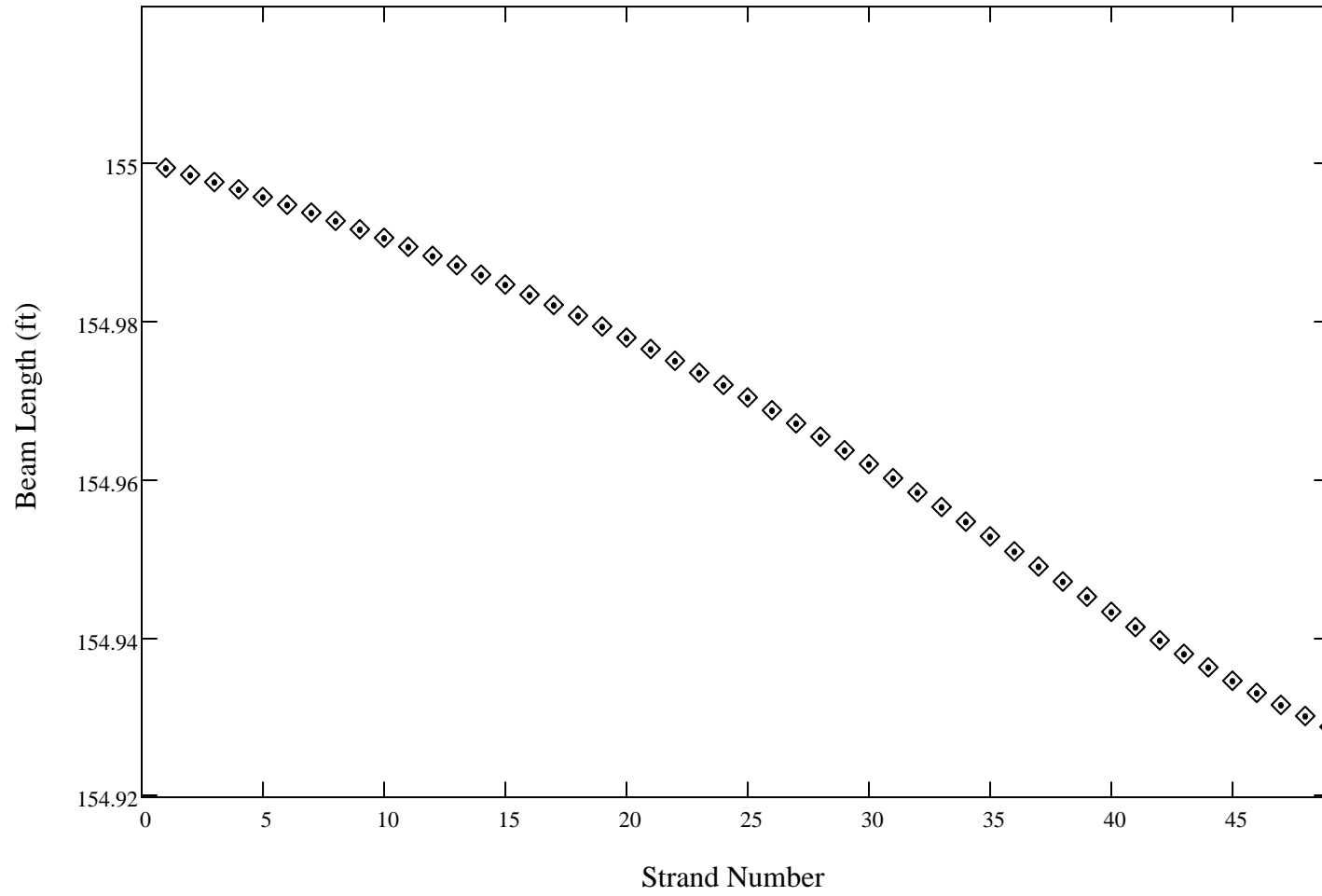
BeamLength =

ft

Beam 1 Length

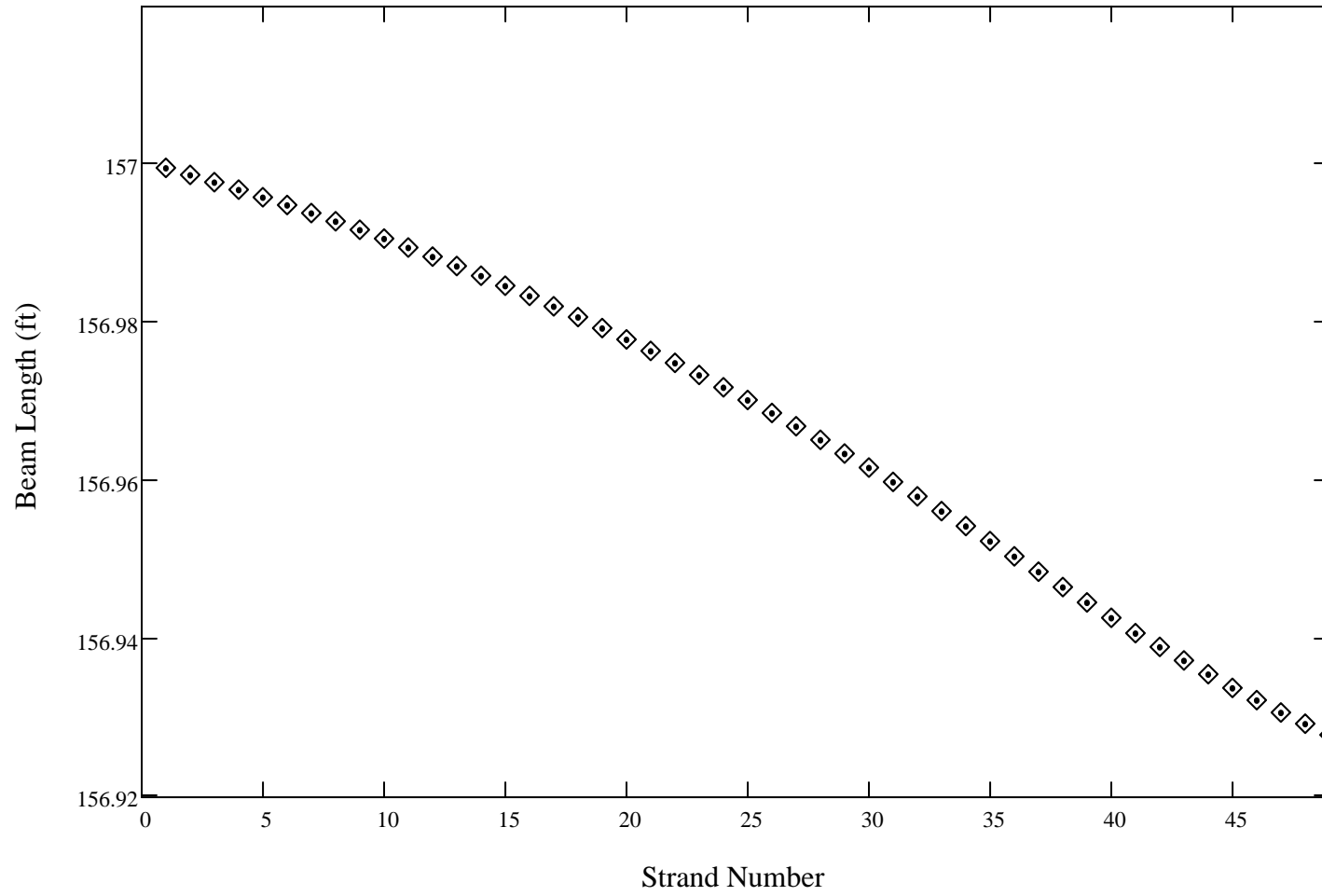


Beam 2 Length

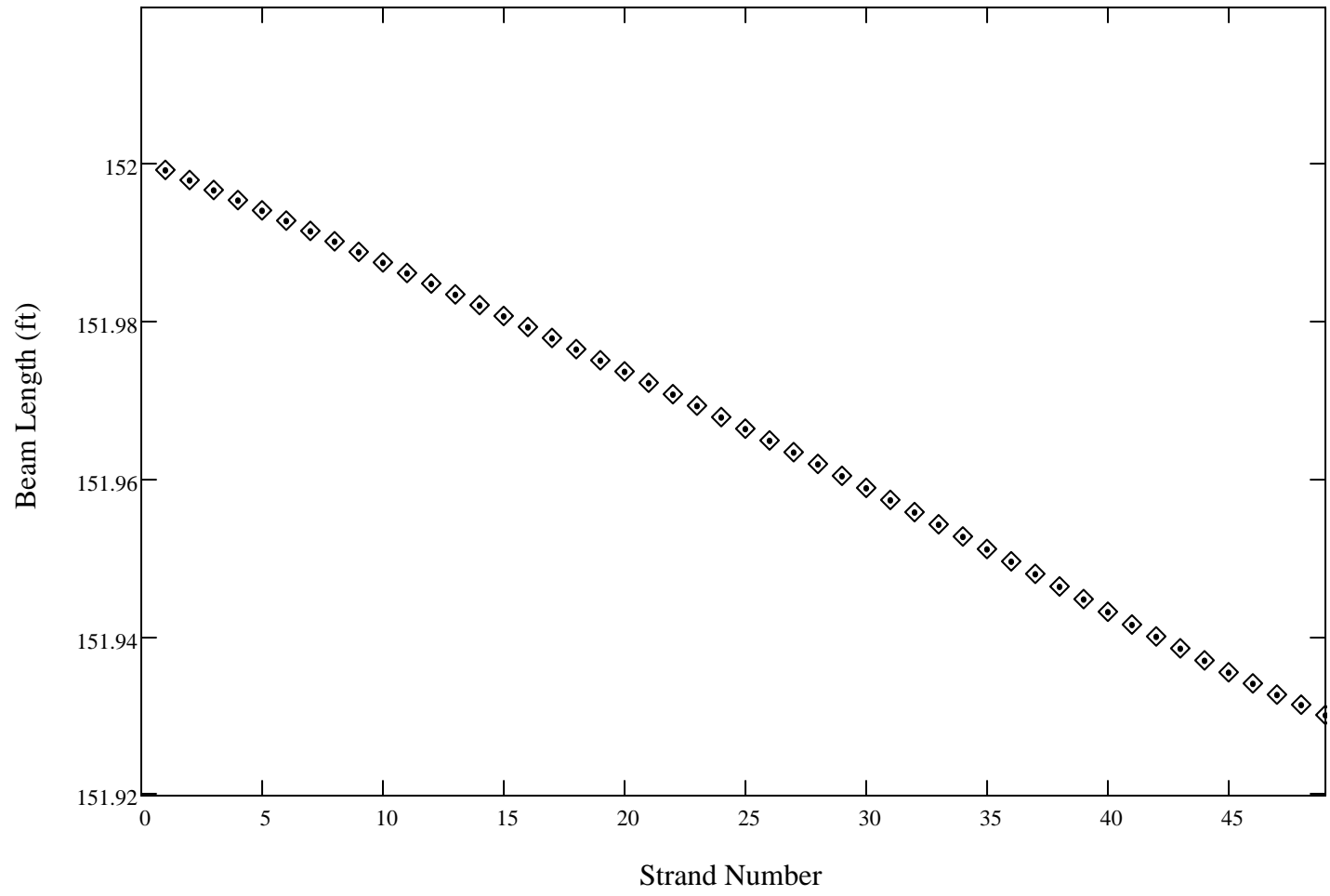




Beam 3 Length



Beam 4 Length



## STRAND RESULTS:

The amount each free strand set stretches after each strand cut

	1	2	3	4	5
1	0.007724	0.00341	0.003495	0.003409	0.007586
2	0.023028	0.010475	0.010655	0.01048	0.022825
3	0.041532	0.014649	0.017946	0.017767	0.03816
4	0.063387	0.019221	0.020733	0.020581	0.059492
5	0.086575	0.022463	0.024369	0.023083	0.081067
6	0.110045	0.025784	0.027847	0.025936	0.102894
7	0.132662	0.030085	0.031865	0.028685	0.124982
8	0.156728	0.033414	0.035339	0.035339	0.144075
9	0.179902	0.038633	0.038633	0.03849	0.166713
10	0.207095	0.039343	0.043066	0.041582	0.189644
11	0.230872	0.044429	0.047679	0.044129	0.212877
12	0.254969	0.04916	0.05163	0.05163	0.23277
13	0.279403	0.054665	0.056088	0.054465	0.256649
14	0.304187	0.05926	0.061451	0.057563	0.280873
15	0.32934	0.063686	0.065951	0.065951	0.301442
16	0.35488	0.069312	0.071587	0.068208	0.326407
17	0.380828	0.073727	0.076697	0.076697	0.347473
18	0.407207	0.079377	0.08304	0.08304	0.368804
19	0.434042	0.085126	0.089064	0.085279	0.395033
20	0.456194	0.094517	0.094771	0.094251	0.416933
21	0.484029	0.098451	0.10211	0.10211	0.439139
22	0.512413	0.1035	0.109356	0.109137	0.461668
23	0.535544	0.115318	0.116059	0.115911	0.48454
24	0.559044	0.124157	0.124398	0.124363	0.507776
25	0.582936	0.133049	0.133462	0.132319	0.5314
26	0.607252	0.141379	0.143445	0.140138	0.55544
27	0.632024	0.149081	0.154375	0.147782	0.579928

StrandMovXXPerStrand =

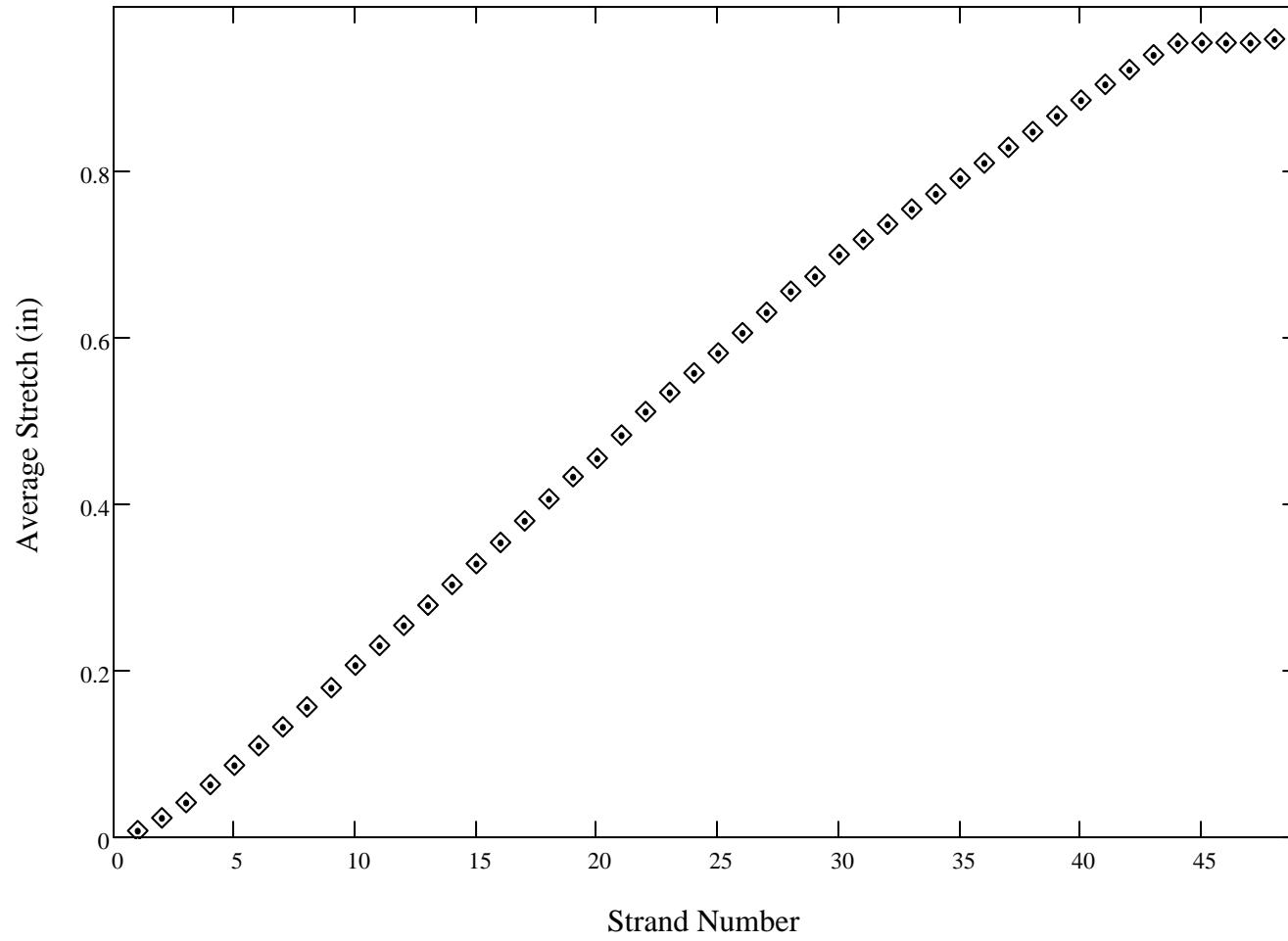
in

	1	2	3	4	5
1	0.007724	0.00341	0.003495	0.003409	0.007586
2	0.023028	0.010475	0.010655	0.01048	0.022825
3	0.041532	0.014649	0.017946	0.017767	0.03816
4	0.063387	0.019221	0.020733	0.020581	0.059492
5	0.086575	0.022463	0.024369	0.023083	0.081067
6	0.110045	0.025784	0.027847	0.025936	0.102894
7	0.132662	0.030085	0.031865	0.028685	0.124982
8	0.156728	0.033414	0.035339	0.035339	0.144075
9	0.179902	0.038633	0.038633	0.03849	0.166713
10	0.207095	0.039343	0.043066	0.041582	0.189644
11	0.230872	0.044429	0.047679	0.044129	0.212877
12	0.254969	0.04916	0.05163	0.05163	0.23277
13	0.279403	0.054665	0.056088	0.054465	0.256649
14	0.304187	0.05926	0.061451	0.057563	0.280873
15	0.32934	0.063686	0.065951	0.065951	0.301442
16	0.35488	0.069312	0.071587	0.068208	0.326407
17	0.380828	0.073727	0.076697	0.076697	0.347473
18	0.407207	0.079377	0.08304	0.08304	0.368804
19	0.434042	0.085126	0.089064	0.085279	0.395033
20	0.456194	0.094517	0.094771	0.094251	0.416933
21	0.484029	0.098451	0.10211	0.10211	0.439139
22	0.512413	0.1035	0.109356	0.109137	0.461668
23	0.535544	0.115318	0.116059	0.115911	0.48454
24	0.559044	0.124157	0.124398	0.124363	0.507776
25	0.582936	0.133049	0.133462	0.132319	0.5314
26	0.607252	0.141379	0.143445	0.140138	0.55544

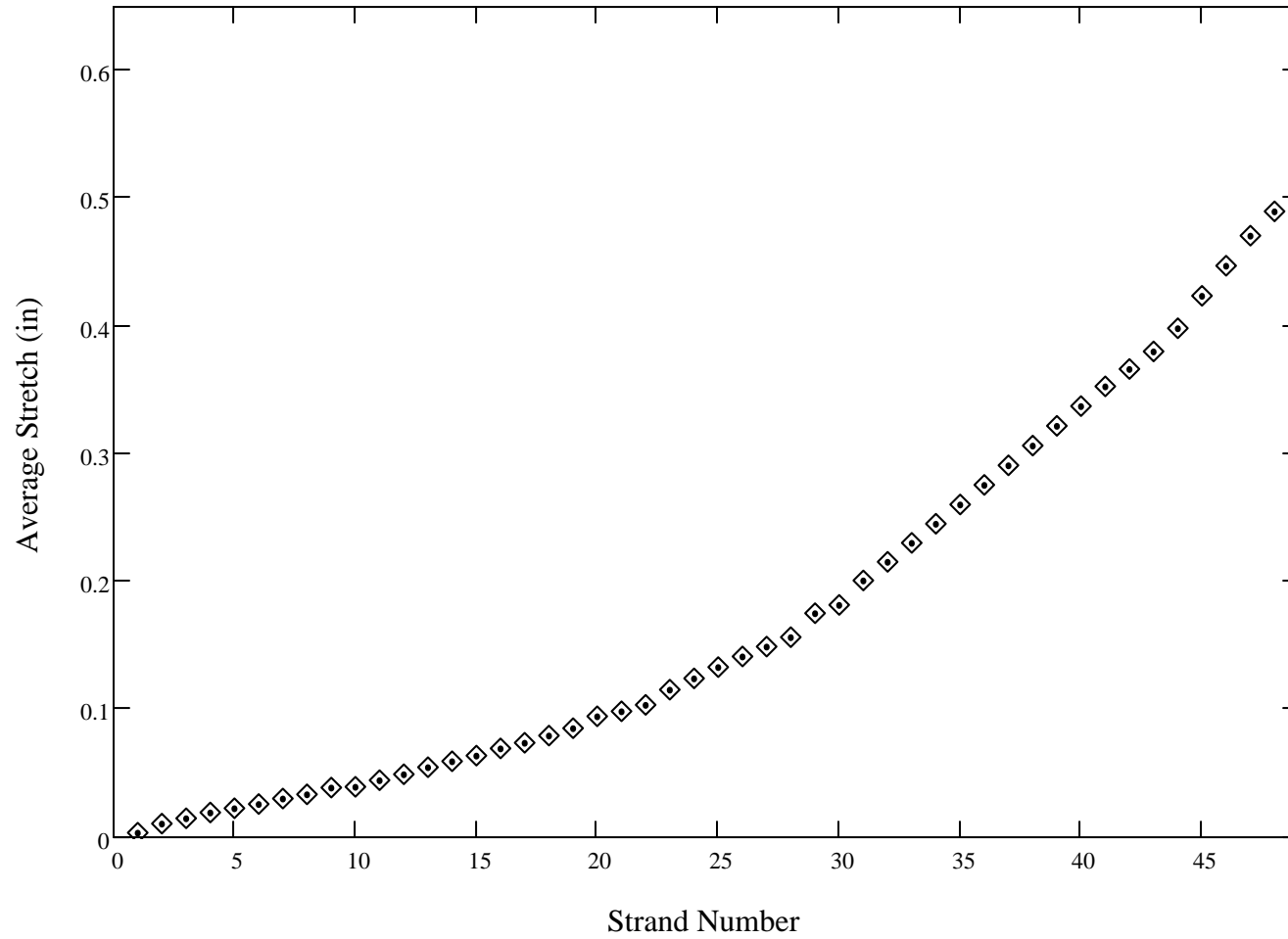
StrandMovXXPerStrand =

in

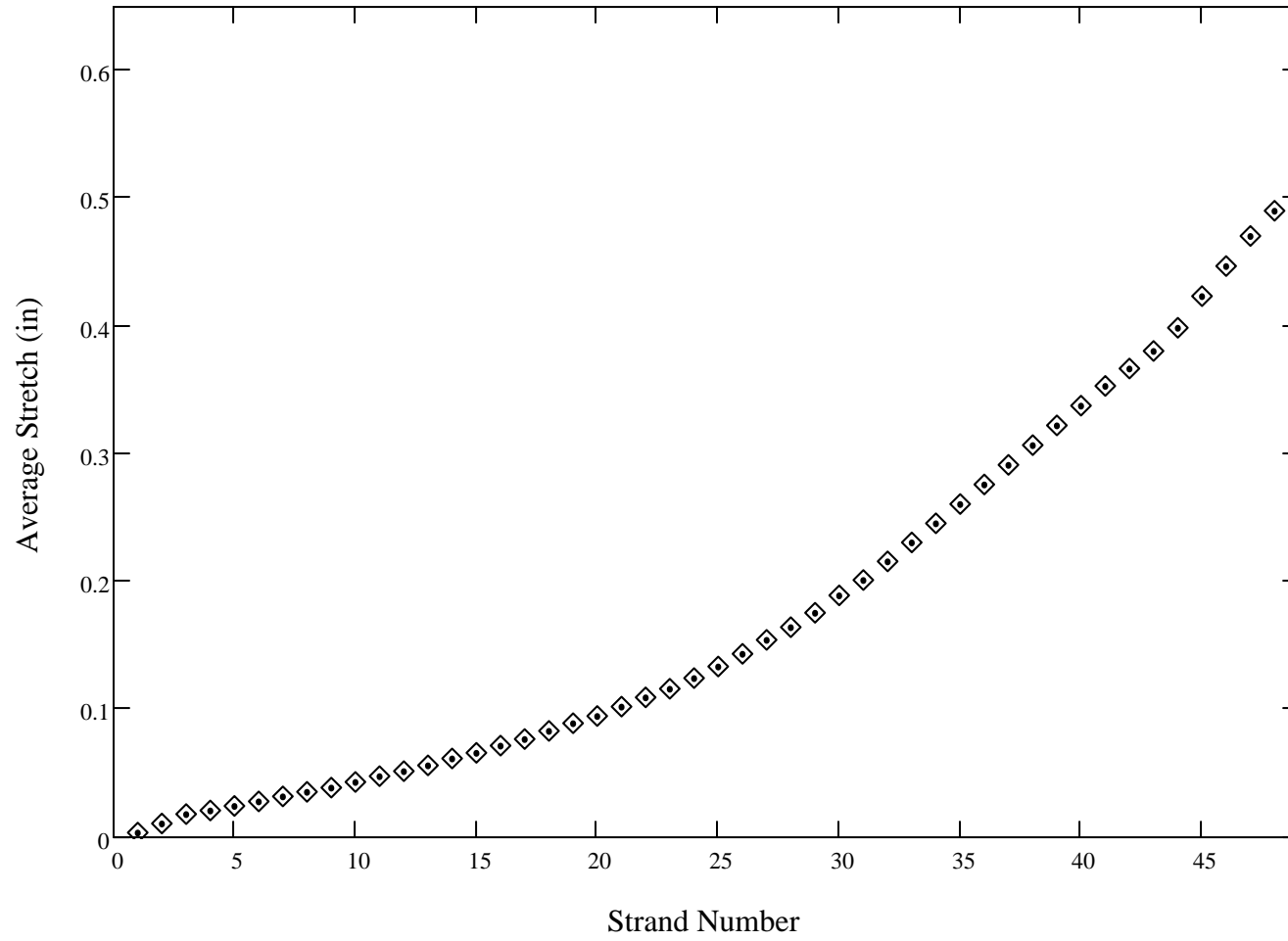
Average Free Strand Set 1 Stretch



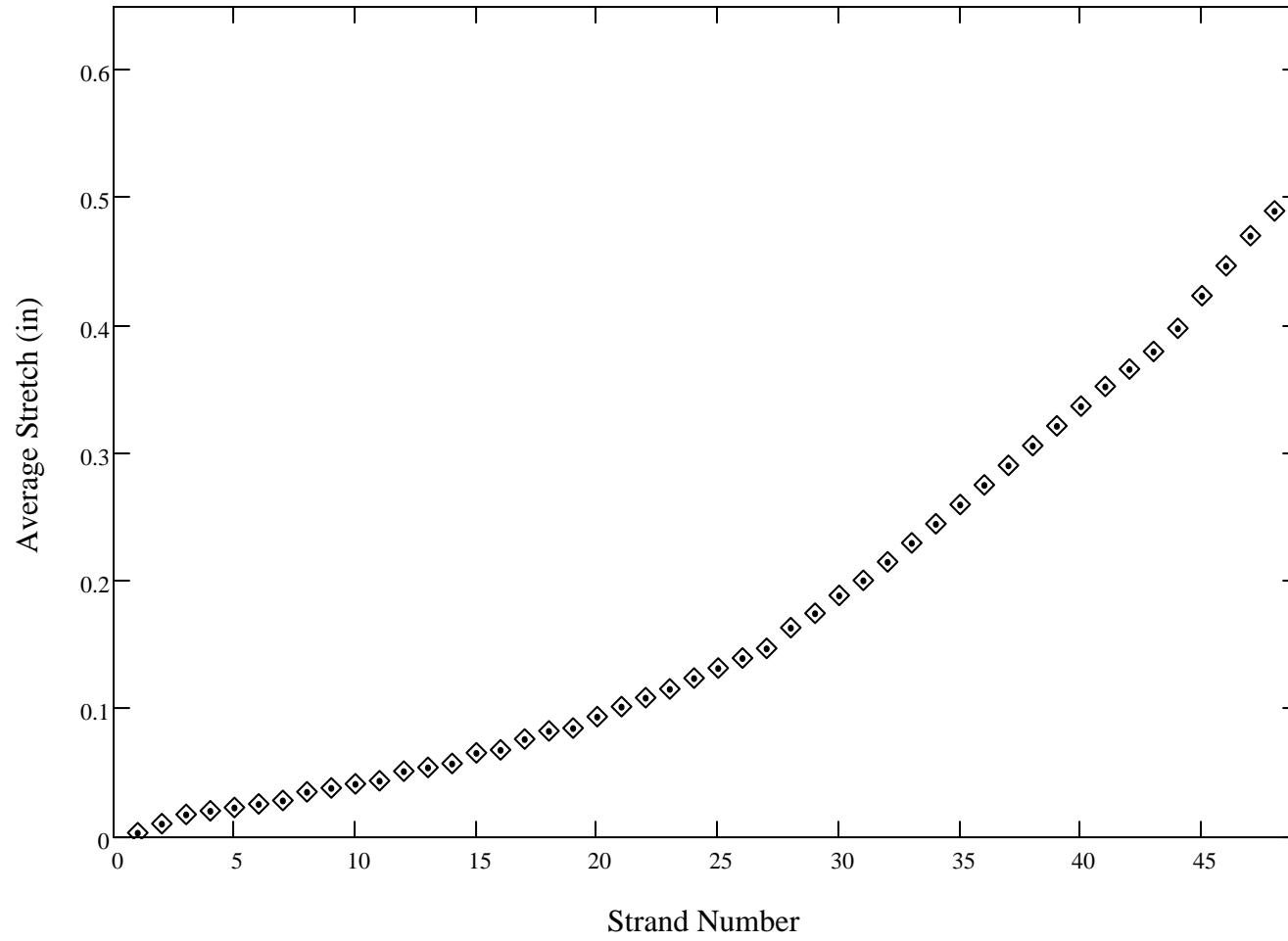
Average Free Strand Set 2 Stretch



Average Free Strand Set 3 Stretch

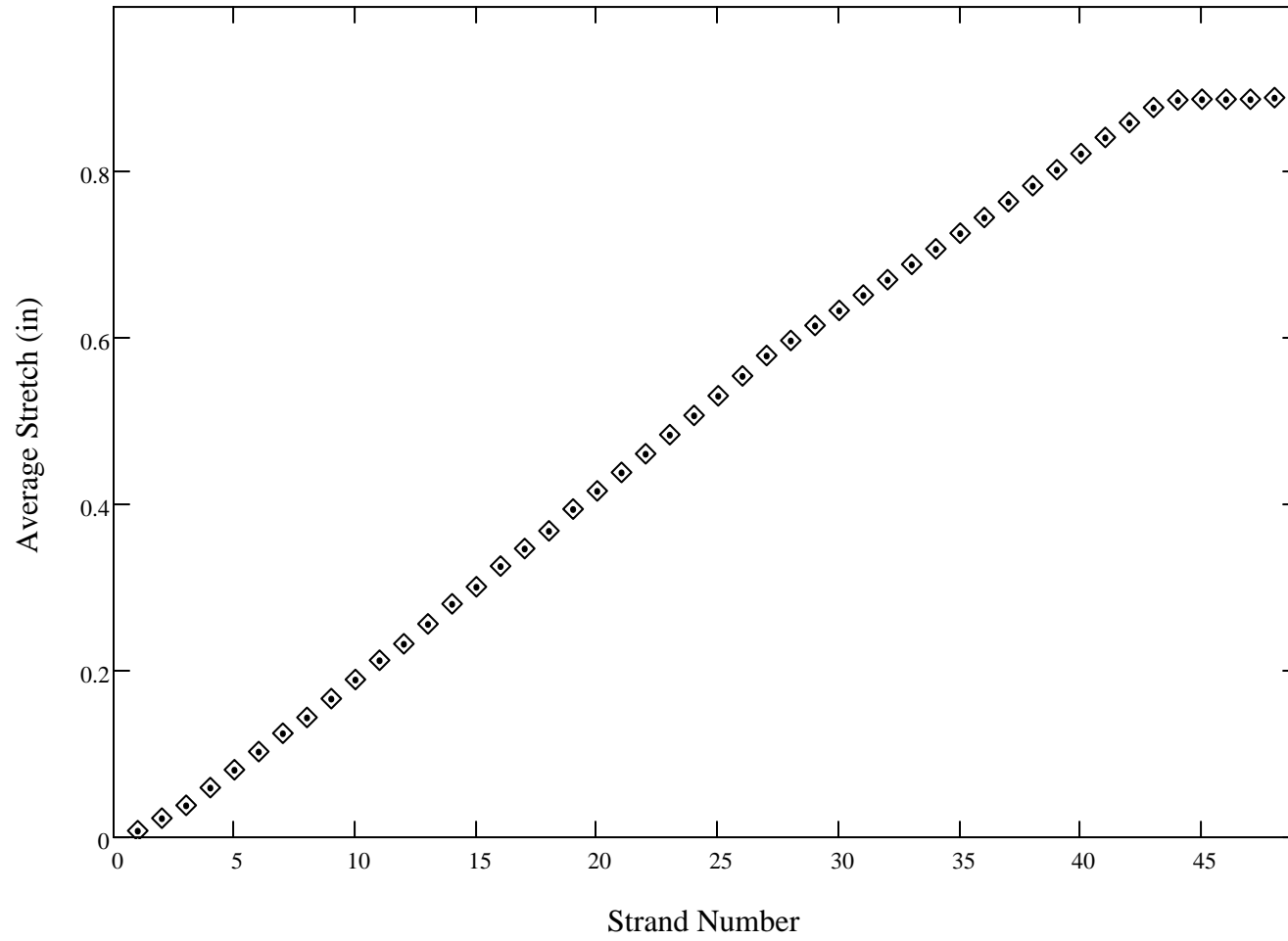


Average Free Strand Set 4 Stretch





Average Free Strand Set 5 Stretch



Average lengths of each Free Strand Set (includes stretching)

```

StraLength := | for q ∈ 1 .. rows(StrandMovXXPerStrand)
                | for c ∈ 1 .. cols(StrandMovXXPerStrand)
                |   outq,c ← AvgStrandLengthsq+1,c + StrandMovXXPerStrandq,c
                | out
    
```

	1	2	3	4	5
1	51.459	5.917	5.917	5.917	46.459
2	51.491	5.98	5.98	5.98	46.491
3	51.525	6.045	6.045	6.045	46.525
4	51.561	6.113	6.113	6.113	46.561
5	51.598	6.184	6.184	6.184	46.598
6	51.637	6.258	6.258	6.258	46.636
7	51.678	6.336	6.336	6.336	46.677
8	51.72	6.417	6.418	6.418	46.719
9	51.765	6.503	6.503	6.503	46.764
10	51.812	6.593	6.593	6.593	46.811
11	51.861	6.688	6.688	6.688	46.86
12	51.913	6.788	6.788	6.788	46.911
13	51.968	6.893	6.894	6.893	46.966
14	52.025	7.005	7.005	7.005	47.023
15	52.086	7.123	7.123	7.123	47.084
16	52.151	7.248	7.248	7.248	47.148
17	52.219	7.381	7.381	7.381	47.216
18	52.292	7.523	7.523	7.523	47.289
19	52.37	7.674	7.674	7.674	47.366
20	52.452	7.835	7.835	7.835	47.449
21	52.54	8.008	8.009	8.009	47.537
22	52.635	8.194	8.194	8.194	47.631

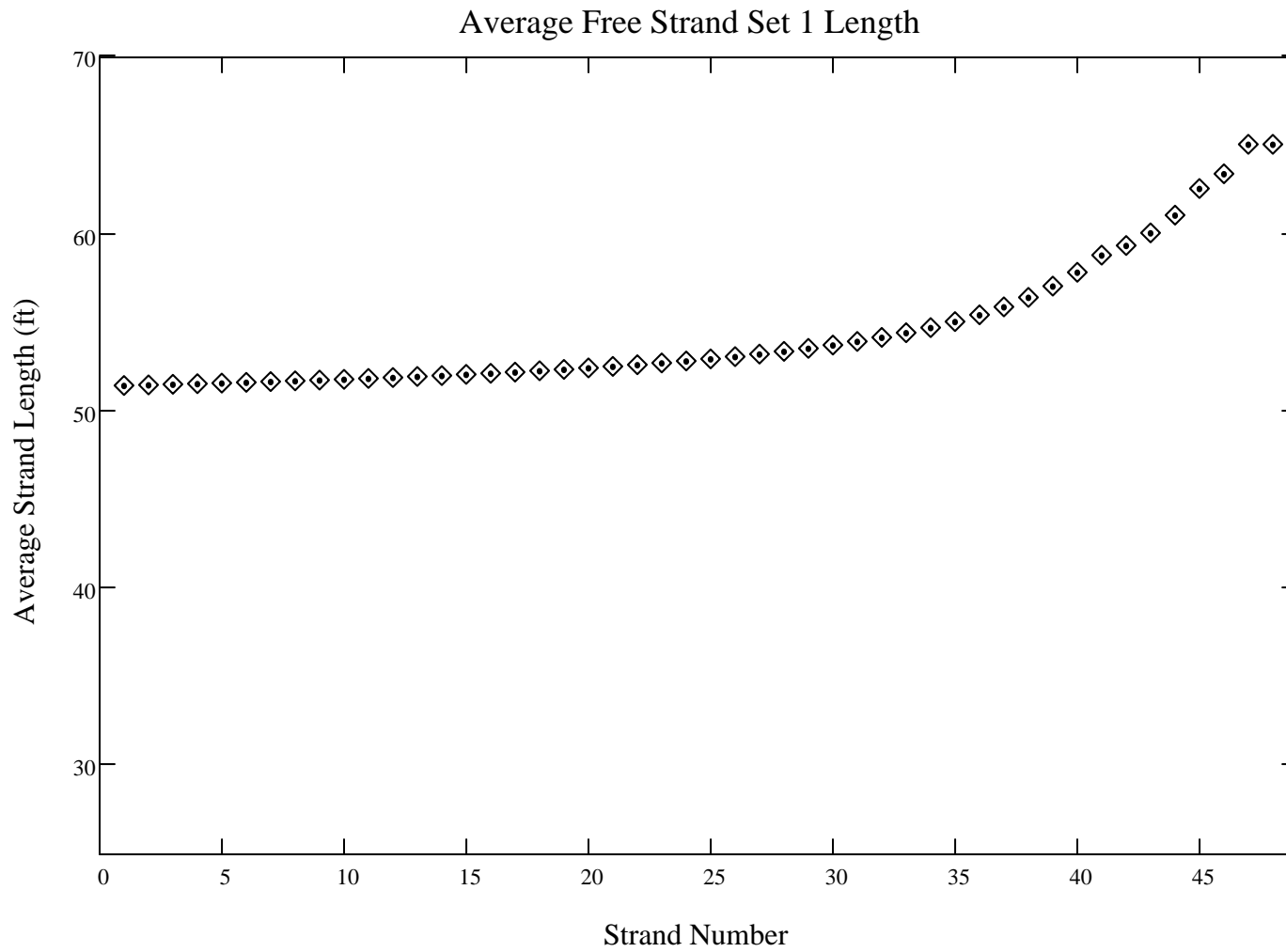
StraLength =

ft

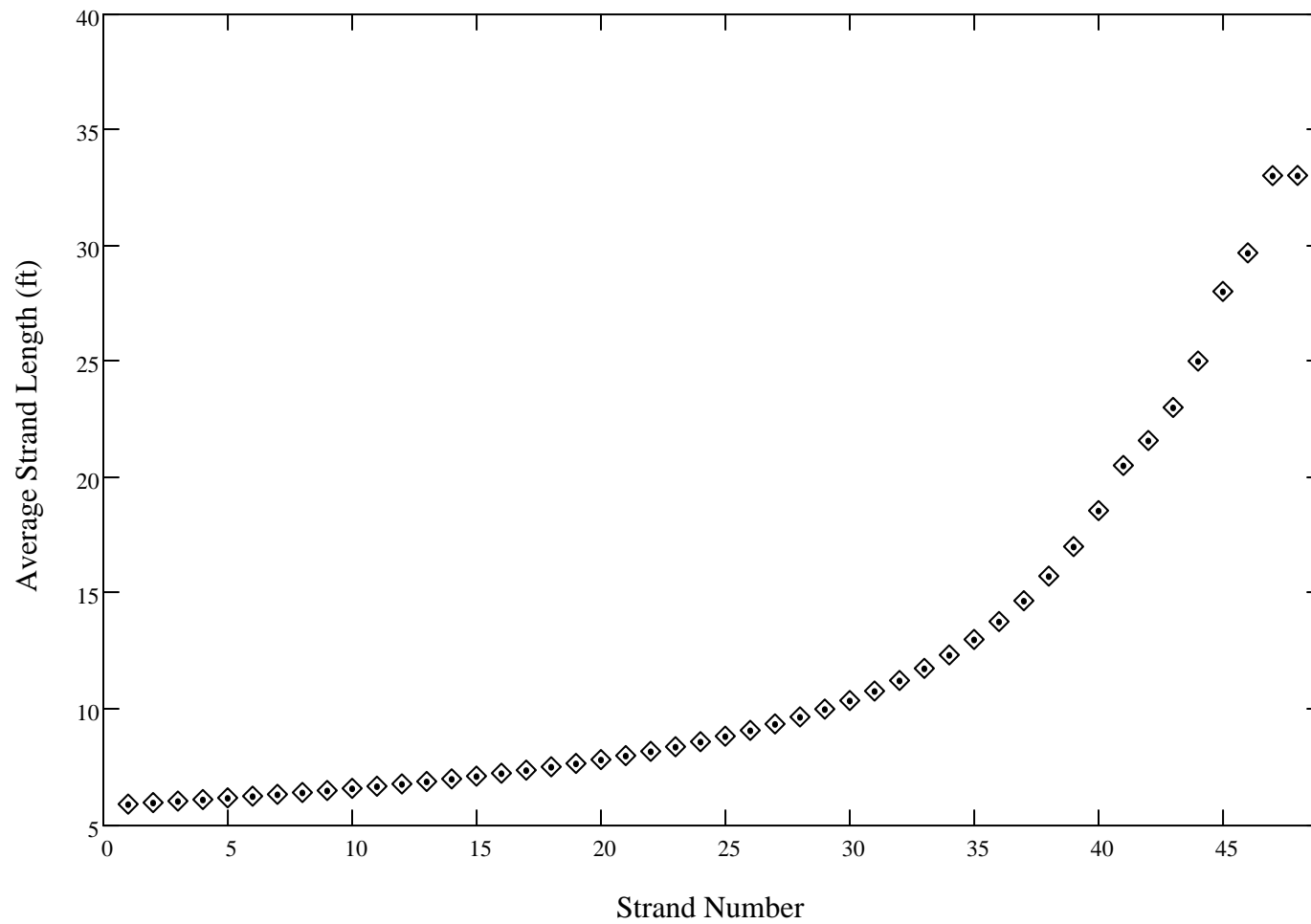
	1	2	3	4	5
1	51.459	5.917	5.917	5.917	46.459
2	51.491	5.98	5.98	5.98	46.491
3	51.525	6.045	6.045	6.045	46.525
4	51.561	6.113	6.113	6.113	46.561
5	51.598	6.184	6.184	6.184	46.598
6	51.637	6.258	6.258	6.258	46.636
7	51.678	6.336	6.336	6.336	46.677
8	51.72	6.417	6.418	6.418	46.719
9	51.765	6.503	6.503	6.503	46.764
10	51.812	6.593	6.593	6.593	46.811
11	51.861	6.688	6.688	6.688	46.86
12	51.913	6.788	6.788	6.788	46.911
13	51.968	6.893	6.894	6.893	46.966
14	52.025	7.005	7.005	7.005	47.023
15	52.086	7.123	7.123	7.123	47.084
16	52.151	7.248	7.248	7.248	47.148
17	52.219	7.381	7.381	7.381	47.216
18	52.292	7.523	7.523	7.523	47.289
19	52.37	7.674	7.674	7.674	47.366
20	52.452	7.835	7.835	7.835	47.449
21	52.54	8.008	8.009	8.009	47.537
22	52.635	8.194	8.194	8.194	47.631
23	52.737	8.394	8.394	8.394	47.733
24	52.847	8.61	8.61	8.61	47.842
25	52.965	8.844	8.844	8.844	47.961
26	53.094	9.099	9.099	9.099	48.09
27	53.234	9.376	9.377	9.376	48.23
28	53.388	9.68	9.68	9.68	48.383
29	53.556	10.015	10.015	10.015	48.551

StraLength =

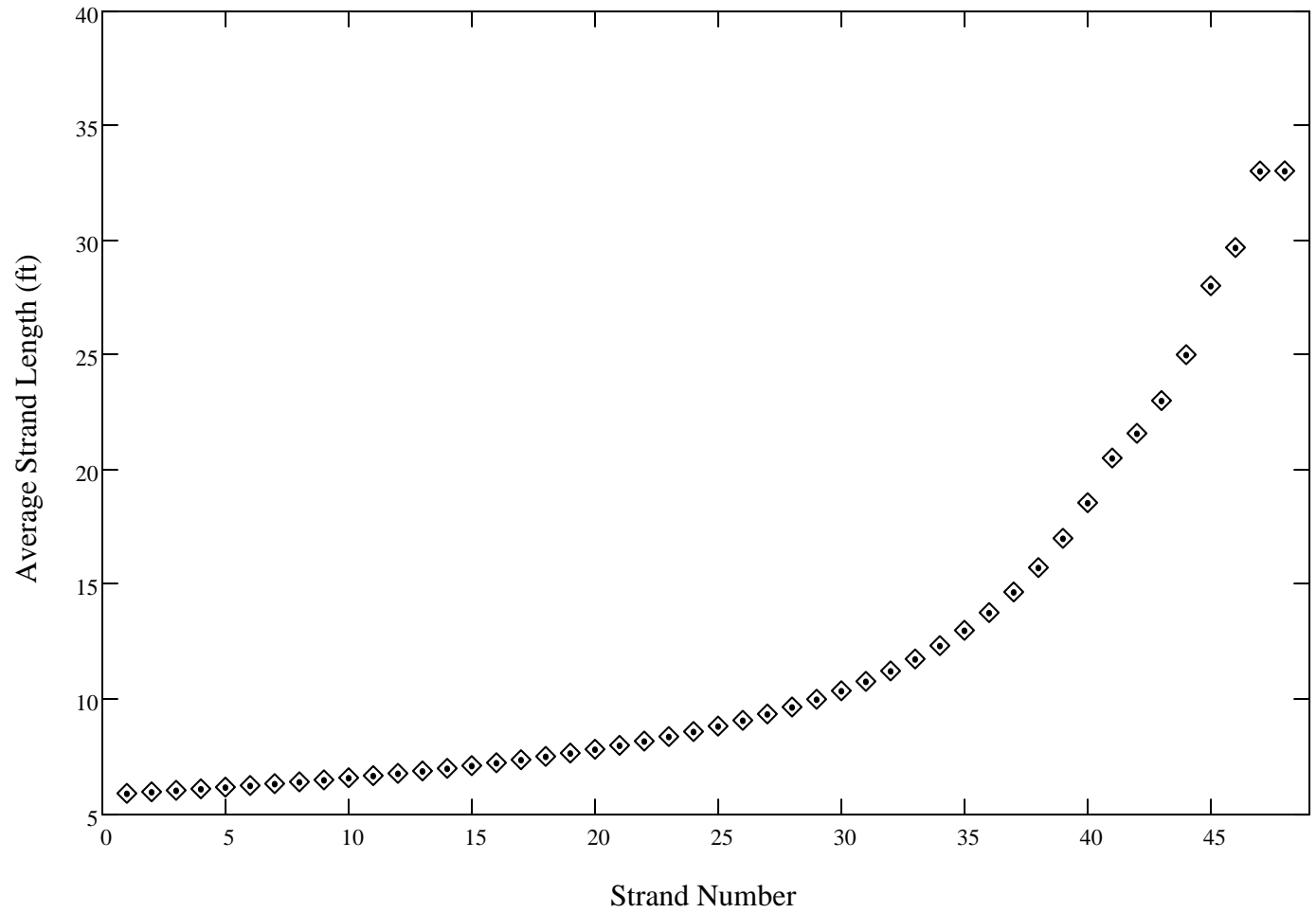
ft



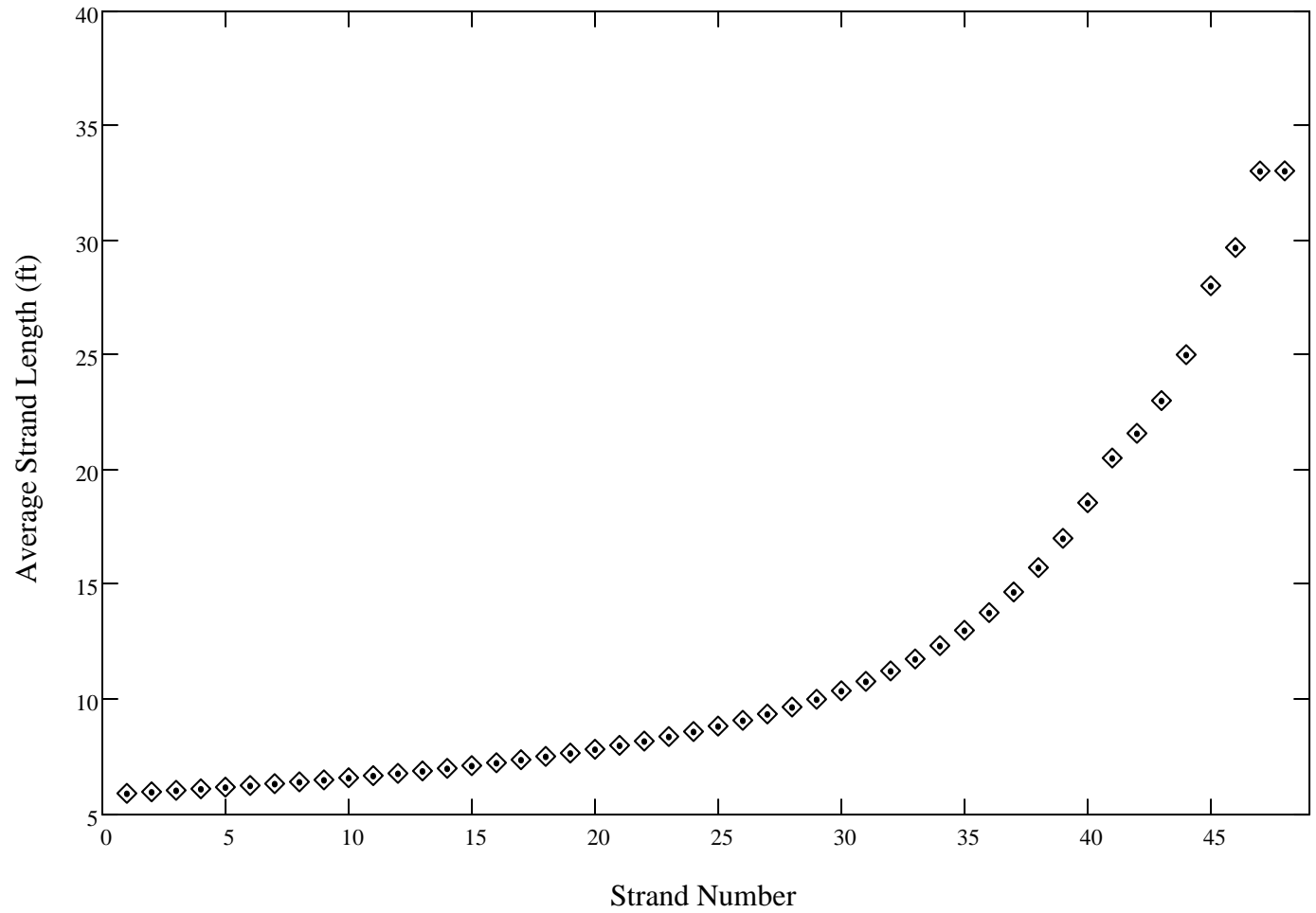
Average Free Strand Set 2 Length

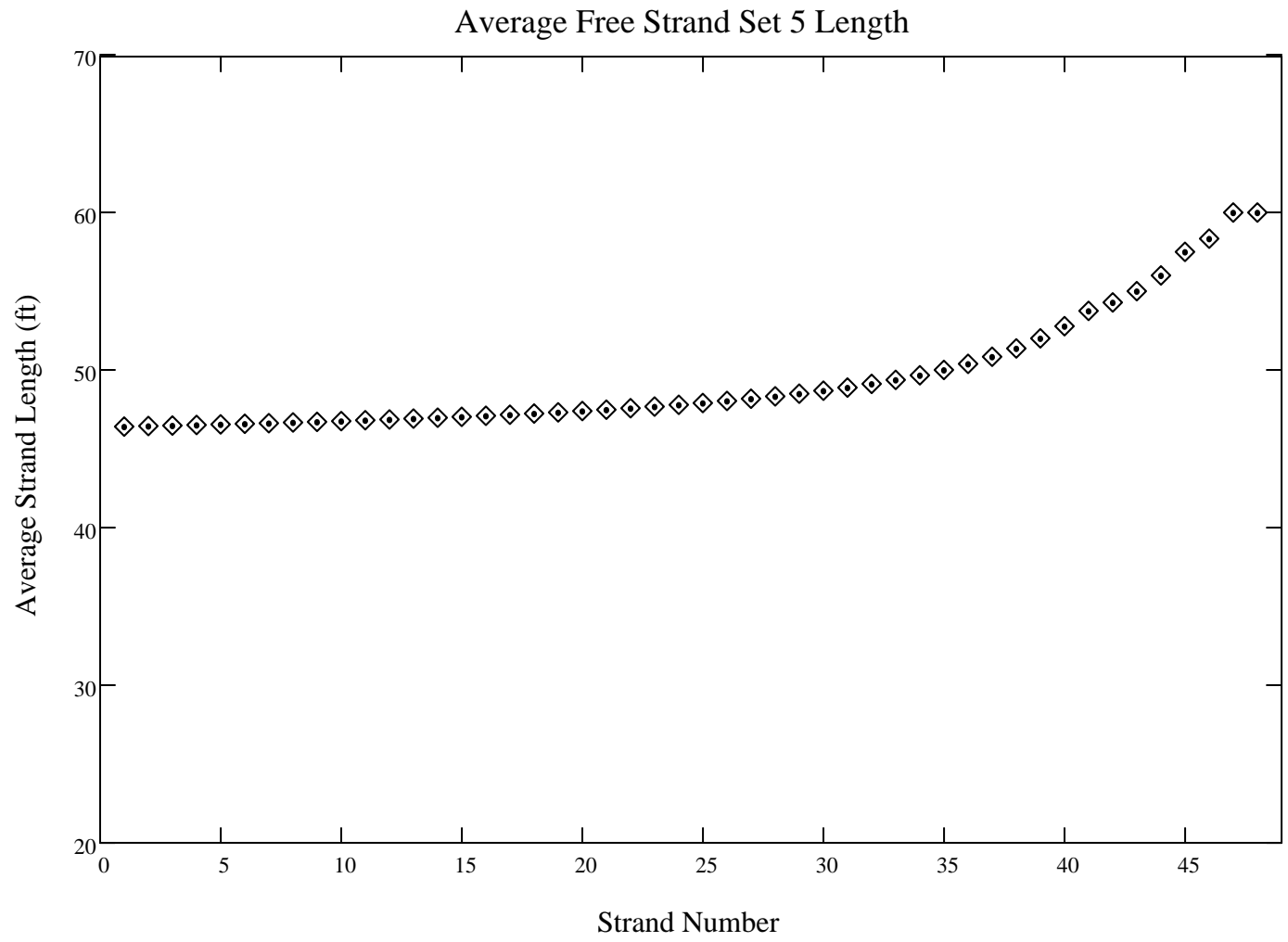


Average Free Strand Set 3 Length



Average Free Strand Set 4 Length







Reverse Transfer Length "RTL"

TensTransferLength := ReverseTransLengthPerStrand

	1	2	3	4	5
1	-0.648	-0.525	-0.521	-0.525	-0.644
2	-0.562	-0.187	-0.178	-0.187	-0.55
3	-0.459	0.007	0.164	0.155	-0.455
4	-0.337	0.214	0.285	0.278	-0.323
5	-0.208	0.355	0.444	0.384	-0.19
6	-0.078	0.495	0.59	0.502	-0.056
7	0.048	0.676	0.757	0.613	0.08
8	0.181	0.808	0.895	0.895	0.197
9	0.309	1.02	1.02	1.013	0.335
10	0.46	1.027	1.19	1.125	0.475
11	0.591	1.222	1.362	1.209	0.617
12	0.723	1.395	1.5	1.5	0.738
13	0.857	1.593	1.652	1.585	0.883
14	0.993	1.746	1.836	1.676	1.029
15	1.13	1.884	1.976	1.976	1.153
16	1.269	2.064	2.154	2.02	1.303
17	1.409	2.186	2.302	2.302	1.428
18	1.552	2.349	2.489	2.489	1.555
19	1.696	2.505	2.653	2.511	1.711
20	1.814	2.785	2.794	2.775	1.84
21	1.963	2.851	2.983	2.983	1.97
22	2.113	2.949	3.155	3.147	2.101
23	2.234	3.268	3.293	3.288	2.233
24	2.357	3.465	3.473	3.471	2.367
25	2.48	3.645	3.658	3.621	2.501
26	2.604	3.787	3.853	3.748	2.637
27	2.73	3.892	4.055	3.852	2.774

TensTransferLength =

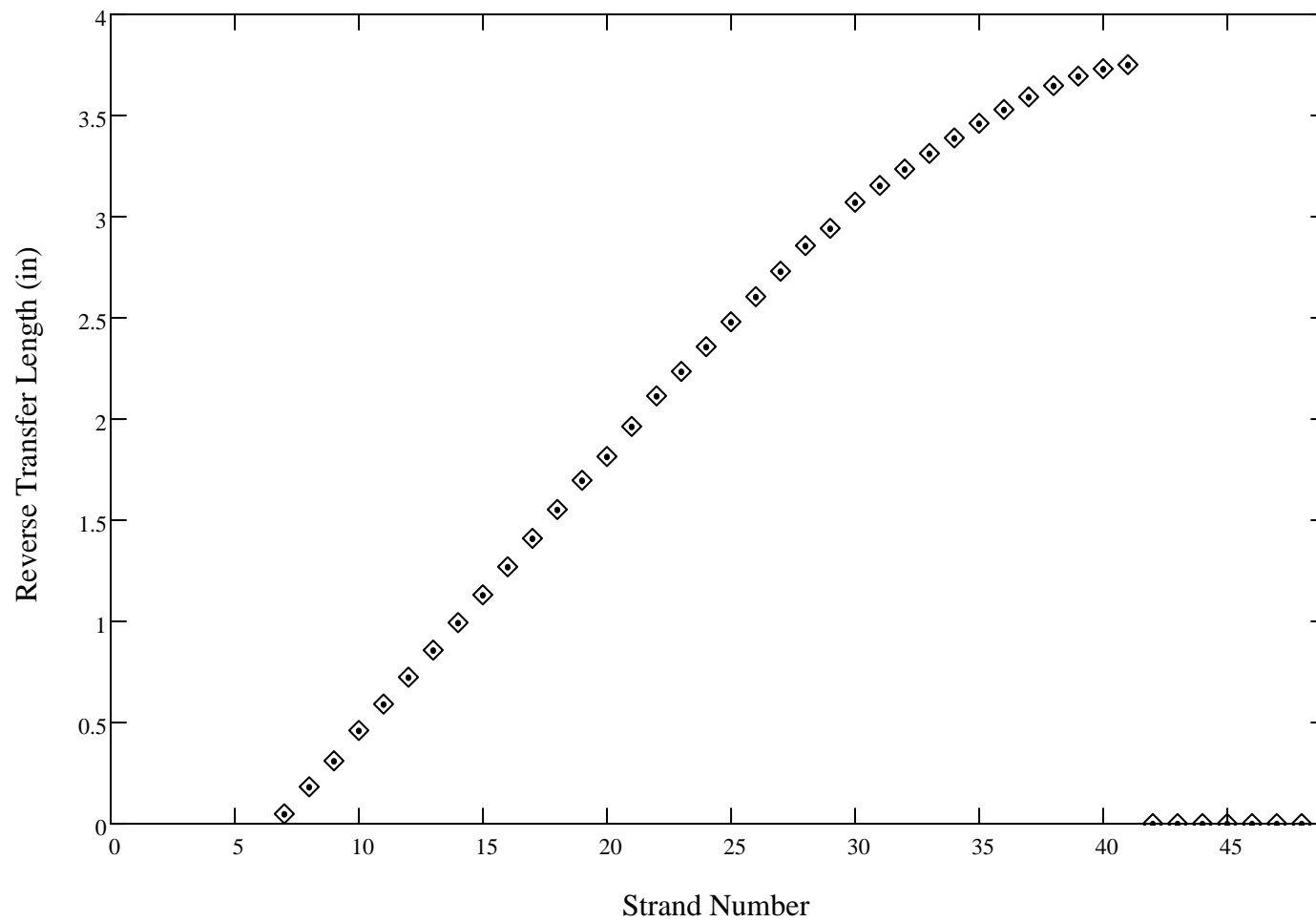
in

	1	2	3	4	5
1	-0.648	-0.525	-0.521	-0.525	-0.644
2	-0.562	-0.187	-0.178	-0.187	-0.55
3	-0.459	0.007	0.164	0.155	-0.455
4	-0.337	0.214	0.285	0.278	-0.323
5	-0.208	0.355	0.444	0.384	-0.19
6	-0.078	0.495	0.59	0.502	-0.056
7	0.048	0.676	0.757	0.613	0.08
8	0.181	0.808	0.895	0.895	0.197
9	0.309	1.02	1.02	1.013	0.335
10	0.46	1.027	1.19	1.125	0.475
11	0.591	1.222	1.362	1.209	0.617
12	0.723	1.395	1.5	1.5	0.738
13	0.857	1.593	1.652	1.585	0.883
14	0.993	1.746	1.836	1.676	1.029
15	1.13	1.884	1.976	1.976	1.153
16	1.269	2.064	2.154	2.02	1.303
17	1.409	2.186	2.302	2.302	1.428
18	1.552	2.349	2.489	2.489	1.555
19	1.696	2.505	2.653	2.511	1.711
20	1.814	2.785	2.794	2.775	1.84
21	1.963	2.851	2.983	2.983	1.97
22	2.113	2.949	3.155	3.147	2.101
23	2.234	3.268	3.293	3.288	2.233
24	2.357	3.465	3.473	3.471	2.367
25	2.48	3.645	3.658	3.621	2.501
26	2.604	3.787	3.853	3.748	2.637
27	2.73	3.892	4.055	3.852	2.774
28	2.856	3.967	4.199	4.189	2.87
29	2.942	4.353	4.363	4.354	2.965

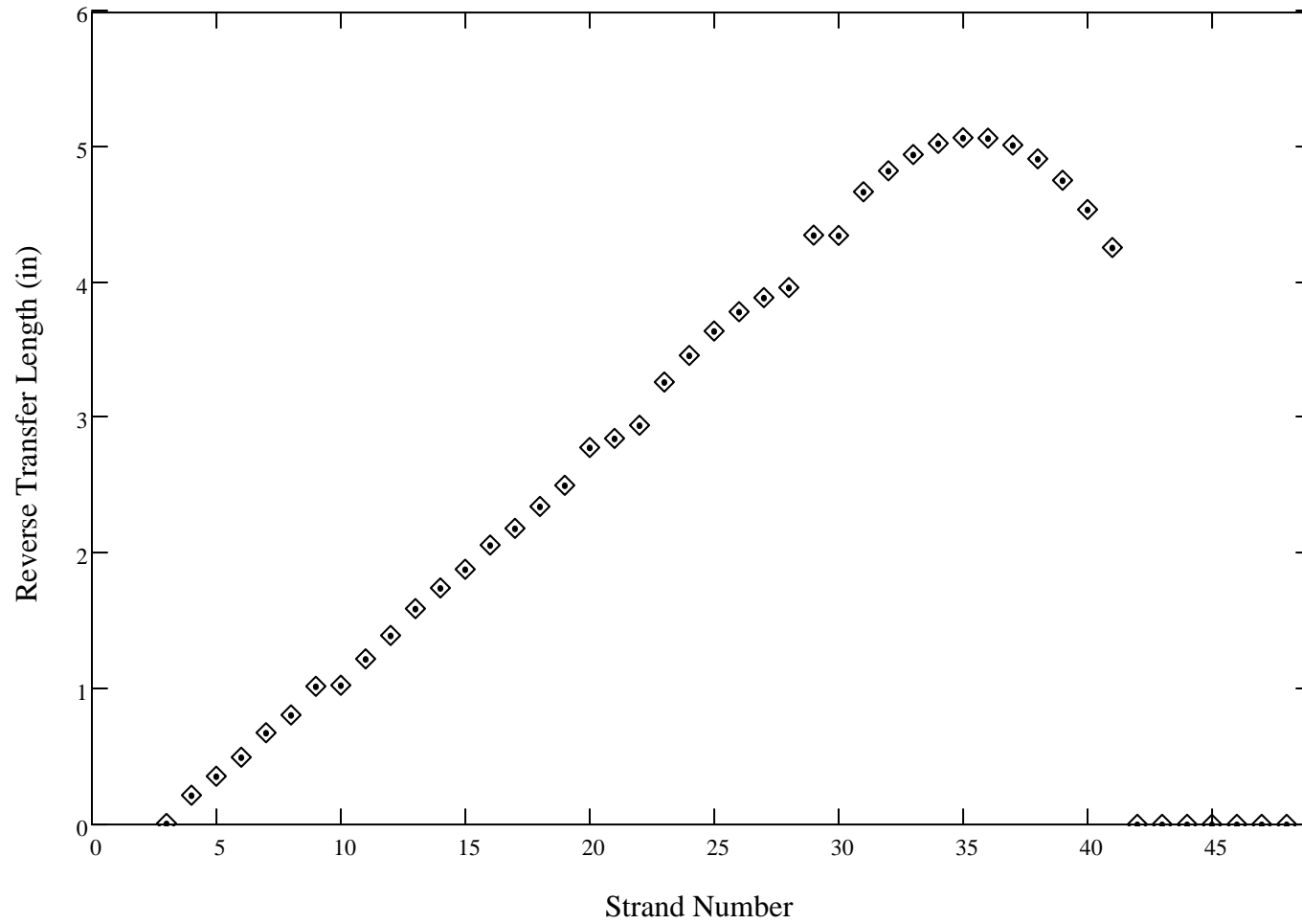
TensTransferLength =

in

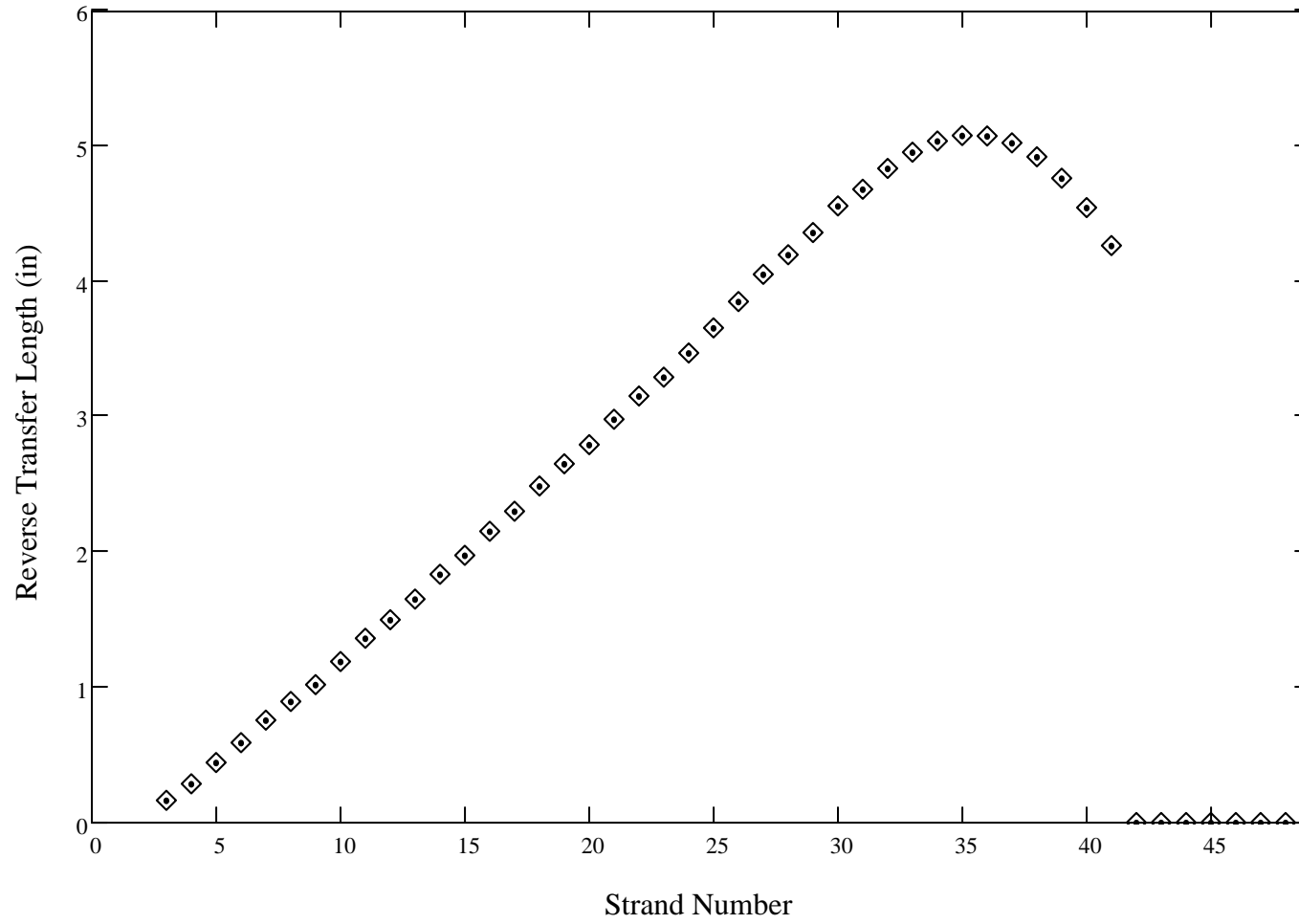
Reverse Transfer Length Strand Set 1



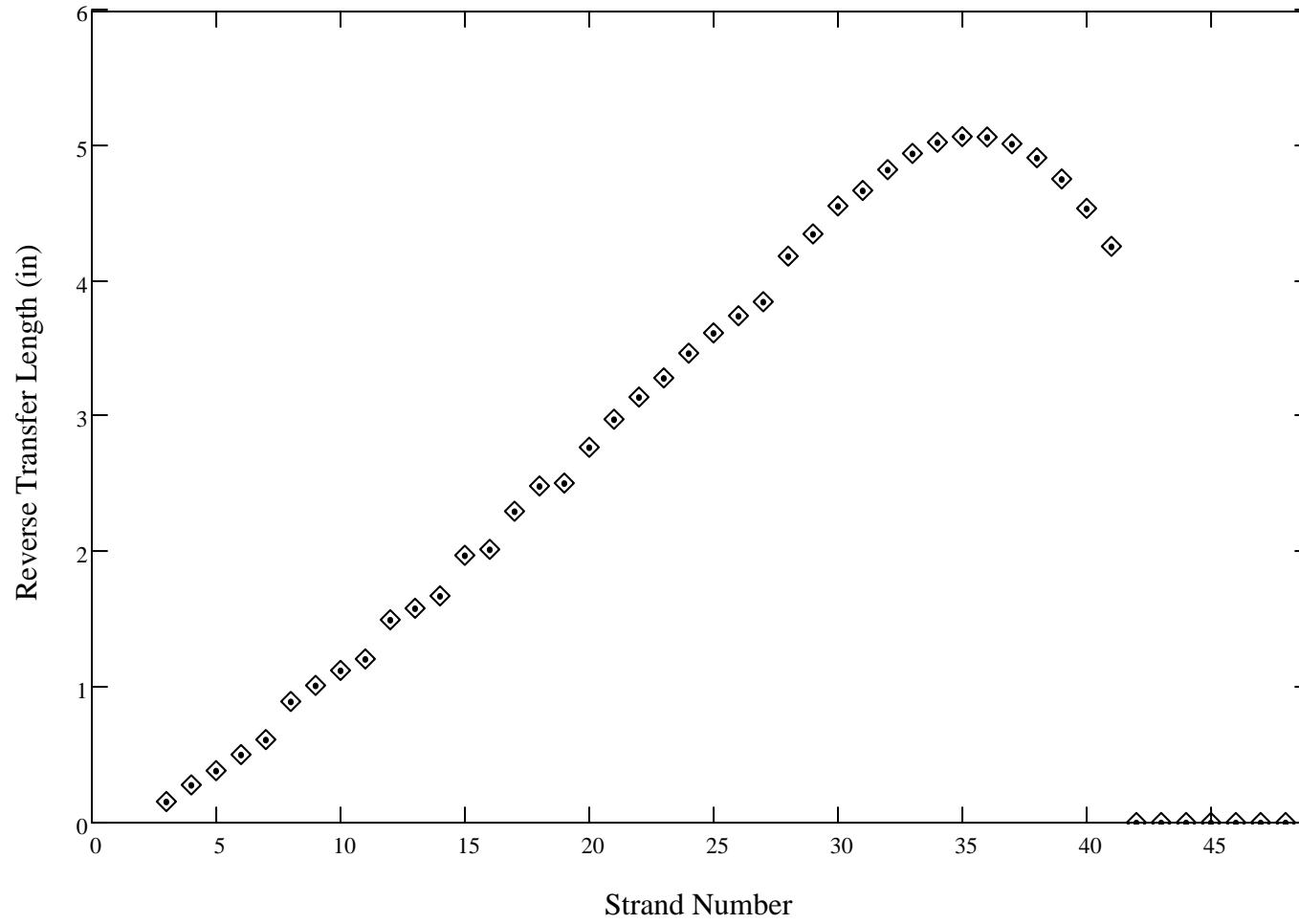
Reverse Transfer Length Strand Set 2



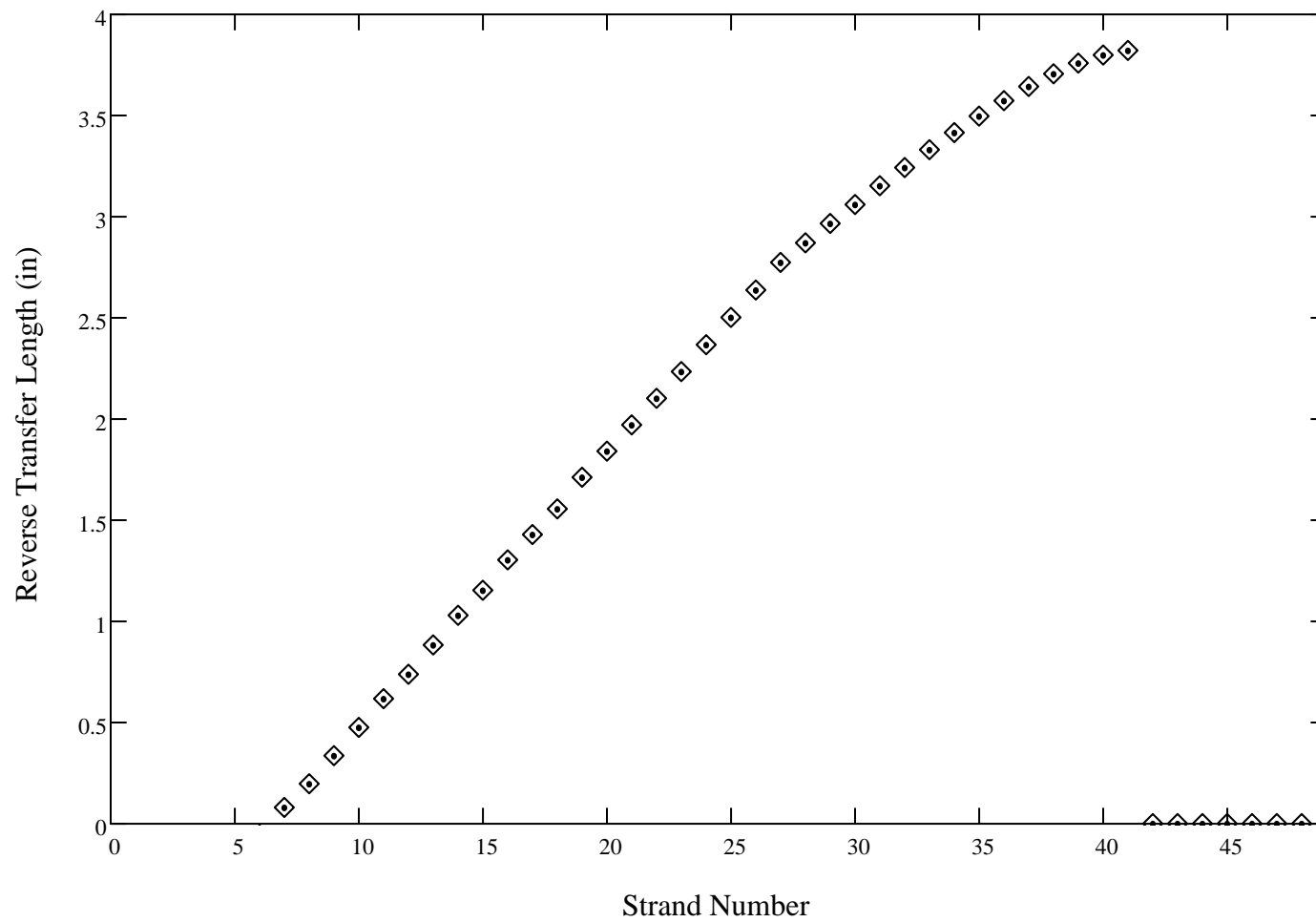
Reverse Transfer Length Strand Set 3



Reverse Transfer Length Strand Set 4



Reverse Transfer Length Strand Set 5



**FORCE RESULTS:**

Unbalanced Tension Pull " $\Delta$ UTP":

Positive number indicates beam wants to move to the right, negative number indicates beam wants to move to the left

	1	2	3	4
1	10.652	0.357	-0.363	-10.317
2	31.928	0.736	-0.717	-30.877
3	38.743	13.066	-0.713	-50.758
4	44.882	5.794	-0.583	-48.959
5	44.797	7.064	-4.767	-45.676
6	44.551	7.383	-6.839	-43.411
7	47.724	6.146	-10.981	-40.472
8	46.487	6.407	0	-51.751
9	51.377	0	-0.457	-49.052
10	40.036	11.473	-4.574	-45.833
11	43.393	9.619	-10.508	-40.7
12	44.944	7.012	0	-50.984
13	47.909	3.872	-4.414	-45.714
14	47.665	5.705	-10.123	-40.946
15	46.4	5.63	0	-50.626
16	47.442	5.396	-8.014	-42.793
17	44.973	6.708	0	-50.575
18	44.683	7.864	0	-52.356
19	43.881	8.022	-7.712	-43.38
20	50.895	0.49	-1.003	-49.027
21	44.996	6.667	0	-51.293
22	40.787	10.056	-0.376	-51.067
23	48.591	1.197	-0.24	-49.601
24	50.094	0.364	-0.052	-49.95
25	50.553	0.584	-1.617	-48.596
26	49.216	2.721	-4.357	-46.227

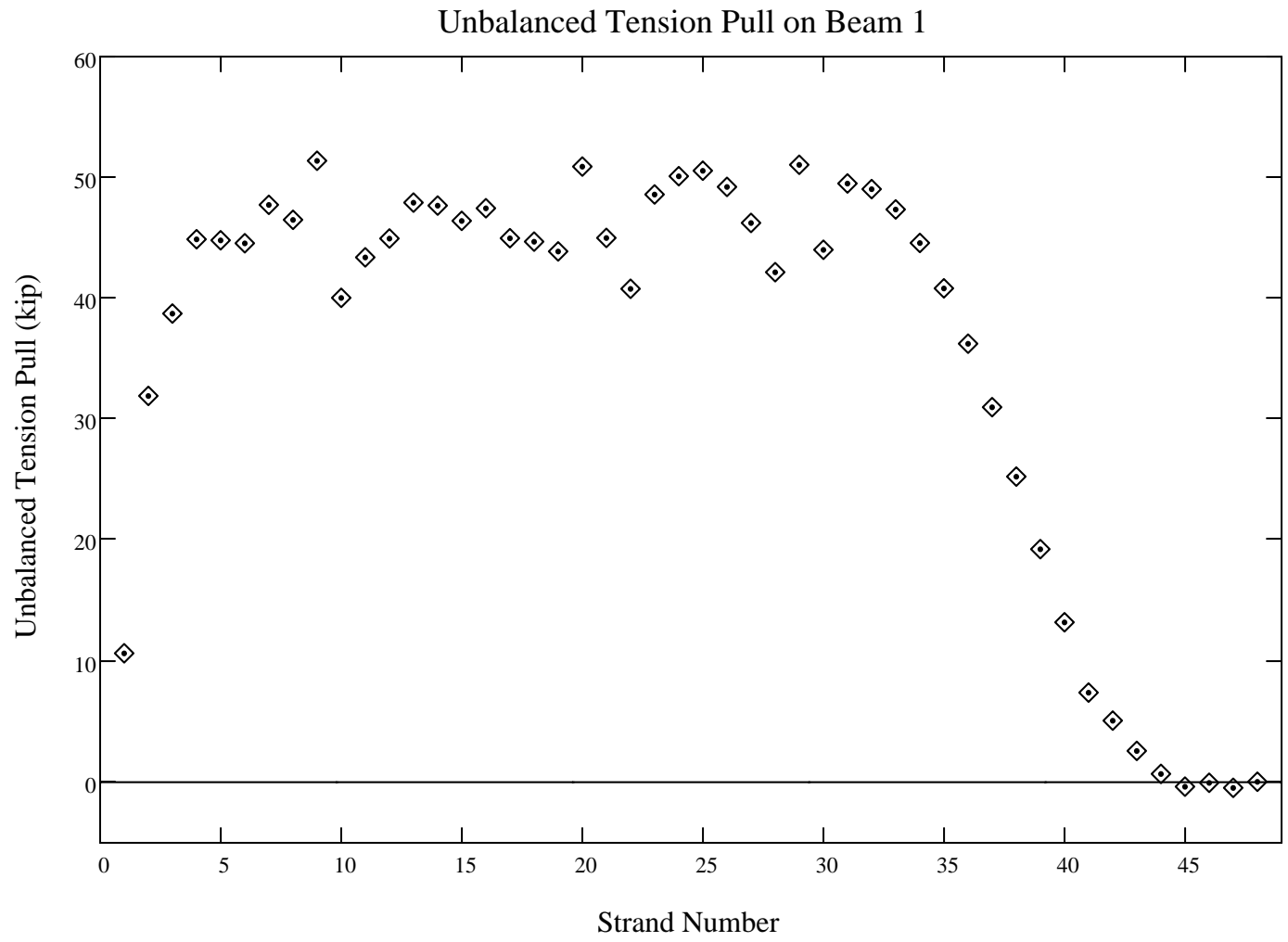
UnbalanceForcePerStrand =

kip

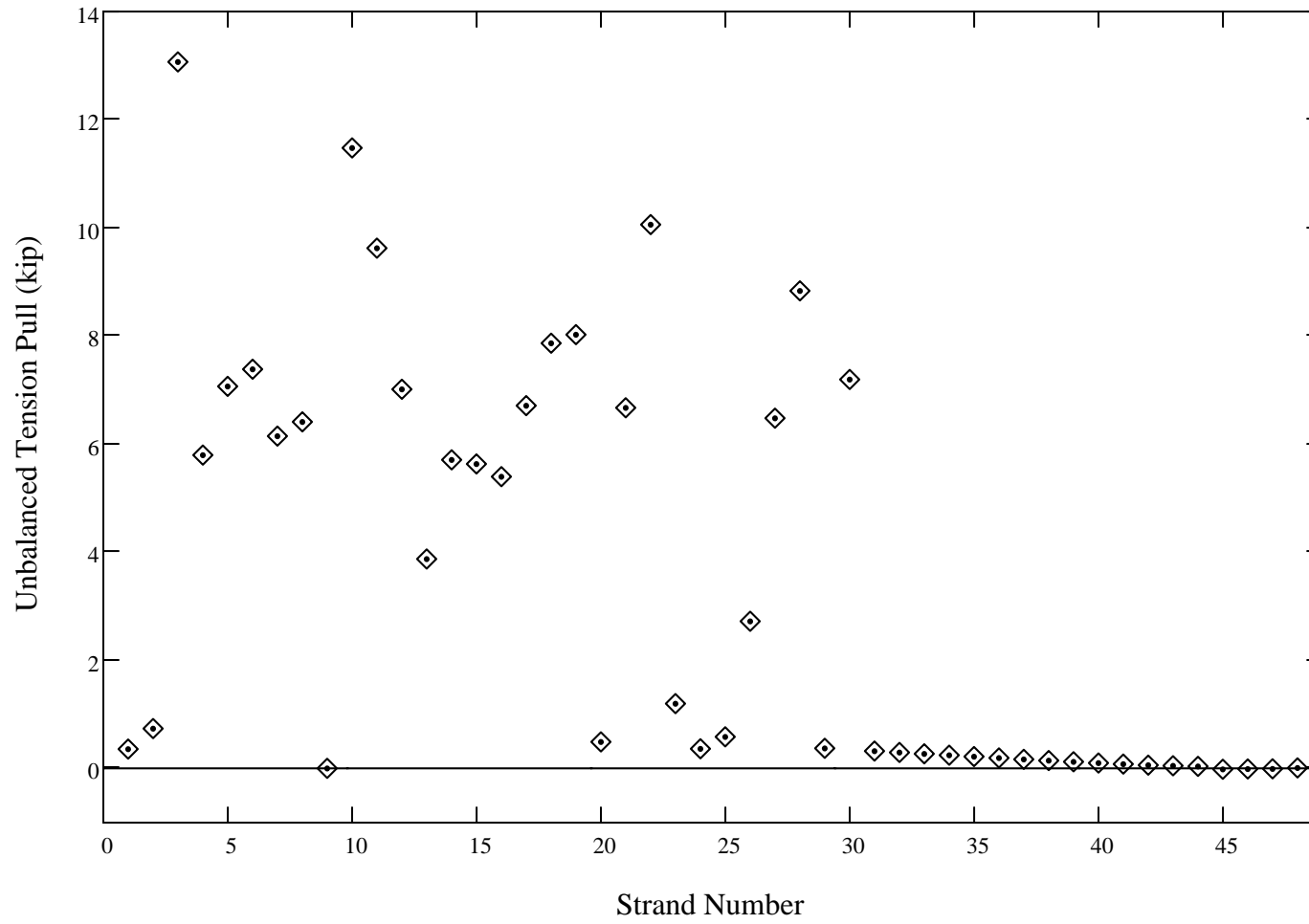


	1	2	3	4
1	10.652	0.357	-0.363	-10.317
2	31.928	0.736	-0.717	-30.877
3	38.743	13.066	-0.713	-50.758
4	44.882	5.794	-0.583	-48.959
5	44.797	7.064	-4.767	-45.676
6	44.551	7.383	-6.839	-43.411
7	47.724	6.146	-10.981	-40.472
8	46.487	6.407	0	-51.751
9	51.377	0	-0.457	-49.052
10	40.036	11.473	-4.574	-45.833
11	43.393	9.619	-10.508	-40.7
12	44.944	7.012	0	-50.984
13	47.909	3.872	-4.414	-45.714
14	47.665	5.705	-10.123	-40.946
15	46.4	5.63	0	-50.626
16	47.442	5.396	-8.014	-42.793
17	44.973	6.708	0	-50.575
18	44.683	7.864	0	-52.356
19	43.881	8.022	-7.712	-43.38
20	50.895	0.49	-1.003	-49.027
21	44.996	6.667	0	-51.293
22	40.787	10.056	-0.376	-51.067
23	48.591	1.197	-0.24	-49.601
24	50.094	0.364	-0.052	-49.95
25	50.553	0.584	-1.617	-48.596

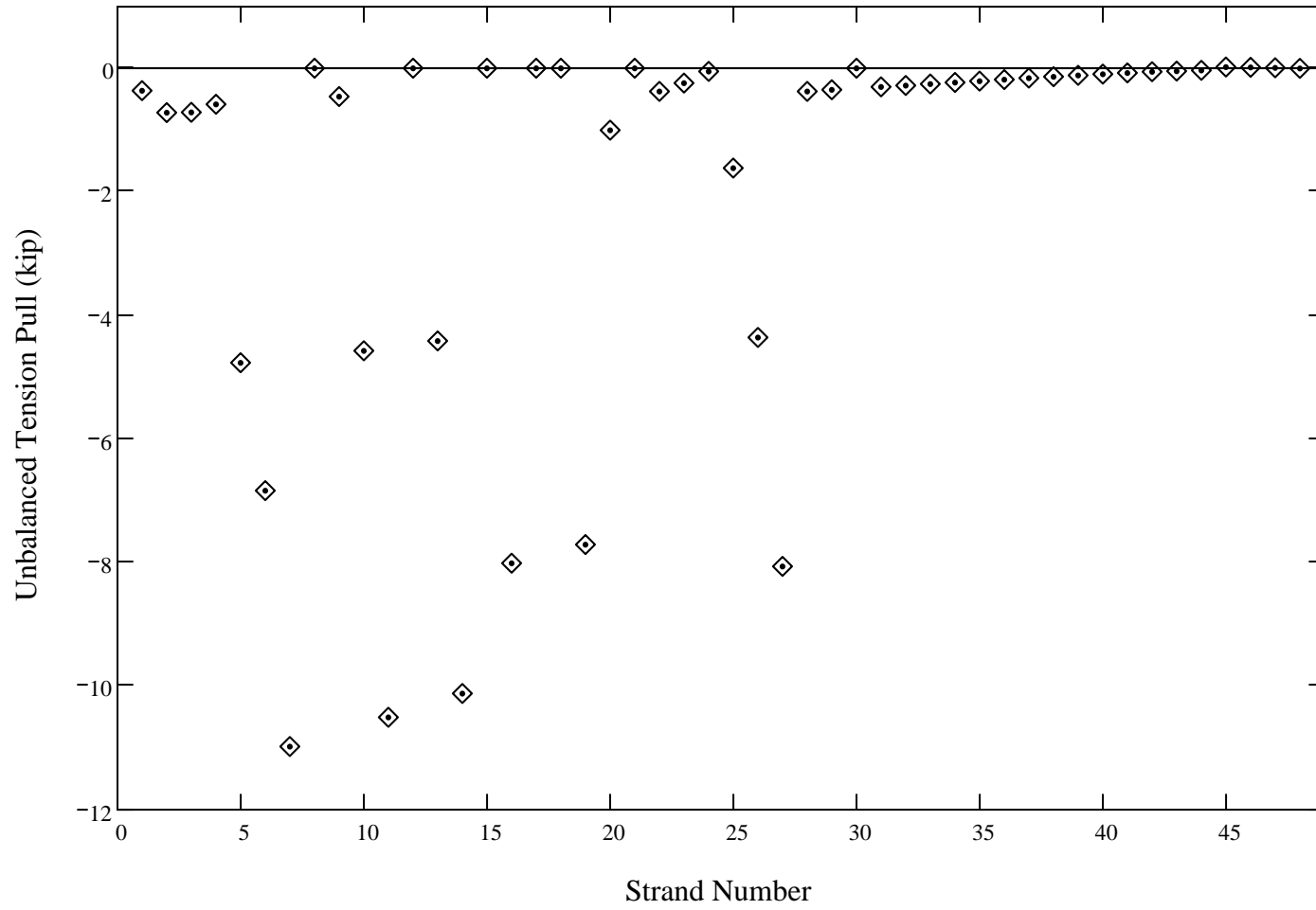
UnbalanceForcePerStrand = kip



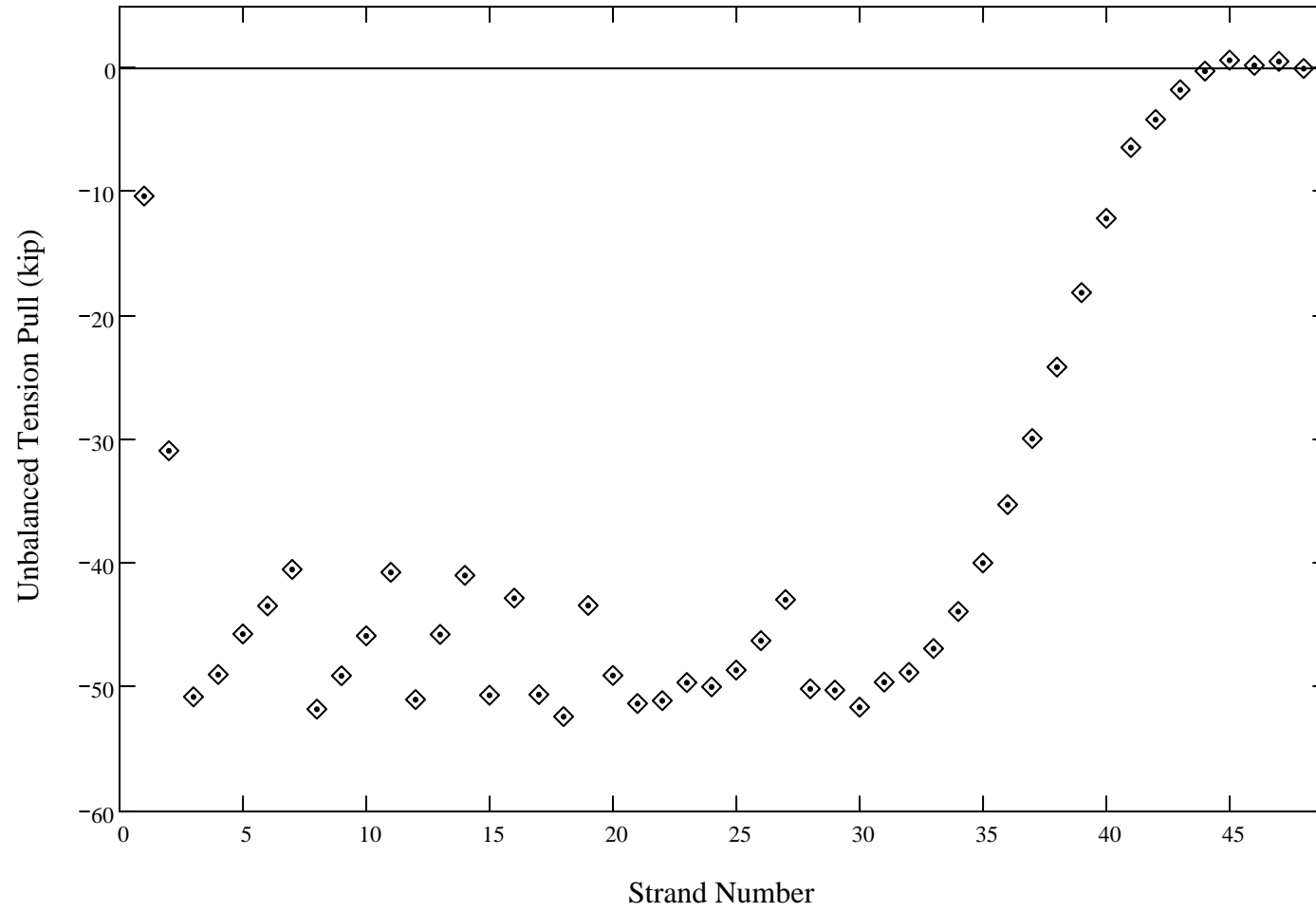
Unbalanced Tension Pull on Beam 2



Unbalanced Tension Pull on Beam 3



Unbalanced Tension Pull on Beam 4



Acting Static Friction Force - negative value indicates that friction is acting in the same direction as the friction at the opposite beam end  
 Any number other than that equal to  $F_s$  is indicating the the friction is forced to change to prevent global motion of the beam

	1	2	3	4	5	6	7	8
1	25.9	15.2	26.8	26.4	26.7	27.1	15.9	26.2
2	25.9	-6	26.8	26	26.4	27.1	-4.6	26.2
3	25.9	-12.8	26.8	13.7	26.4	27.1	-24.5	26.2
4	25.9	-19	26.8	21	26.5	27.1	-22.7	26.2
5	25.9	-18.9	26.8	19.7	22.3	27.1	-19.4	26.2
6	25.9	-18.7	26.8	19.4	20.3	27.1	-17.2	26.2
7	25.9	-21.8	26.8	20.6	16.1	27.1	-14.2	26.2
8	25.9	-20.6	26.8	20.4	27.1	27.1	-25.5	26.2
9	25.9	-25.5	26.8	26.8	26.6	27.1	-22.8	26.2
10	25.9	-14.1	26.8	15.3	22.5	27.1	-19.6	26.2
11	25.9	-17.5	26.8	17.1	16.6	27.1	-14.5	26.2
12	25.9	-19	26.8	19.7	27.1	27.1	-24.7	26.2
13	25.9	-22	26.8	22.9	22.7	27.1	-19.5	26.2
14	25.9	-21.8	26.8	21.1	17	27.1	-14.7	26.2
15	25.9	-20.5	26.8	21.1	27.1	27.1	-24.4	26.2
16	25.9	-21.5	26.8	21.4	19.1	27.1	-16.5	26.2
17	25.9	-19.1	26.8	20.1	27.1	27.1	-24.3	26.2
18	25.9	-18.8	26.8	18.9	27.1	27.1	-26.1	26.2
19	25.9	-18	26.8	18.7	19.4	27.1	-17.1	26.2
20	25.9	-25	26.8	26.3	26.1	27.1	-22.8	26.2
21	25.9	-19.1	26.8	20.1	27.1	27.1	-25	26.2
22	25.9	-14.9	26.8	16.7	26.7	27.1	-24.8	26.2
23	25.9	-22.7	26.8	25.6	26.9	27.1	-23.4	26.2
24	25.9	-24.2	26.8	26.4	27.1	27.1	-23.7	26.2
25	25.9	-24.7	26.8	26.2	25.5	27.1	-22.4	26.2
26	25.9	-23.3	26.8	24	22.7	27.1	-20	26.2
27	25.9	-20.3	26.8	20.3	19	27.1	-16.7	26.2

FrictionValuePerStrand =

kip

	1	2	3	4	5	6	7	8
1	25.9	15.2	26.8	26.4	26.7	27.1	15.9	26.2
2	25.9	-6	26.8	26	26.4	27.1	-4.6	26.2
3	25.9	-12.8	26.8	13.7	26.4	27.1	-24.5	26.2
4	25.9	-19	26.8	21	26.5	27.1	-22.7	26.2
5	25.9	-18.9	26.8	19.7	22.3	27.1	-19.4	26.2
6	25.9	-18.7	26.8	19.4	20.3	27.1	-17.2	26.2
7	25.9	-21.8	26.8	20.6	16.1	27.1	-14.2	26.2
8	25.9	-20.6	26.8	20.4	27.1	27.1	-25.5	26.2
9	25.9	-25.5	26.8	26.8	26.6	27.1	-22.8	26.2
10	25.9	-14.1	26.8	15.3	22.5	27.1	-19.6	26.2
11	25.9	-17.5	26.8	17.1	16.6	27.1	-14.5	26.2
12	25.9	-19	26.8	19.7	27.1	27.1	-24.7	26.2
13	25.9	-22	26.8	22.9	22.7	27.1	-19.5	26.2
14	25.9	-21.8	26.8	21.1	17	27.1	-14.7	26.2
15	25.9	-20.5	26.8	21.1	27.1	27.1	-24.4	26.2
16	25.9	-21.5	26.8	21.4	19.1	27.1	-16.5	26.2
17	25.9	-19.1	26.8	20.1	27.1	27.1	-24.3	26.2
18	25.9	-18.8	26.8	18.9	27.1	27.1	-26.1	26.2
19	25.9	-18	26.8	18.7	19.4	27.1	-17.1	26.2
20	25.9	-25	26.8	26.3	26.1	27.1	-22.8	26.2
21	25.9	-19.1	26.8	20.1	27.1	27.1	-25	26.2
22	25.9	-14.9	26.8	16.7	26.7	27.1	-24.8	26.2
23	25.9	-22.7	26.8	25.6	26.9	27.1	-23.4	26.2
24	25.9	-24.2	26.8	26.4	27.1	27.1	-23.7	26.2

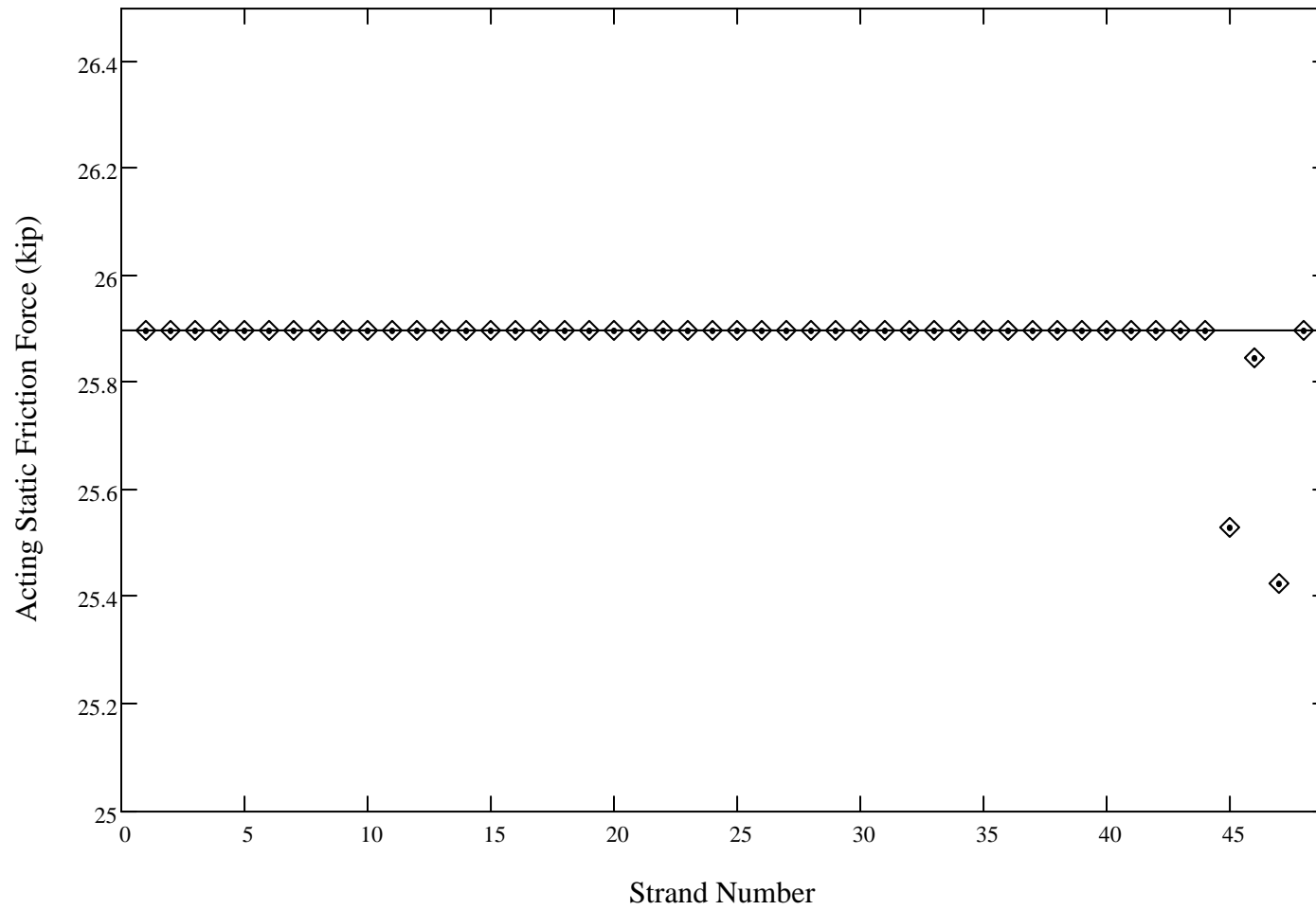
FrictionValuePerStrand =

kip

Maximum Static friction force Fs - (shown as a reference)

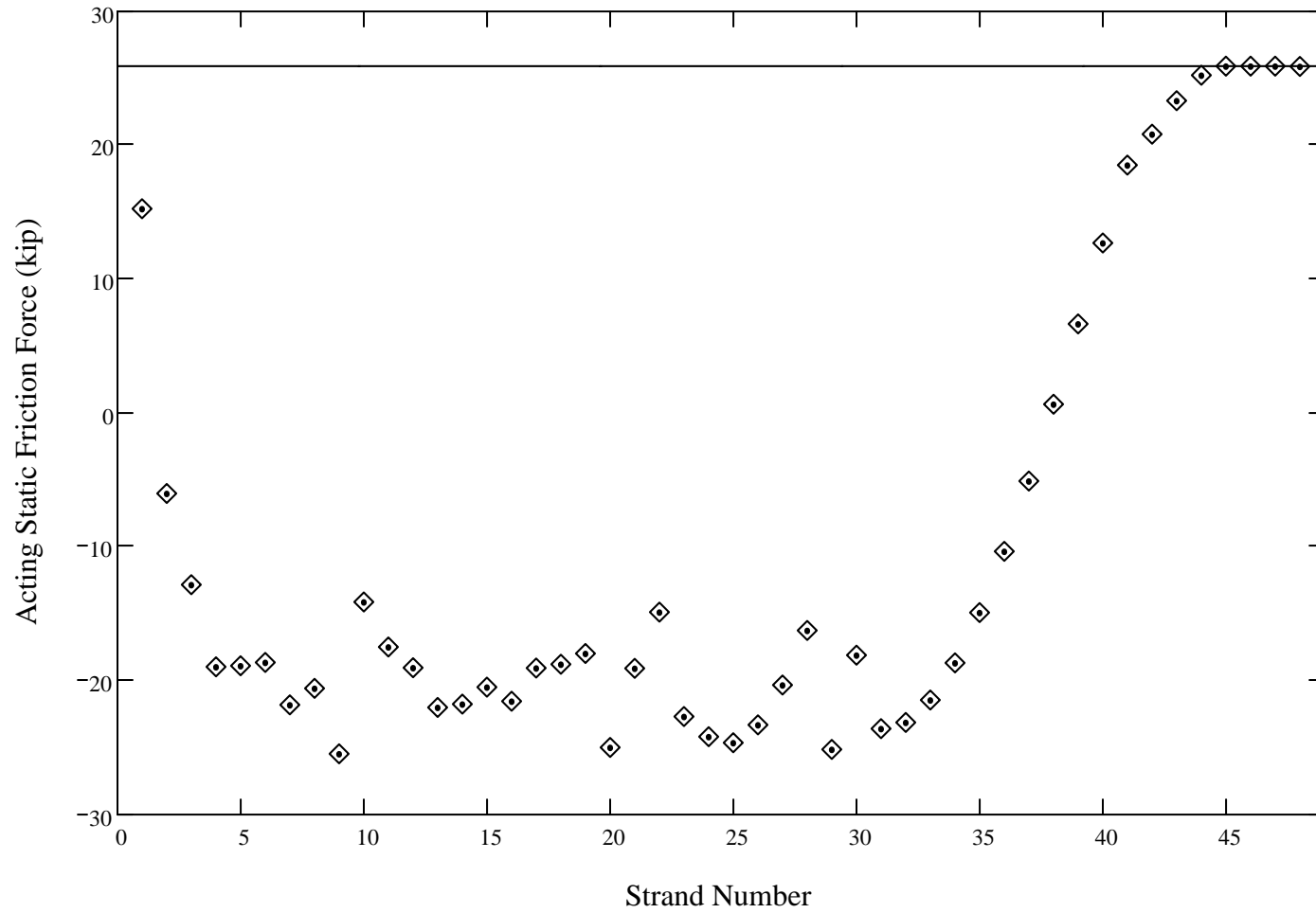
FRfw = (25.898 25.898 26.762 26.762 27.107 27.107 26.244 26.244) kip

Acting Static Friction Force B 1 End 1

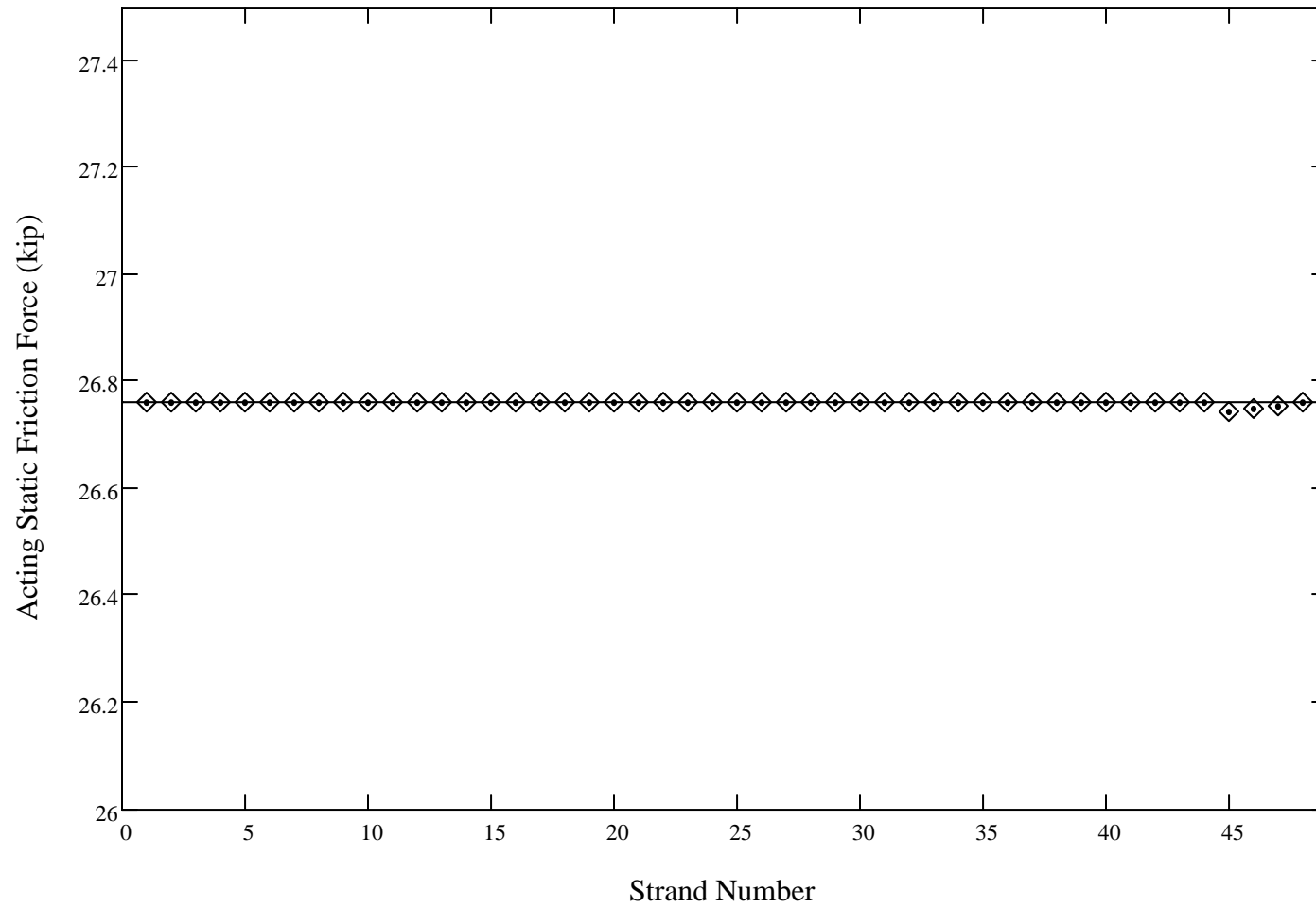




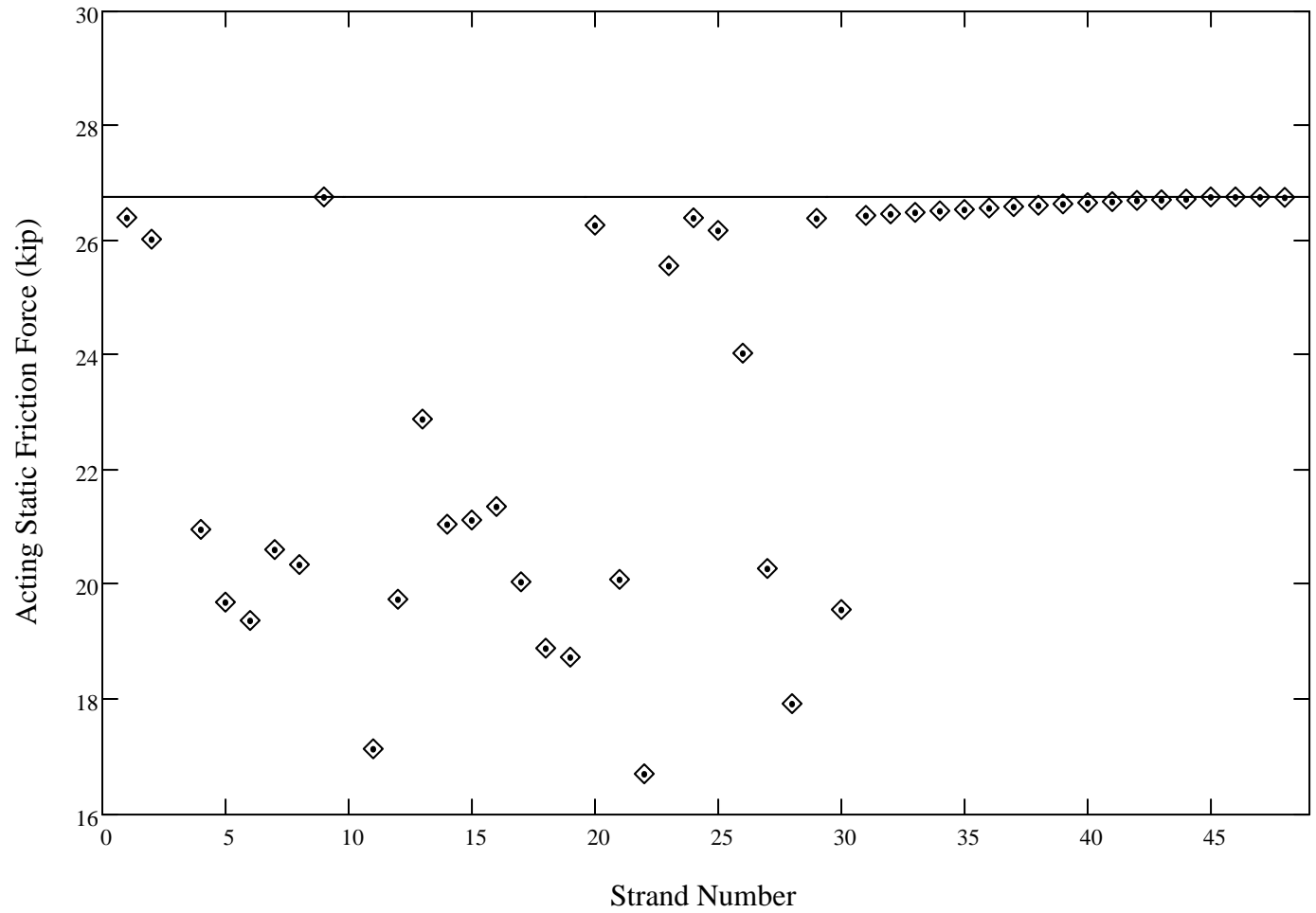
Acting Static Friction Force B 1 End 2

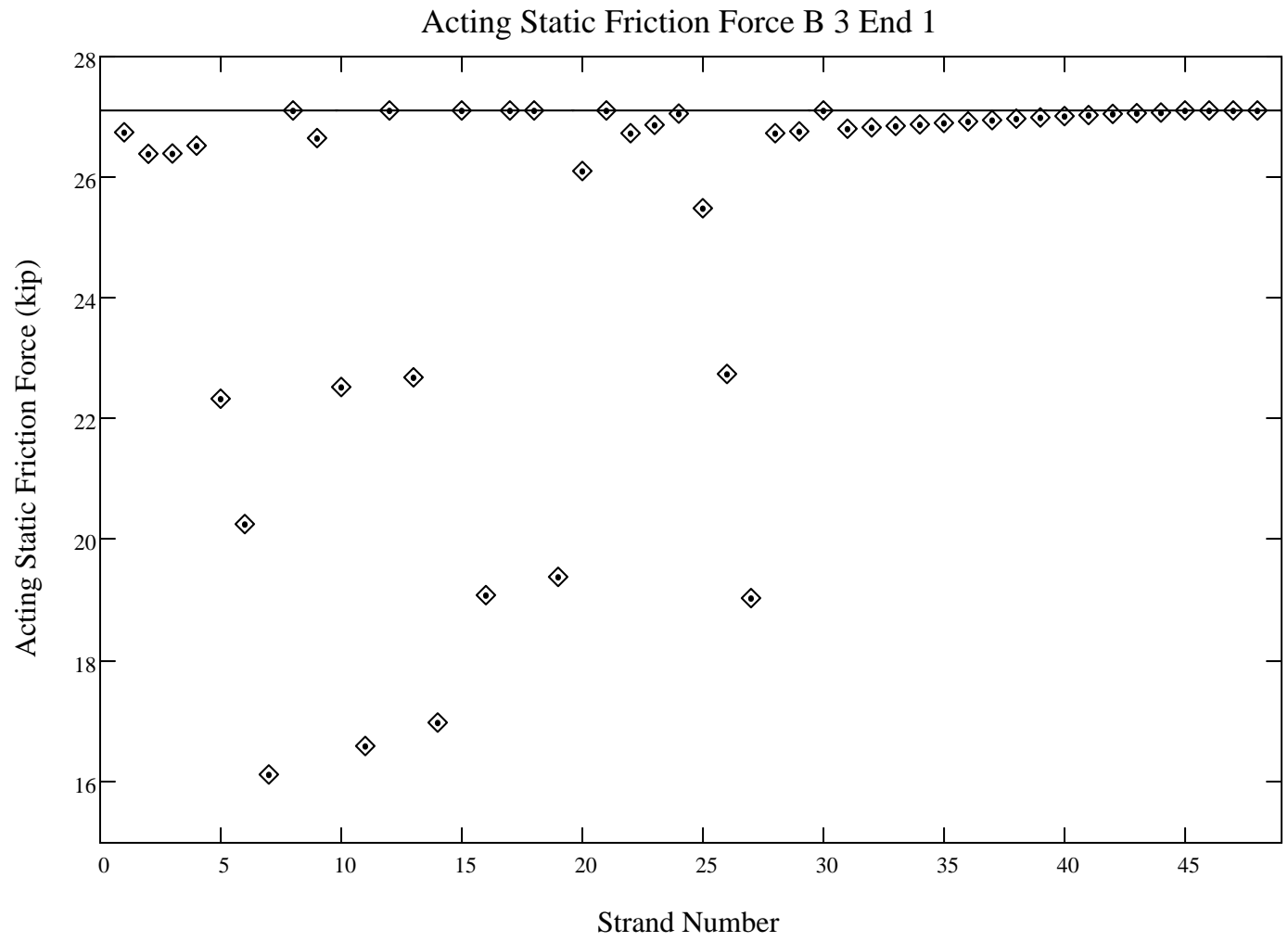


Acting Static Friction Force B 2 End 1

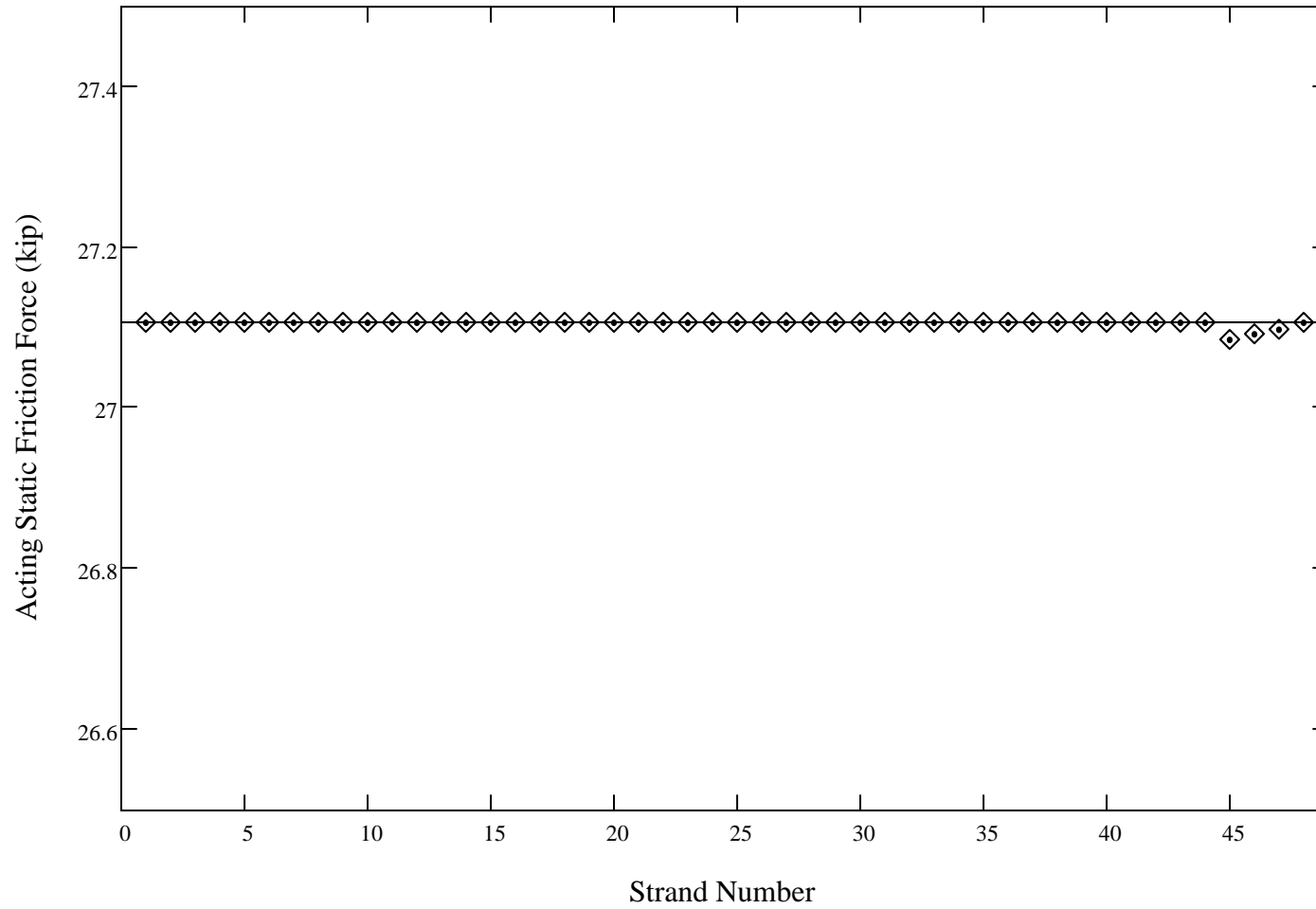


Acting Static Friction Force B 2 End 2

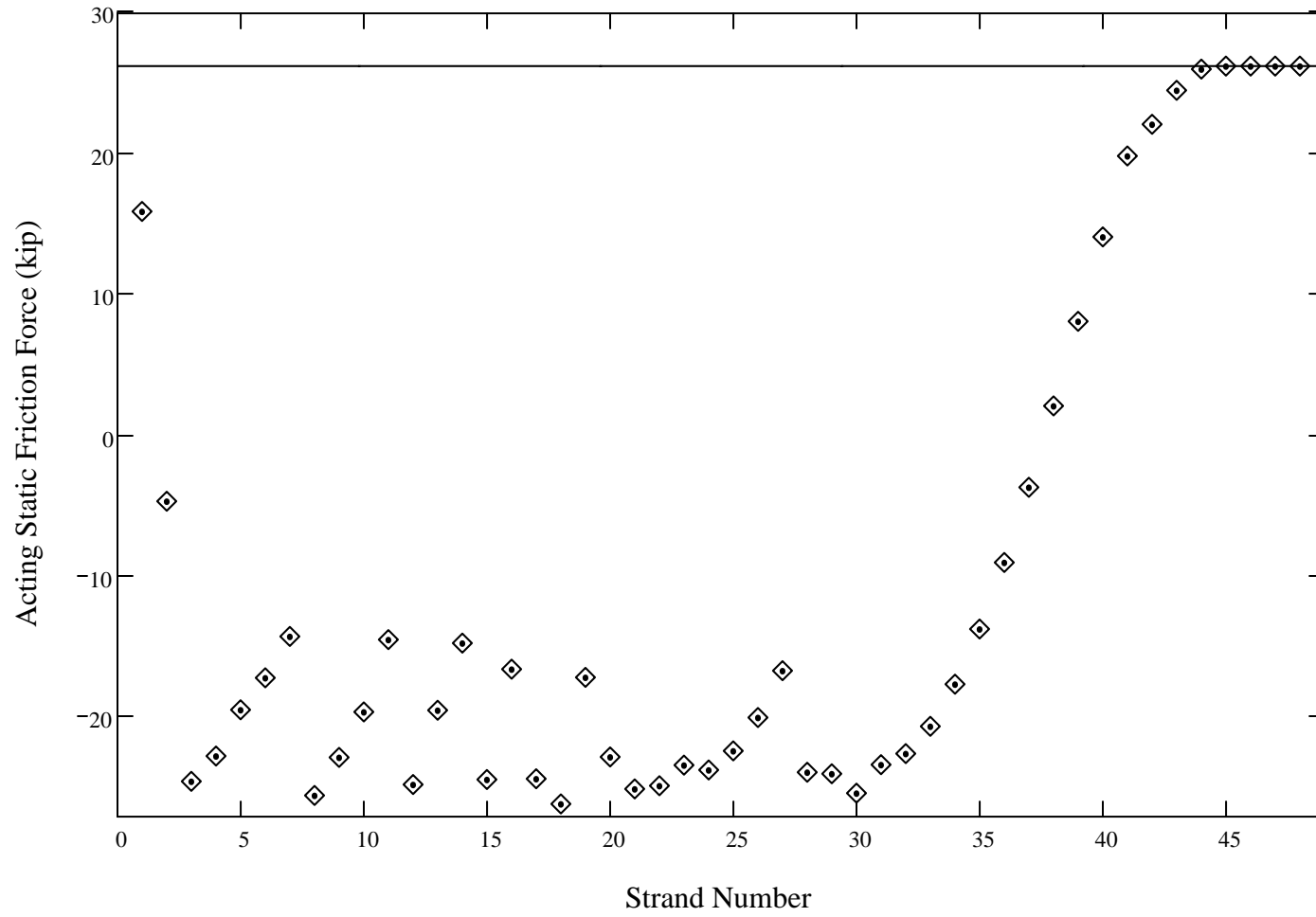




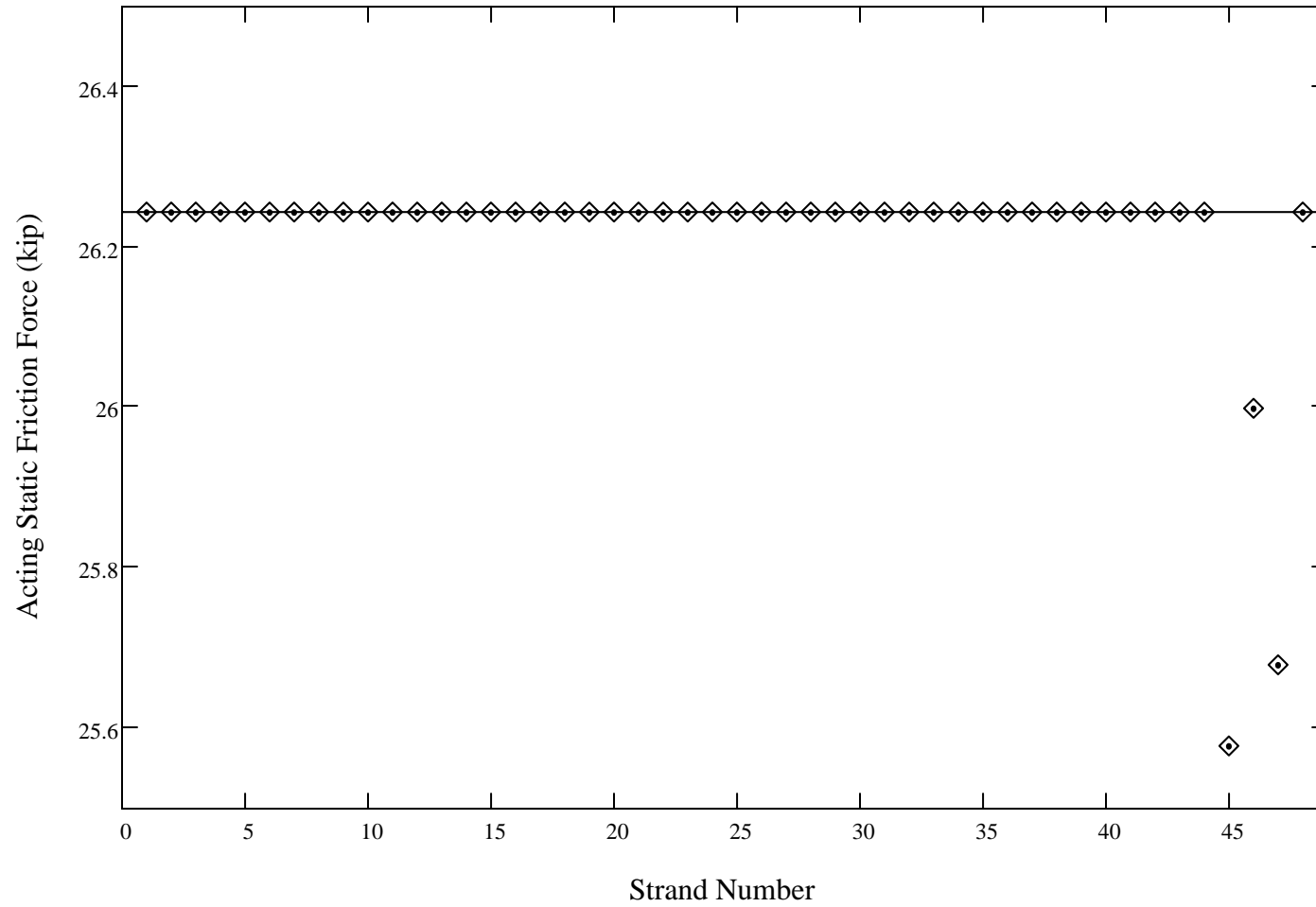
Acting Static Friction Force B 3 End 2



Acting Static Friction Force B 4 End 1



Acting Static Friction Force B 4 End 2



Total Tension Pull Force in Each Free Strand Set

	1	2	3	4	5
1	-56.25	-45.6	-45.24	-45.61	-55.92
2	-47.81	-15.88	-15.15	-15.86	-46.74
3	-38.2	0.54	13.61	12.9	-37.86
4	-27.45	17.43	23.23	22.64	-26.32
5	-16.56	28.23	35.3	30.53	-15.15
6	-6.04	38.51	45.9	39.06	-4.35
7	3.64	51.36	57.51	46.53	6.06
8	13.44	59.93	66.34	66.34	14.59
9	22.39	73.77	73.77	73.31	24.26
10	32.43	72.46	83.94	79.36	33.53
11	40.6	83.99	93.61	83.1	42.4
12	48.39	93.34	100.35	100.35	49.36
13	55.81	103.72	107.59	103.17	57.46
14	62.84	110.5	116.21	106.08	65.14
15	69.48	115.88	121.51	121.51	70.88
16	75.72	123.16	128.56	120.54	77.75
17	81.57	126.54	133.25	133.25	82.67
18	87	131.69	139.55	139.55	87.19
19	92.03	135.91	143.93	136.22	92.84
20	95.15	146.05	146.54	145.53	96.51
21	99.39	144.39	151.05	151.05	99.76
22	103.2	143.99	154.04	153.67	102.6
23	105.07	153.66	154.86	154.62	105.02
24	106.55	156.64	157.01	156.96	107.01
25	107.64	158.19	158.78	157.16	108.56
26	108.33	157.54	160.26	155.91	109.68
27	108.61	154.85	161.32	153.26	110.35

TensPullPerStrand =

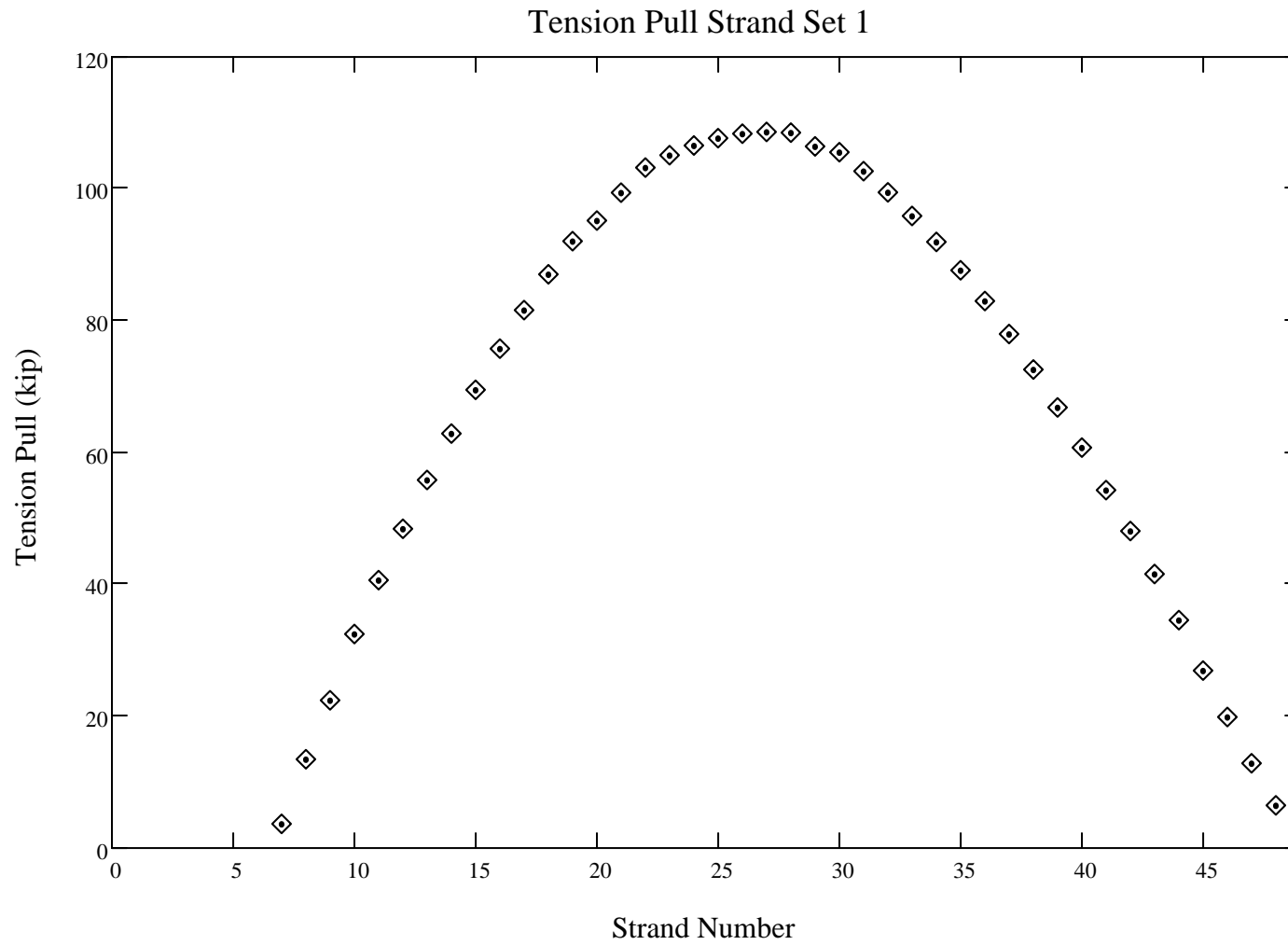
kip

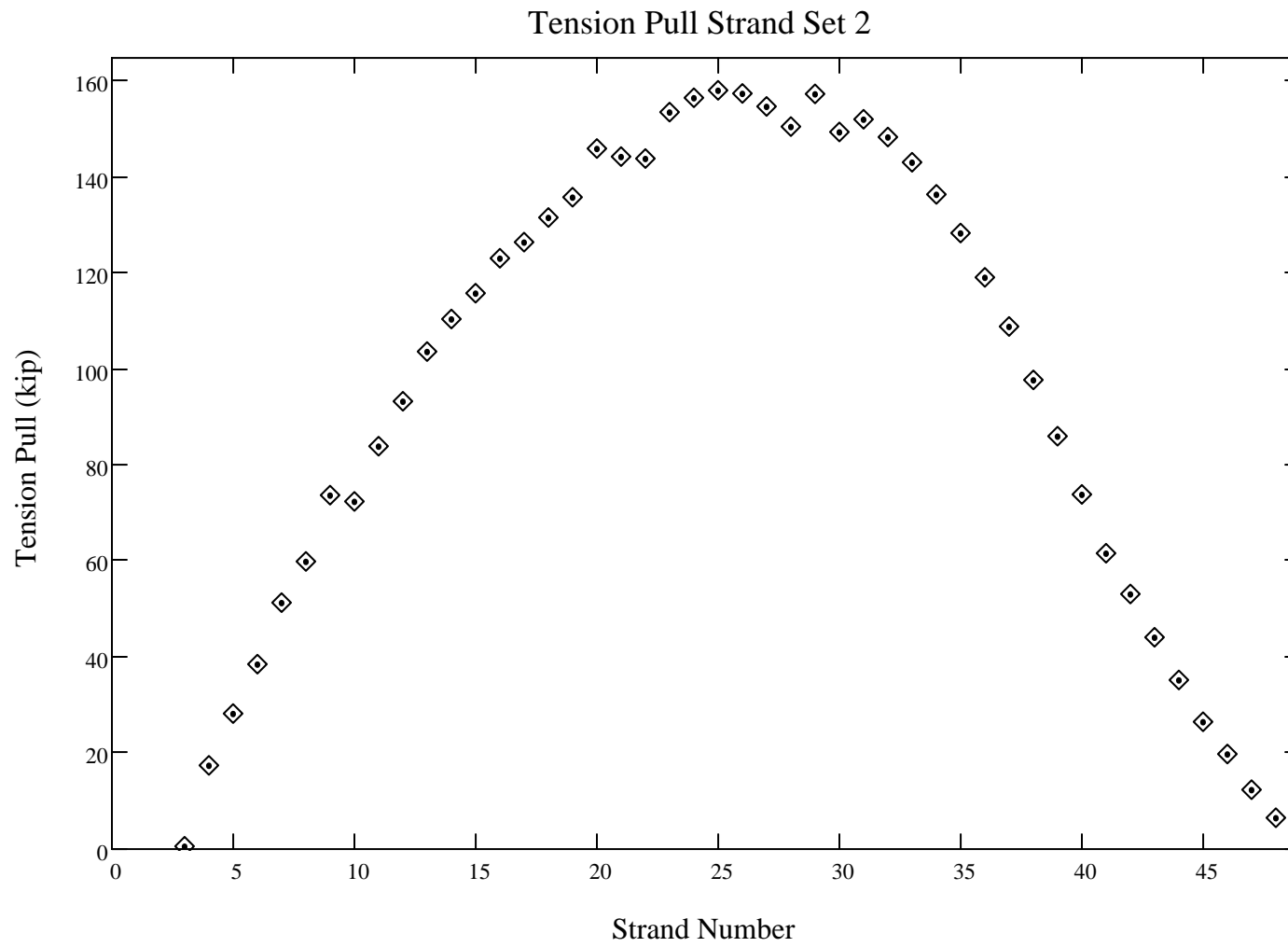


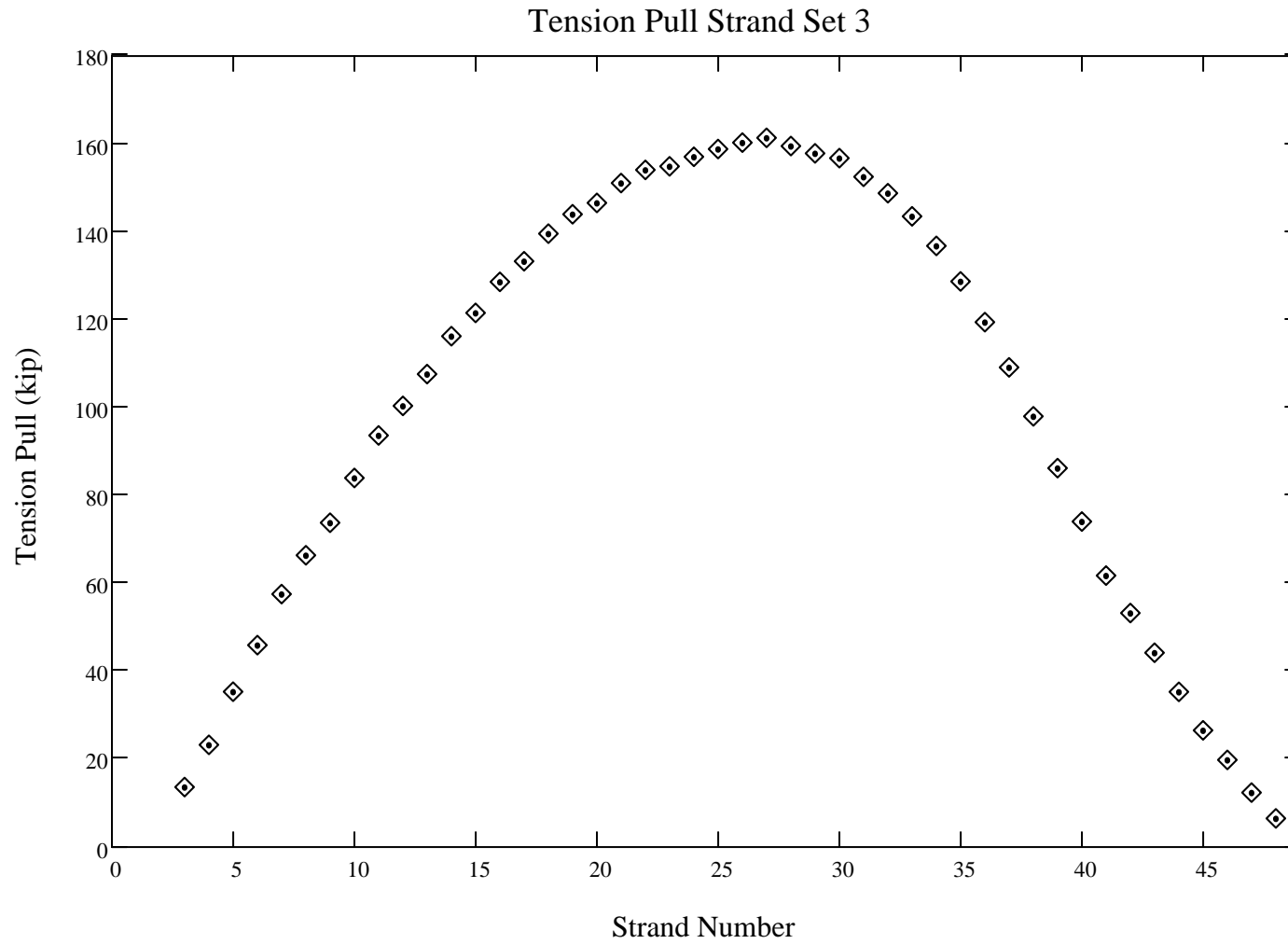
	1	2	3	4	5
1	-56.252	-45.601	-45.243	-45.606	-55.923
2	-47.81	-15.882	-15.146	-15.863	-46.74
3	-38.198	0.545	13.61	12.898	-37.86
4	-27.45	17.432	23.226	22.643	-26.316
5	-16.563	28.233	35.297	30.529	-15.146
6	-6.037	38.513	45.896	39.057	-4.354
7	3.64	51.364	57.509	46.528	6.056
8	13.444	59.931	66.339	66.339	14.587
9	22.389	73.766	73.766	73.309	24.257
10	32.428	72.464	83.937	79.363	33.53
11	40.598	83.991	93.61	83.102	42.402
12	48.392	93.336	100.348	100.348	49.365
13	55.806	103.716	107.588	103.174	57.459
14	62.836	110.501	116.206	106.083	65.137
15	69.476	115.875	121.506	121.506	70.88
16	75.721	123.163	128.559	120.545	77.752
17	81.565	126.538	133.246	133.246	82.671
18	87.004	131.686	139.55	139.55	87.194
19	92.031	135.912	143.934	136.222	92.842
20	95.151	146.047	146.537	145.534	96.507
21	99.39	144.387	151.054	151.054	99.761
22	103.199	143.986	154.042	153.666	102.599
23	105.069	153.66	154.857	154.617	105.016
24	106.551	156.644	157.008	156.956	107.006
25	107.638	158.191	158.775	157.158	108.562

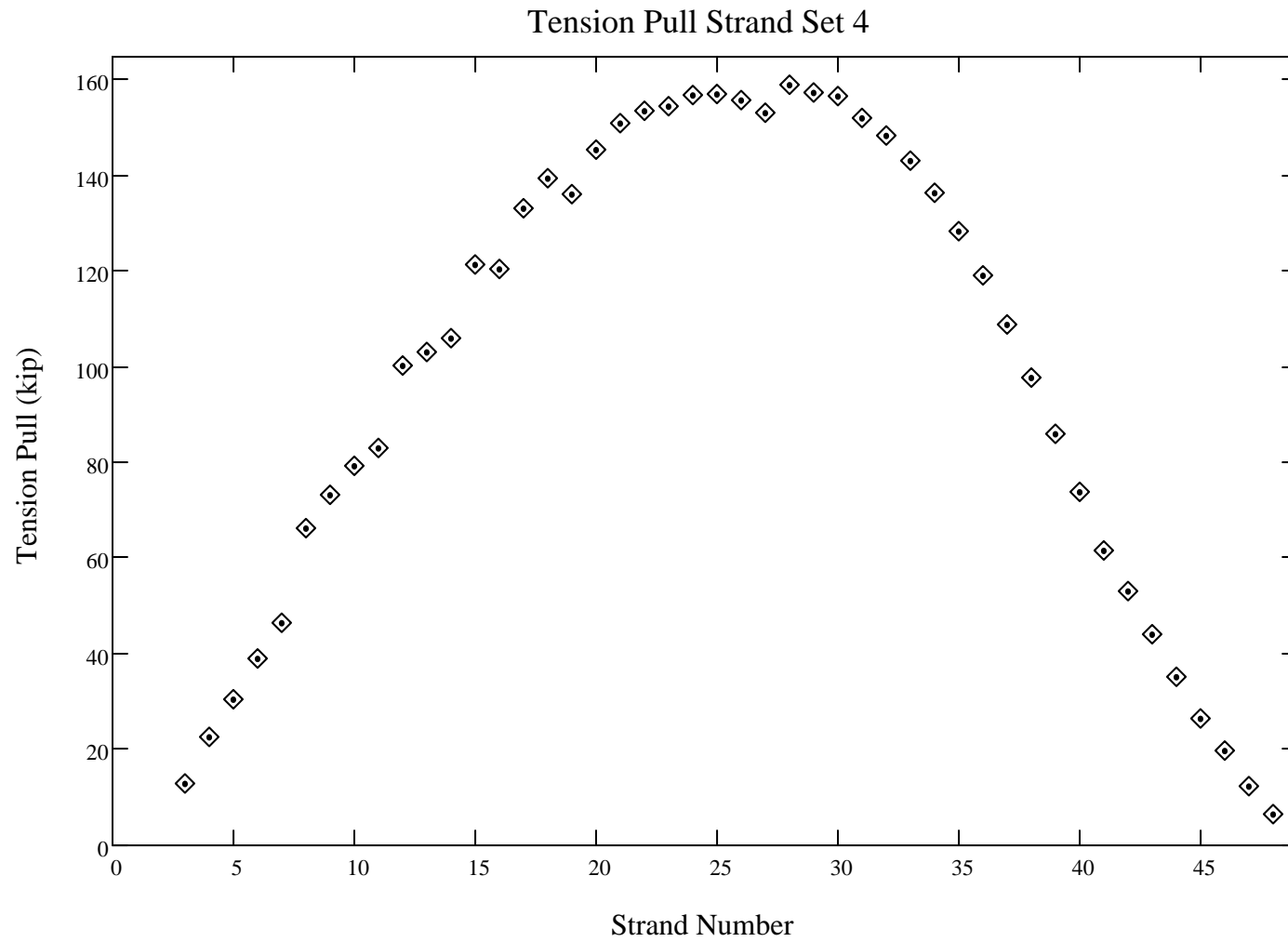
TensPullPerStrand =

kip

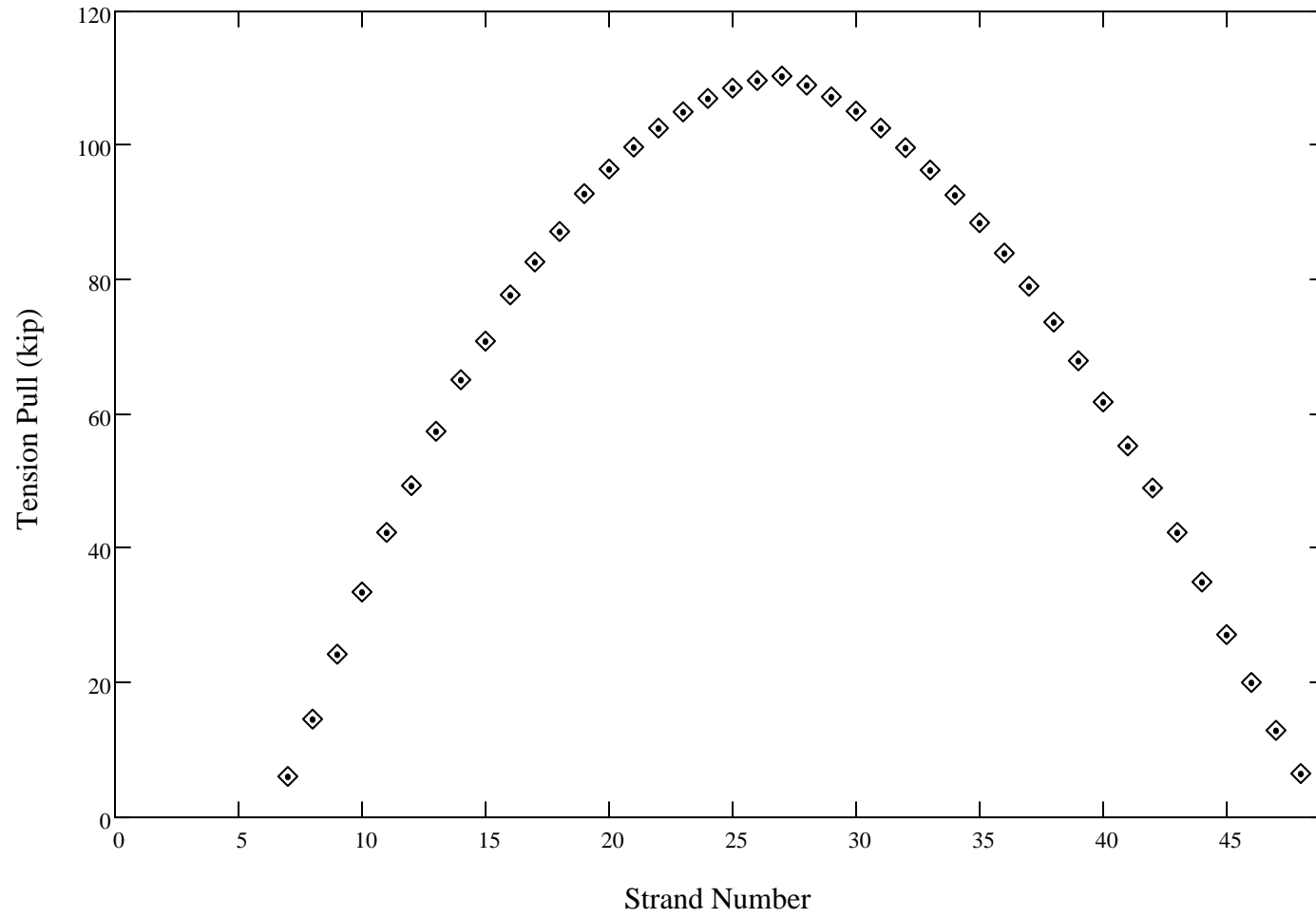








Tension Pull Strand Set 5



Tension Pull in free strand sets "TP"  
 (By Strand Set)

	1	2	3	4	5
1	-46.877	-38	-37.703	-38.005	-46.602
2	-39.672	-13.178	-12.568	-13.163	-38.784
3	-31.555	0.45	11.243	10.655	-31.275
4	-22.57	14.333	19.097	18.617	-21.638
5	-13.552	23.1	28.879	24.978	-12.393
6	-4.914	31.348	37.357	31.791	-3.544
7	2.946	41.58	46.555	37.666	4.903
8	10.821	48.237	53.395	53.395	11.741
9	17.911	59.013	59.013	58.647	19.406
10	25.776	57.6	66.719	63.084	26.652
11	32.051	66.309	73.903	65.607	33.475
12	37.929	73.155	78.651	78.651	38.691
13	43.405	80.668	83.68	80.246	44.691
14	48.473	85.244	89.645	81.836	50.248
15	53.129	88.611	92.916	92.916	54.203
16	57.364	93.305	97.393	91.322	58.903
17	61.174	94.904	99.934	99.934	62.003
18	64.551	97.703	103.537	103.537	64.693
19	67.489	99.669	105.552	99.896	68.084
20	68.903	105.758	106.113	105.387	69.884
21	70.993	103.133	107.895	107.895	71.258
22	72.621	101.323	108.4	108.135	72.199
23	72.74	106.38	107.209	107.043	72.703
24	72.454	106.518	106.766	106.73	72.764
25	71.759	105.461	105.85	104.772	72.375
26	70.648	102.745	104.52	101.678	71.53
27	69.116	98.539	102.66	97.529	70.224

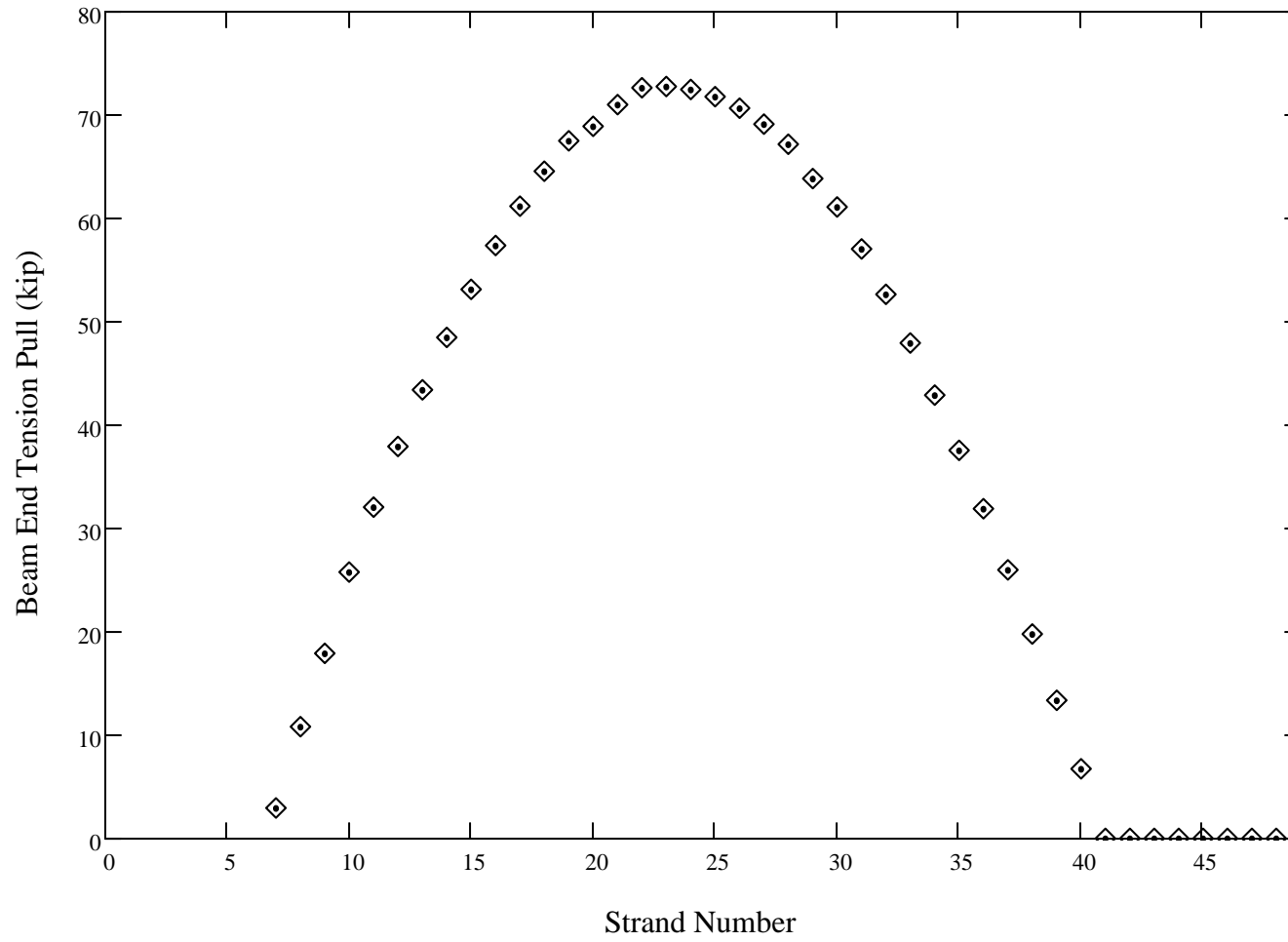
TensPullEndPerStrand =

kip

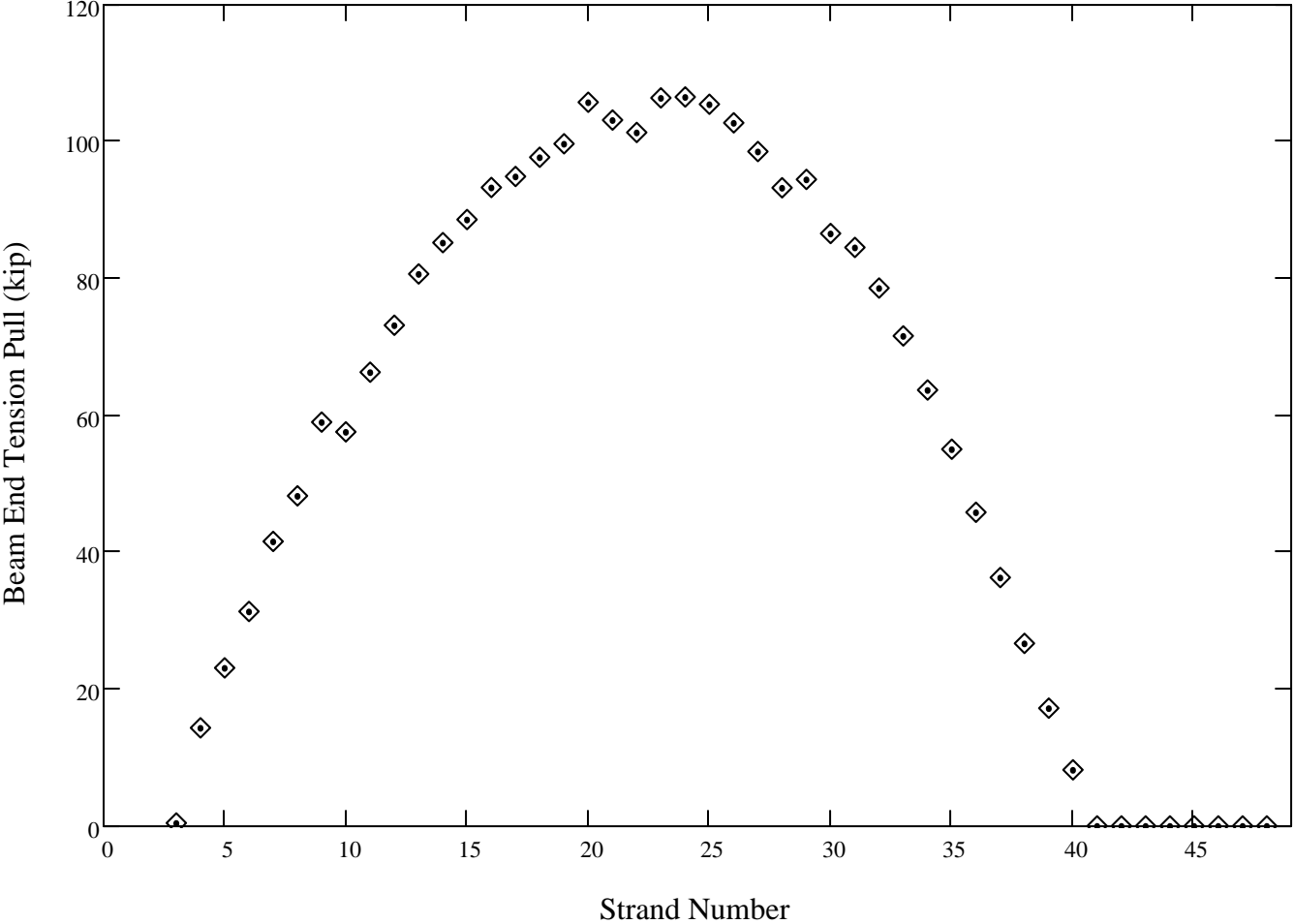
	1	2	3	4	5		
1	-46.877	-38	-37.703	-38.005	-46.602		
2	-39.672	-13.178	-12.568	-13.163	-38.784		
3	-31.555	0.45	11.243	10.655	-31.275		
4	-22.57	14.333	19.097	18.617	-21.638		
5	-13.552	23.1	28.879	24.978	-12.393		
6	-4.914	31.348	37.357	31.791	-3.544		
7	2.946	41.58	46.555	37.666	4.903		
8	10.821	48.237	53.395	53.395	11.741		
9	17.911	59.013	59.013	58.647	19.406		
10	25.776	57.6	66.719	63.084	26.652		
11	32.051	66.309	73.903	65.607	33.475		
12	37.929	73.155	78.651	78.651	38.691		
TensPullEndPerStrand =	13	43.405	80.668	83.68	80.246	44.691	kip
	14	48.473	85.244	89.645	81.836	50.248	
	15	53.129	88.611	92.916	92.916	54.203	
	16	57.364	93.305	97.393	91.322	58.903	
	17	61.174	94.904	99.934	99.934	62.003	
	18	64.551	97.703	103.537	103.537	64.693	
	19	67.489	99.669	105.552	99.896	68.084	
	20	68.903	105.758	106.113	105.387	69.884	
	21	70.993	103.133	107.895	107.895	71.258	
	22	72.621	101.323	108.4	108.135	72.199	
	23	72.74	106.38	107.209	107.043	72.703	
	24	72.454	106.518	106.766	106.73	72.764	
	25	71.759	105.461	105.85	104.772	72.375	
	26	70.648	102.745	104.52	101.678	71.53	



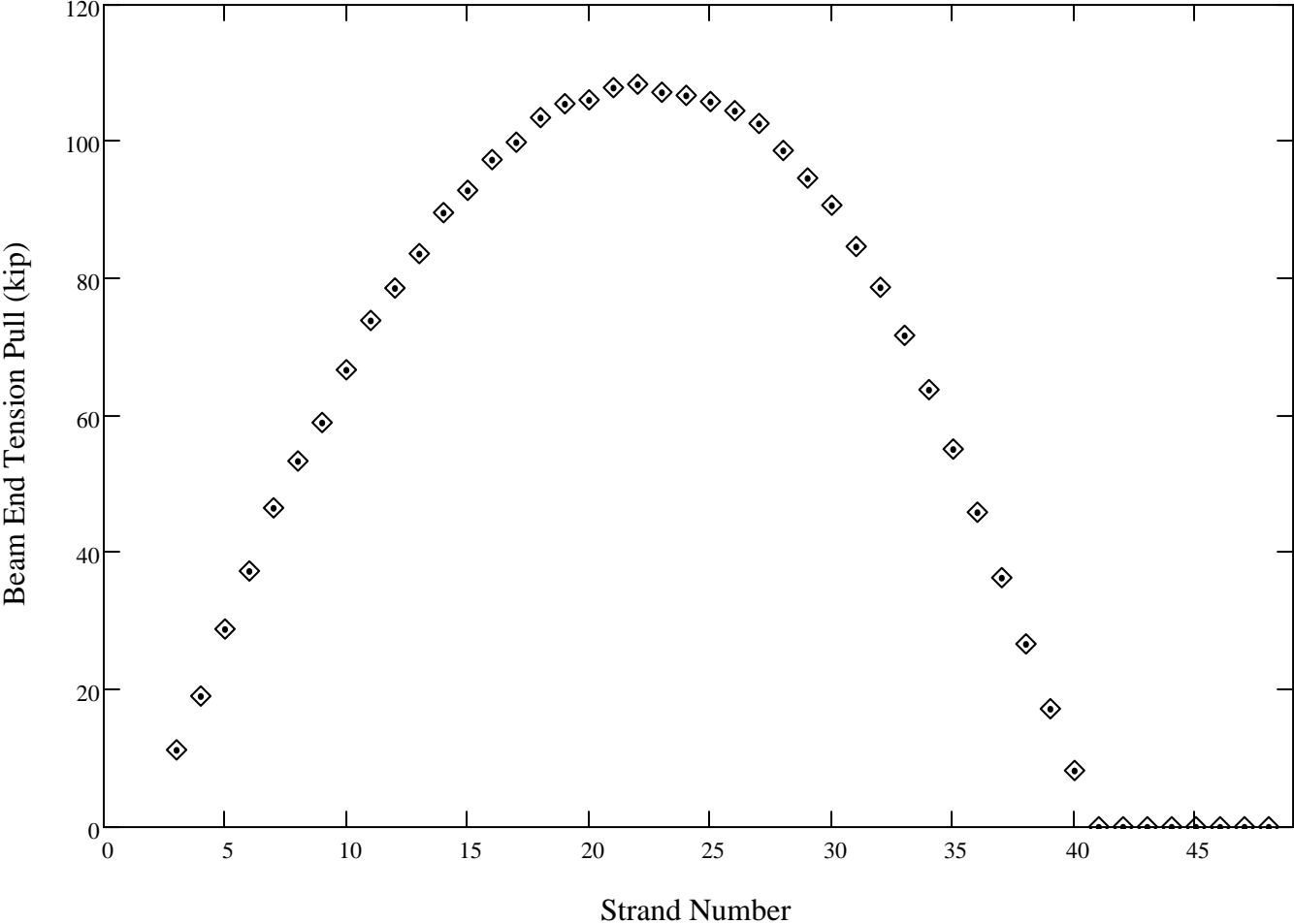
Beam End Tension Pull Strand Set 1



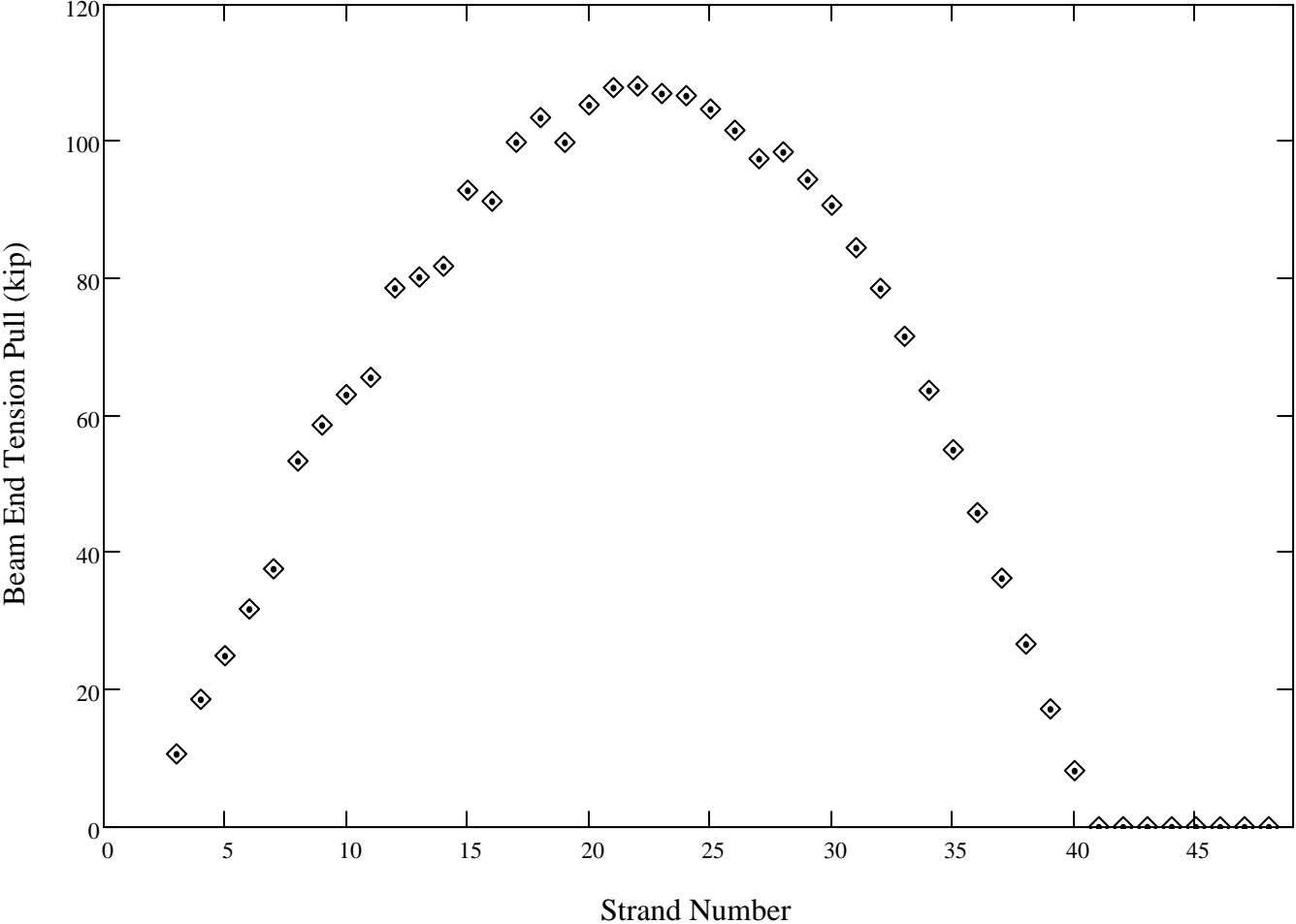
Beam End Tension Pull Strand Set 2



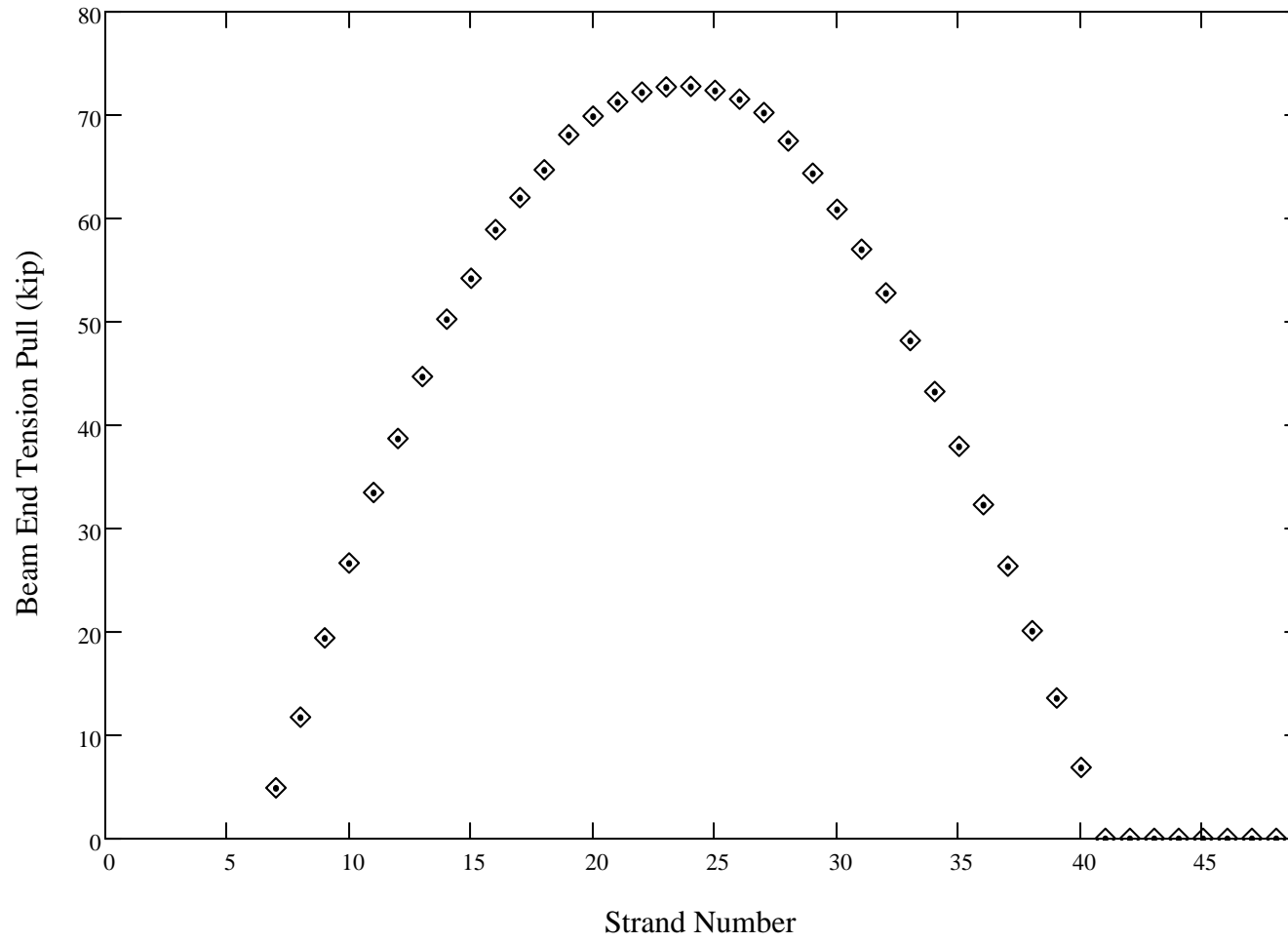
Beam End Tension Pull Strand Set 3



Beam End Tension Pull Strand Set 4



Beam End Tension Pull Strand Set 5



Transferred Prestress Force Linearly Interpolated at Reverse Transfer Length "CRTL"

	1	2	3	4	5	6	7	8
1	-1.172	-0.95	-0.95	-0.943	-0.943	-0.95	-0.95	-1.165
2	-2.034	-0.676	-0.676	-0.645	-0.645	-0.675	-0.675	-1.989
3	-2.491	0.036	0.036	0.888	0.888	0.841	0.841	-2.469
4	-2.44	1.55	1.55	2.065	2.065	2.013	2.013	-2.339
5	-1.882	3.208	3.208	4.011	4.011	3.469	3.469	-1.721
6	-0.842	5.374	5.374	6.404	6.404	5.45	5.45	-0.608
7	0.607	8.561	8.561	9.585	9.585	7.755	7.755	1.009
8	2.623	11.694	11.694	12.944	12.944	12.944	12.944	2.846
9	5.038	16.597	16.597	16.597	16.597	16.494	16.494	5.458
10	8.315	18.581	18.581	21.522	21.522	20.35	20.35	8.597
11	11.752	24.313	24.313	27.098	27.098	24.056	24.056	12.274
12	15.695	30.271	30.271	32.545	32.545	32.545	32.545	16.01
13	20.152	37.453	37.453	38.851	38.851	37.257	37.257	20.749
14	25.134	44.2	44.2	46.482	46.482	42.433	42.433	26.055
15	30.651	51.122	51.122	53.606	53.606	53.606	53.606	31.271
16	36.713	59.715	59.715	62.332	62.332	58.446	58.446	37.698
17	43.331	67.223	67.223	70.787	70.787	70.787	70.787	43.919
18	50.518	76.463	76.463	81.029	81.029	81.029	81.029	50.629
19	58.286	86.077	86.077	91.158	91.158	86.274	86.274	58.8
20	65.622	100.722	100.722	101.06	101.06	100.369	100.369	66.557
21	74.543	108.29	108.29	113.29	113.29	113.29	113.29	74.821
22	84.088	117.322	117.322	125.515	125.515	125.209	125.209	83.599
23	92.946	135.93	135.93	136.989	136.989	136.777	136.777	92.899
24	102.288	150.378	150.378	150.728	150.728	150.678	150.678	102.725
25	112.123	164.783	164.783	165.391	165.391	163.707	163.707	113.086
26	122.456	178.092	178.092	181.168	181.168	176.243	176.243	123.986
27	133.295	190.04	190.04	197.988	197.988	188.091	188.091	135.432

CompAtRTLPerStrand =

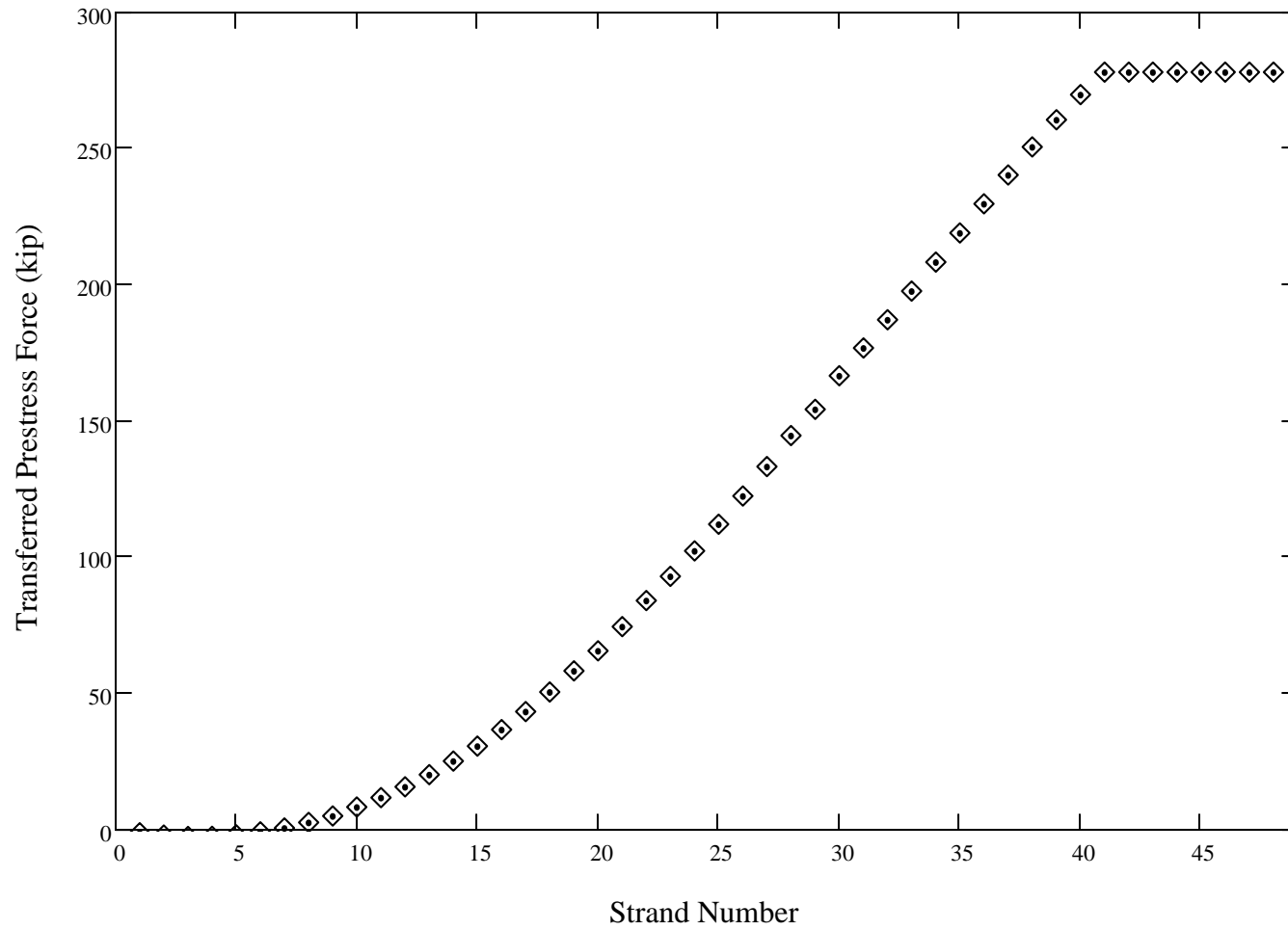
kip

	1	2	3	4	5	6	7	8
1	-1.172	-0.95	-0.95	-0.943	-0.943	-0.95	-0.95	-1.165
2	-2.034	-0.676	-0.676	-0.645	-0.645	-0.675	-0.675	-1.989
3	-2.491	0.036	0.036	0.888	0.888	0.841	0.841	-2.469
4	-2.44	1.55	1.55	2.065	2.065	2.013	2.013	-2.339
5	-1.882	3.208	3.208	4.011	4.011	3.469	3.469	-1.721
6	-0.842	5.374	5.374	6.404	6.404	5.45	5.45	-0.608
7	0.607	8.561	8.561	9.585	9.585	7.755	7.755	1.009
8	2.623	11.694	11.694	12.944	12.944	12.944	12.944	2.846
9	5.038	16.597	16.597	16.597	16.597	16.494	16.494	5.458
10	8.315	18.581	18.581	21.522	21.522	20.35	20.35	8.597
11	11.752	24.313	24.313	27.098	27.098	24.056	24.056	12.274
12	15.695	30.271	30.271	32.545	32.545	32.545	32.545	16.01
13	20.152	37.453	37.453	38.851	38.851	37.257	37.257	20.749
14	25.134	44.2	44.2	46.482	46.482	42.433	42.433	26.055
15	30.651	51.122	51.122	53.606	53.606	53.606	53.606	31.271
16	36.713	59.715	59.715	62.332	62.332	58.446	58.446	37.698
17	43.331	67.223	67.223	70.787	70.787	70.787	70.787	43.919
18	50.518	76.463	76.463	81.029	81.029	81.029	81.029	50.629
19	58.286	86.077	86.077	91.158	91.158	86.274	86.274	58.8
20	65.622	100.722	100.722	101.06	101.06	100.369	100.369	66.557
21	74.543	108.29	108.29	113.29	113.29	113.29	113.29	74.821
22	84.088	117.322	117.322	125.515	125.515	125.209	125.209	83.599
23	92.946	135.93	135.93	136.989	136.989	136.777	136.777	92.899
24	102.288	150.378	150.378	150.728	150.728	150.678	150.678	102.725
25	112.123	164.783	164.783	165.391	165.391	163.707	163.707	113.086

CompAtRTLPerStrand =

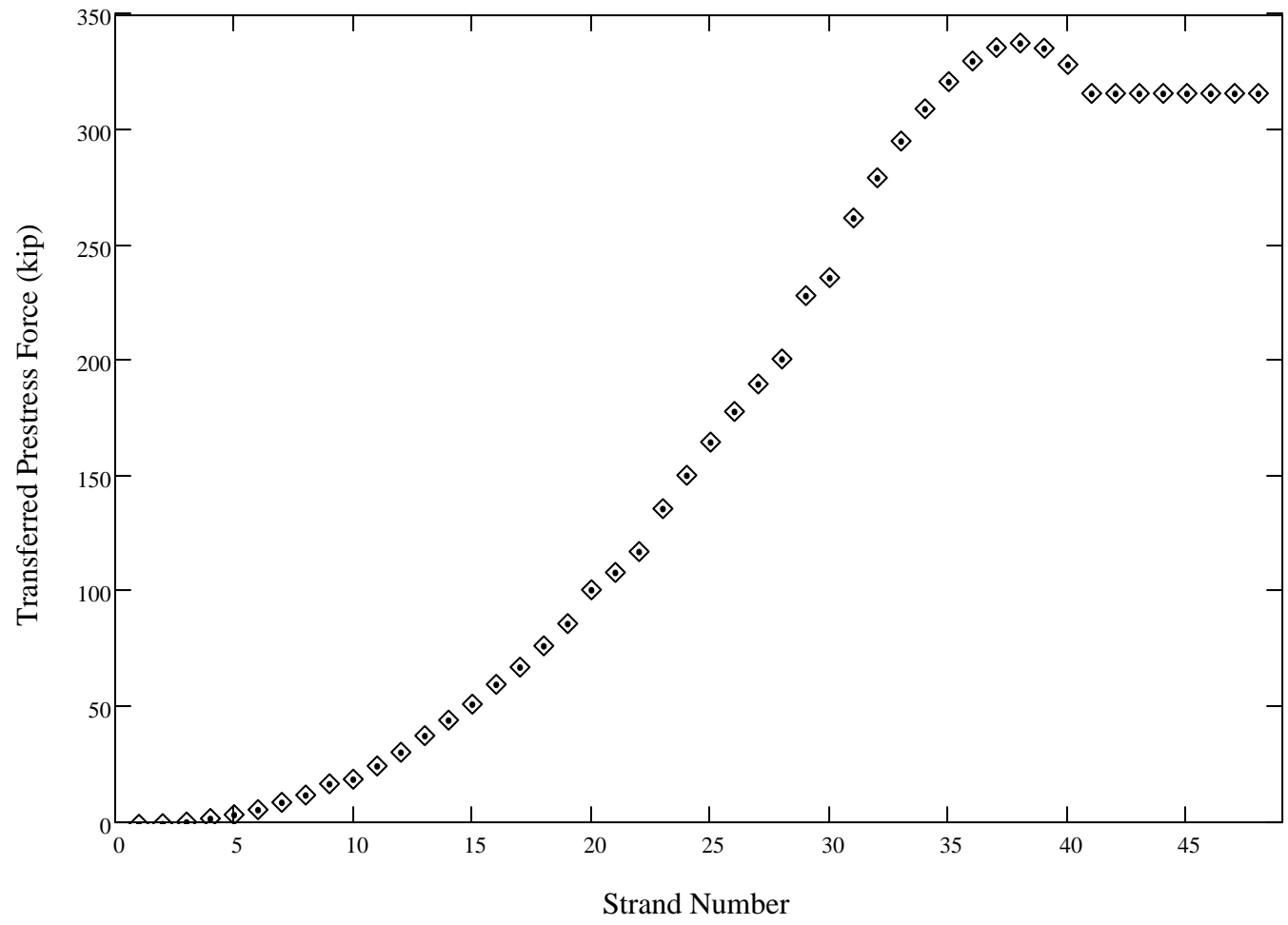
kip

Transferred Prestress at RTL B 1 End 1

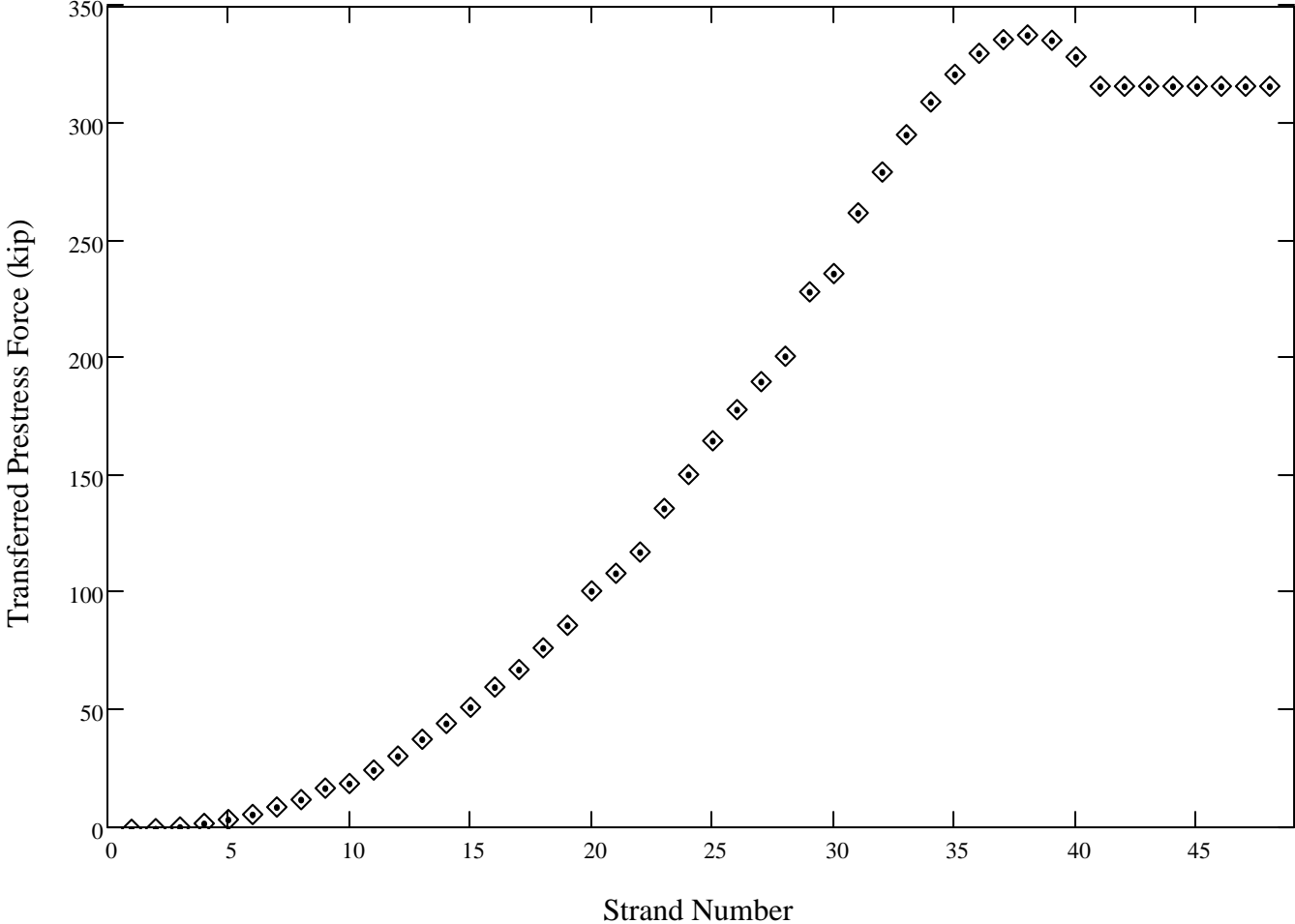




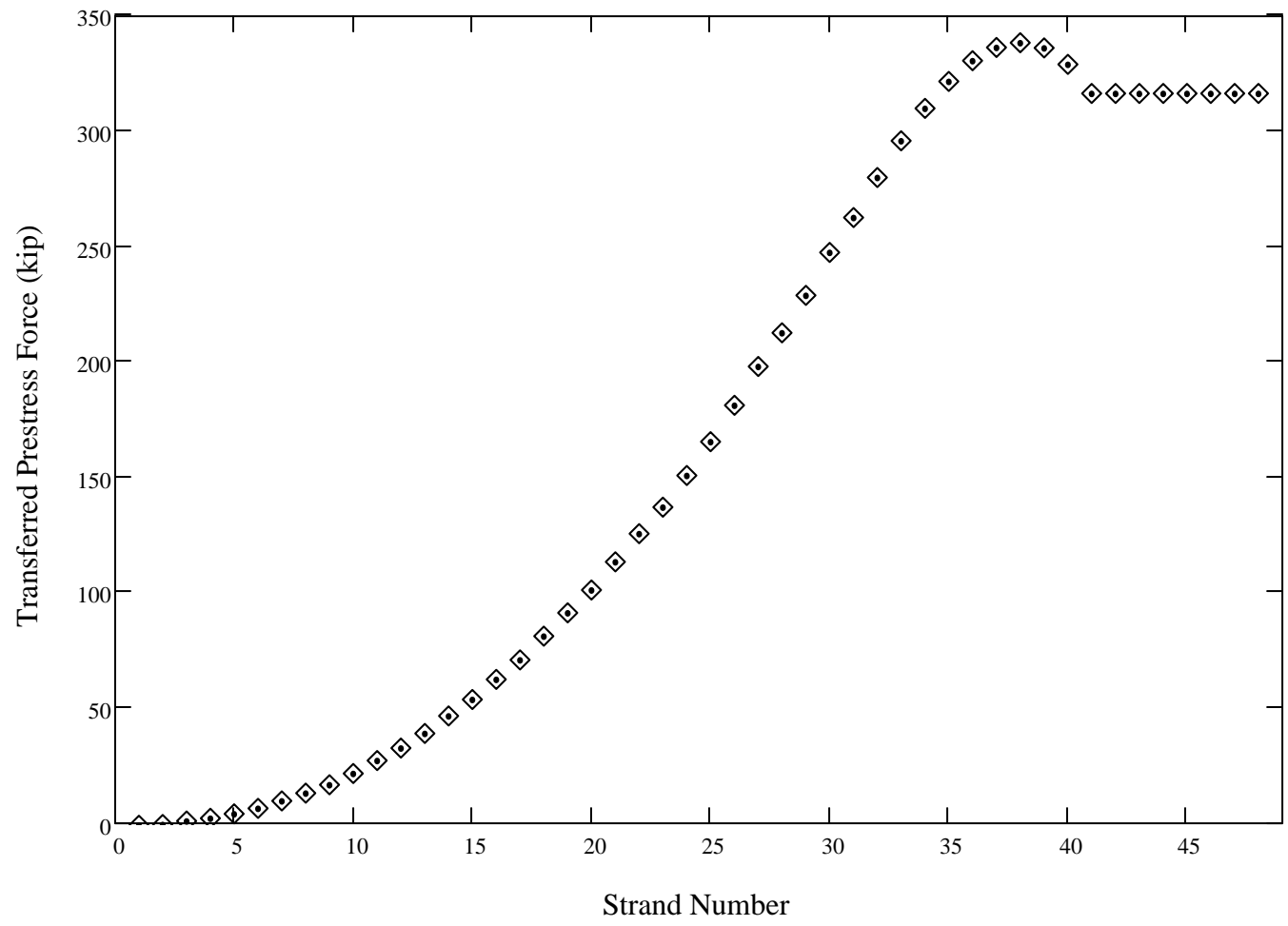
Transferred Prestress at RTL B 1 End 2



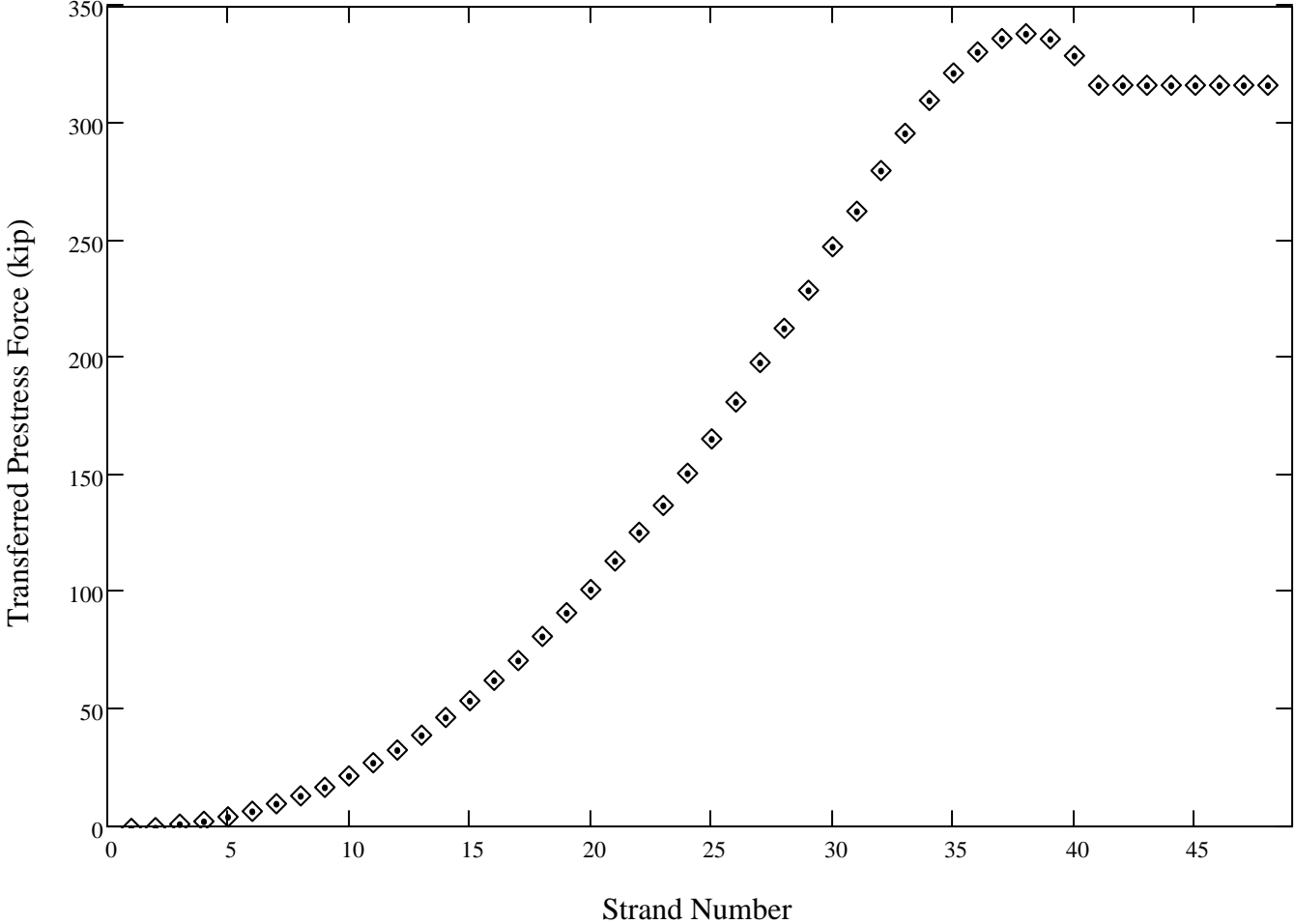
Transferred Prestress at RTL B 2 End 1



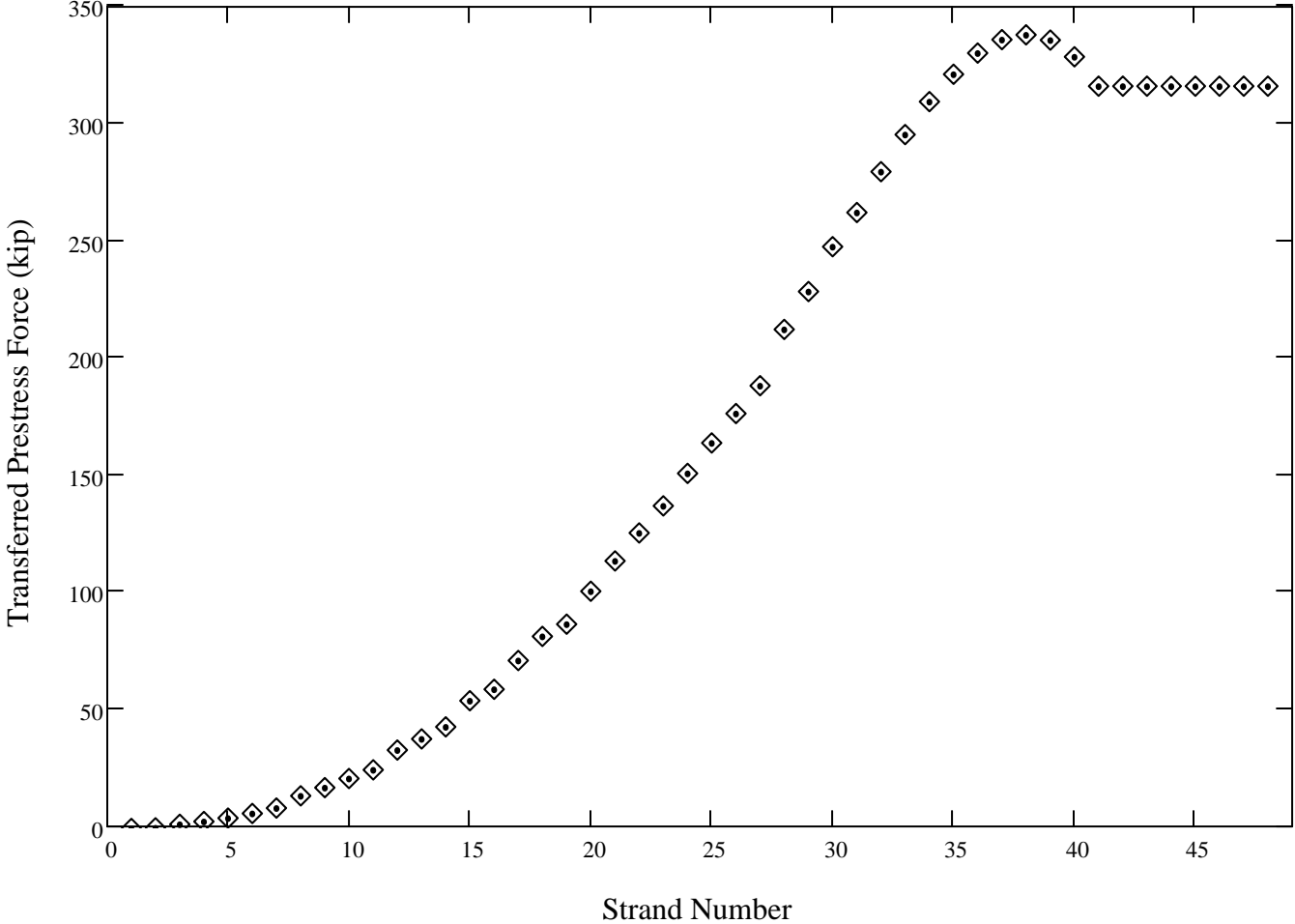
Transferred Prestress at RTL B 2 End 2



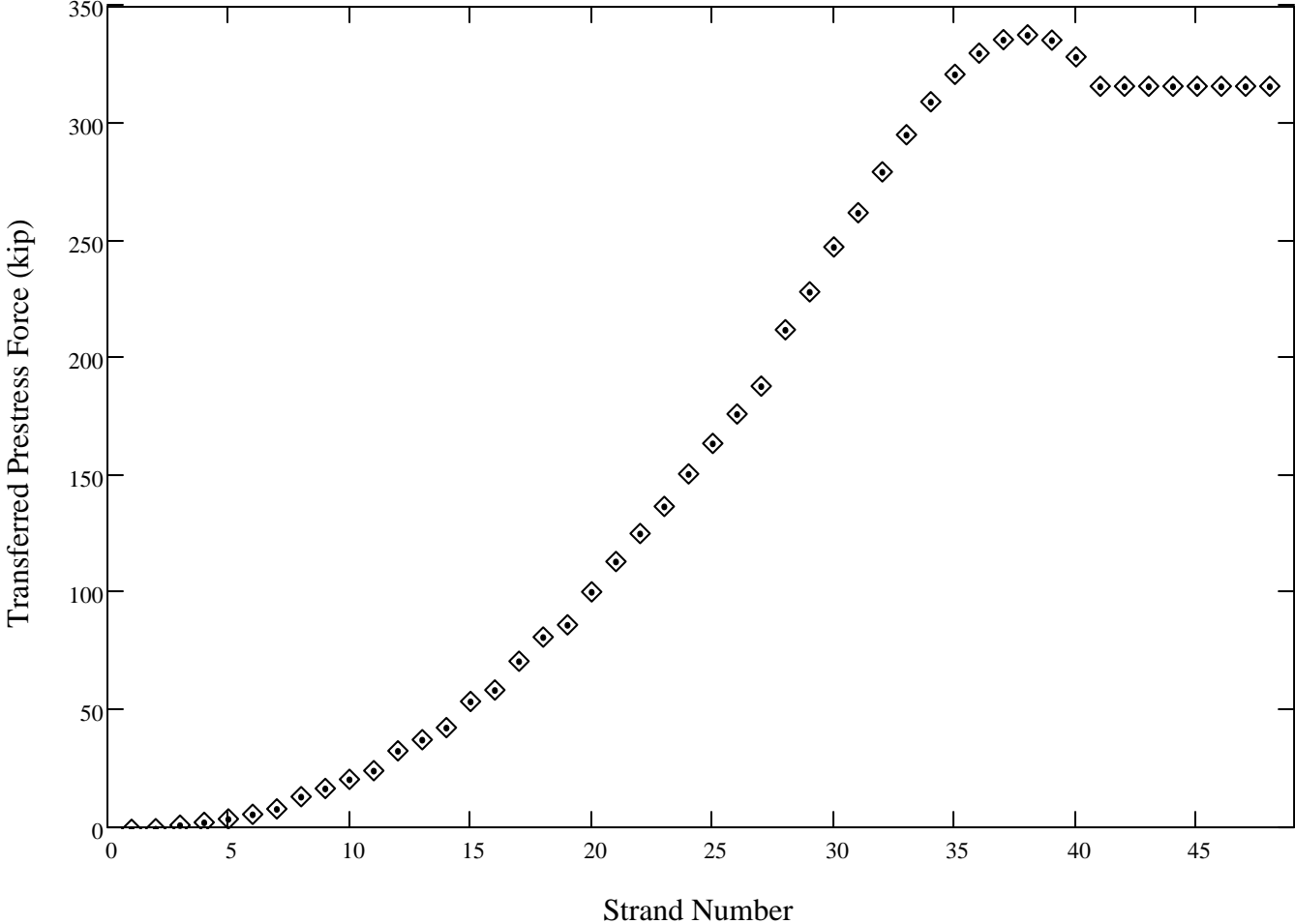
Transferred Prestress at RTL B 3 End 1



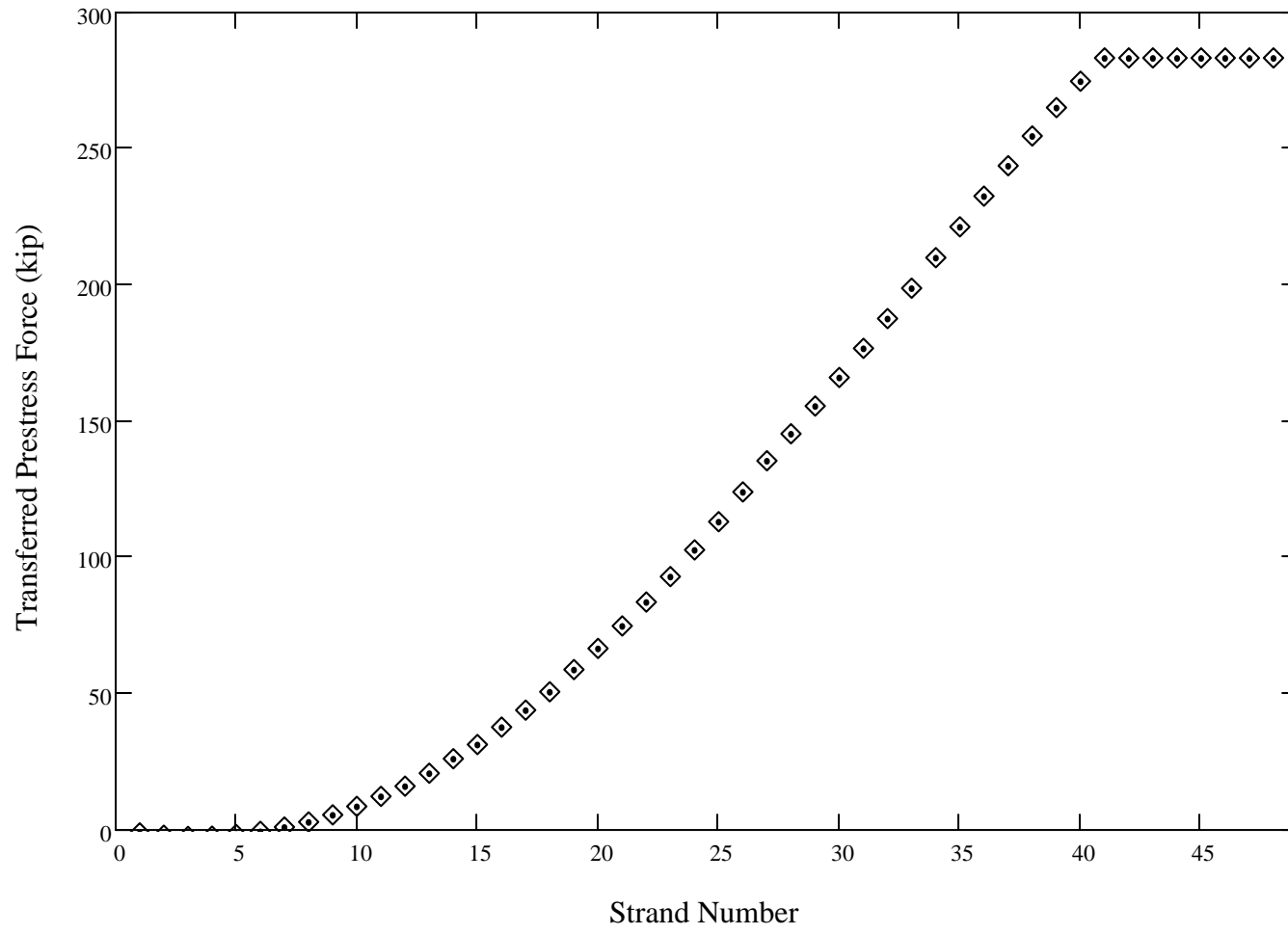
Transferred Prestress at RTL B 3 End 2



Transferred Prestress at RTL B 4 End 1



Transferred Prestress at RTL B 4 End 2



Intermediate calculations to determine cracking potential of each beam end:

$$\text{CTNoTS} := \left| \begin{array}{l} \text{for } k \in 1 \dots \text{cols}(\text{CrackPredictorPerStrand}) \\ \quad \text{for } j \in 1 \dots \text{rows}(\text{CrackPredictorPerStrand}) \\ \quad \quad \text{out}_{j,k} \leftarrow \frac{\text{CrackPredictorPerStrand}_{j,k} - \text{ConcAllowableTension}}{\text{Abf}} \\ \text{out} \end{array} \right.$$

Accounts for eccentricity of friction force

$$\text{CTNoTS2} := \left| \begin{array}{l} \text{for } k \in 1 \dots \text{cols}(\text{CTNoTS}) \\ \quad \text{for } j \in 1 \dots \text{rows}(\text{CTNoTS}) \\ \quad \quad \text{out}_{j,k} \leftarrow \text{CTNoTS}_{j,k} - \frac{\text{FrictionValuePerStrand}_{j,k} \cdot \text{Frice}^2}{\text{IBottom}} \\ \text{out} \end{array} \right.$$



Increases the size of the matrix

```

TensTransferLengthw :=
  for q ∈ 1 .. rows(TensTransferLength)
  |
  | if DebondLengthq,1 = 0ft
  |   |
  |   | outq,1 ← TensTransferLengthq,1
  |   | outq,2 cols(TensTransferLength)-2 ← TensTransferLengthq, cols(TensTransferLength)
  |   |
  |   | if DebondLengthq,1 ≠ 0ft
  |   |   |
  |   |   | outq,1 ← outq-1,1
  |   |   | outq,2 cols(TensTransferLength)-2 ← outq-1,2 cols(TensTransferLength)-2
  |   |
  |   | for j ∈ 2 .. cols(TensTransferLength) - 1
  |   |   |
  |   |   | for z ∈ 1 .. rows(TensTransferLength)
  |   |   |   |
  |   |   |   | if DebondLengthz,1 = 0ft
  |   |   |   |   |
  |   |   |   |   | outz,2·j-2 ← TensTransferLengthz,j
  |   |   |   |   | outz,2·j-1 ← TensTransferLengthz,j
  |   |   |   |   |
  |   |   |   |   | if DebondLengthz,1 ≠ 0ft
  |   |   |   |   |   |
  |   |   |   |   |   | outz,2·j-2 ← outz-1,2·j-2
  |   |   |   |   |   | outz,2·j-1 ← outz-1,2·j-1
  |   |   |   |
  |   |   |
  |   |
  | out

```

Accounts for eccentricity of bearing force (assuming that it acts at the very end of the beam)

$$\text{CTNoTS3} := \begin{array}{l} \text{for } k \in 1 \dots \text{cols}(\text{CTNoTS2}) \\ \text{for } j \in 1 \dots \text{rows}(\text{CTNoTS2}) \\ \text{out}_{j,k} \leftarrow \text{CTNoTS2}_{j,k} - \frac{\text{Bearingw}_{1,k} \cdot \text{TensTransferLengthw}_{j,k} \cdot \text{Frice}}{\text{IBottom}} \\ \text{out} \end{array}$$

Cracking Potential of each beam end bottom flange = Actual Stresses/ Allowable Tension Stress

$$\text{CrackingPotential} := \begin{array}{l} \text{for } k \in 1 \dots \text{cols}(\text{CTNoTS2}) \\ \text{for } j \in 1 \dots \text{rows}(\text{CTNoTS2}) \\ \text{out}_{j,k} \leftarrow \frac{\text{CTNoTS3}_{j,k} \cdot -1}{\text{ConcTensStrength}} \\ \text{out} \end{array}$$

Cracking Criterion: Above 1.0 indicates vertical crack probable

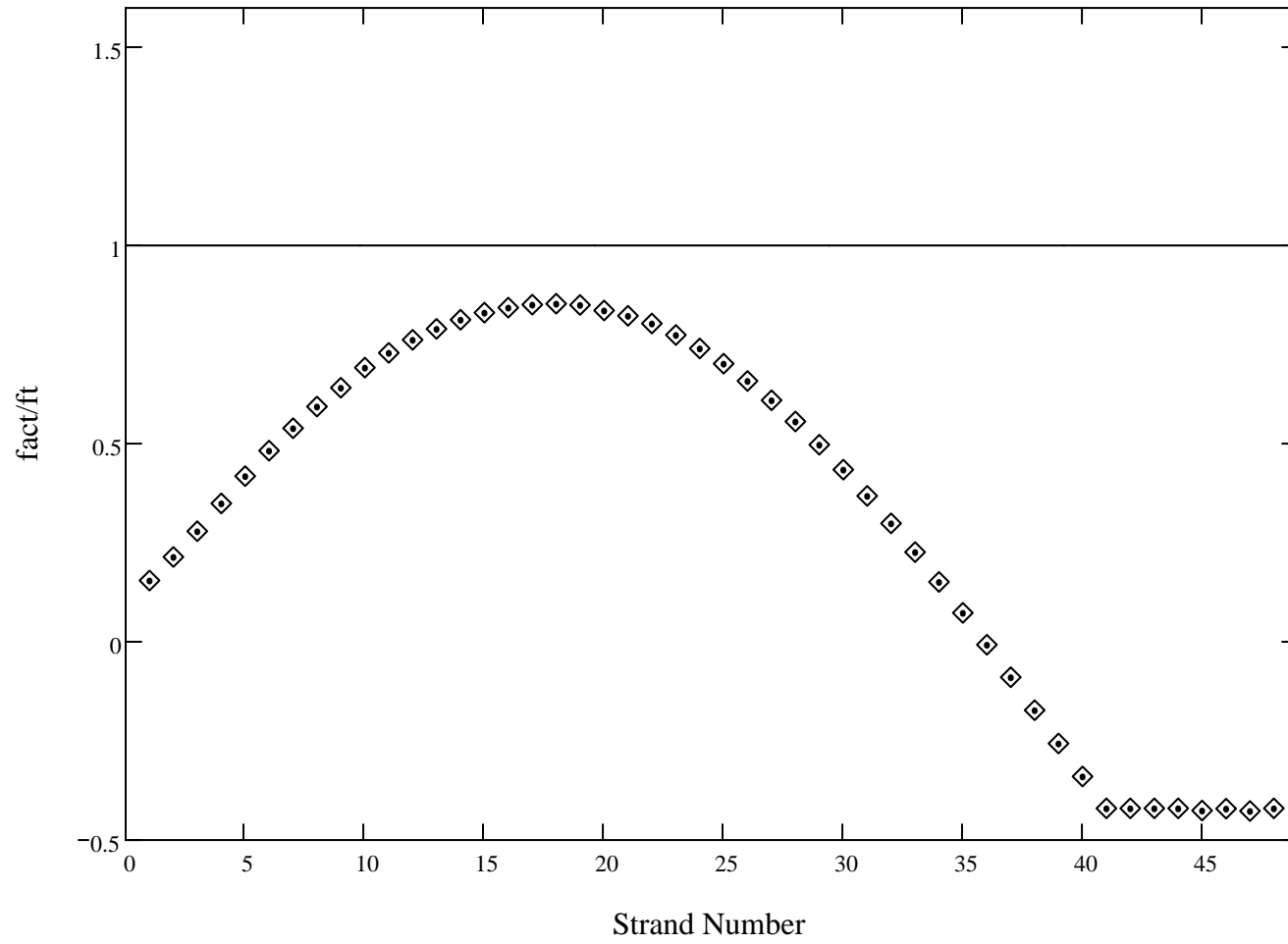
CP = Actual Stress/ Allowable Stress

	1	2	3	4	5	6	7	8
1	0.1543	0.0102	0.2375	0.2327	0.2383	0.2432	0.0226	0.162
2	0.2137	-0.2215	0.4329	0.4229	0.4298	0.4395	-0.1939	0.2262
3	0.2788	-0.2533	0.5383	0.3597	0.6139	0.6236	-0.409	0.2868
4	0.3491	-0.2724	0.6431	0.5633	0.675	0.683	-0.3143	0.3622
5	0.4178	-0.2076	0.707	0.6089	0.6626	0.7287	-0.2036	0.4328
6	0.482	-0.1454	0.7651	0.6616	0.6807	0.7763	-0.1114	0.4987
7	0.5387	-0.1394	0.8355	0.7486	0.6605	0.8154	-0.0141	0.5597
8	0.5936	-0.0731	0.8778	0.7864	0.9233	0.9233	-0.1332	0.6073
9	0.641	-0.1029	0.9468	0.9468	0.9468	0.9534	-0.0498	0.6589
10	0.6915	0.106	0.9291	0.7622	0.9096	0.976	0.0365	0.7054
11	0.7291	0.0878	0.9791	0.8377	0.8298	0.9839	0.1465	0.7467
12	0.7618	0.0903	1.0136	0.9094	1.0597	1.0597	0.0152	0.7753
CrackingPotential = 13	0.7897	0.0657	1.0493	0.9911	0.9908	1.057	0.1173	0.8066
14	0.8126	0.0836	1.0632	0.9764	0.8991	1.0527	0.2078	0.8325
15	0.8304	0.1119	1.067	0.9803	1.1041	1.1041	0.0641	0.8462
16	0.8432	0.0989	1.0757	0.9916	0.9509	1.0756	0.1919	0.8617
17	0.8507	0.1354	1.0636	0.9576	1.1036	1.1036	0.0628	0.8652
18	0.8529	0.132	1.0553	0.9292	1.0987	1.0987	0.0213	0.864
19	0.8497	0.131	1.0391	0.9085	0.9275	1.0527	0.1545	0.8633
20	0.836	-0.0109	1.0389	1.0308	1.0336	1.0502	0.0377	0.8515
21	0.8223	0.0609	0.9932	0.8811	1.0278	1.0278	-0.0311	0.8347
22	0.8028	0.1003	0.9491	0.777	0.9843	0.9908	-0.0645	0.8127
23	0.7739	-0.0788	0.9277	0.9068	0.9401	0.9443	-0.0825	0.7856
24	0.7401	-0.1565	0.8811	0.8746	0.8954	0.8963	-0.1384	0.7532
25	0.7015	-0.2226	0.8252	0.8146	0.8089	0.8382	-0.1703	0.7155
26	0.6579	-0.262	0.7598	0.7093	0.692	0.7726	-0.1892	0.6724

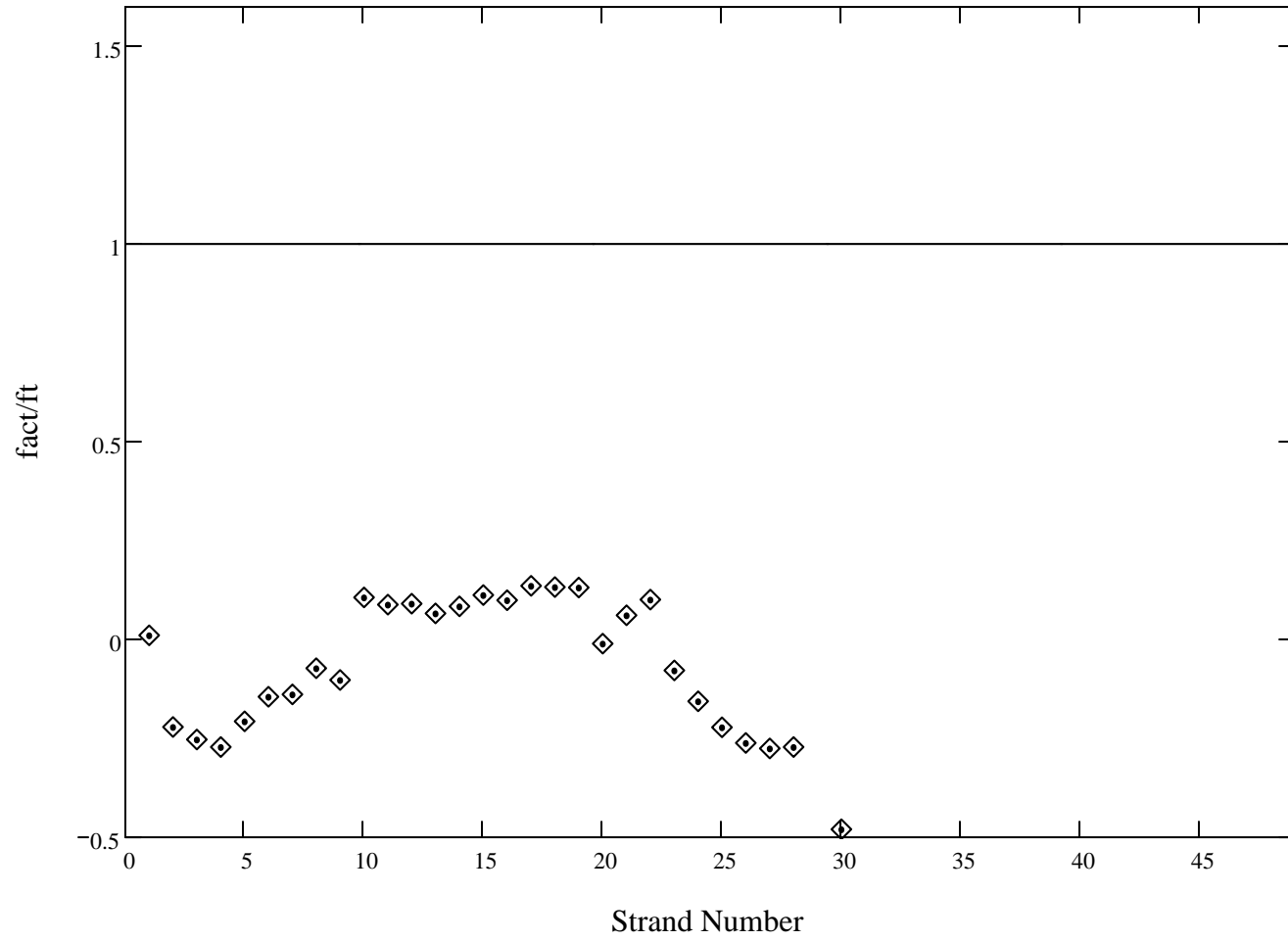
CrackingPotential =

	1	2	3	4	5	6	7	8
1	0.1543	0.0102	0.2375	0.2327	0.2383	0.2432	0.0226	0.162
2	0.2137	-0.2215	0.4329	0.4229	0.4298	0.4395	-0.1939	0.2262
3	0.2788	-0.2533	0.5383	0.3597	0.6139	0.6236	-0.409	0.2868
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5	0.4178	-0.2076	0.707	0.6089	0.6626	0.7287	-0.2036	0.4328
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9	0.641	-0.1029	0.9468	0.9468	0.9468	0.9534	-0.0498	0.6589
10	0.6915	0.106	0.9291	0.7622	0.9096	0.976	0.0365	0.7054
11	0.7291	0.0878	0.9791	0.8377	0.8298	0.9839	0.1465	0.7467
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13	0.7897	0.0657	1.0493	0.9911	0.9908	1.057	0.1173	0.8066
14	0.8126	0.0836	1.0632	0.9764	0.8991	1.0527	0.2078	0.8325
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16	0.8432	0.0989	1.0757	0.9916	0.9509	1.0756	0.1919	0.8617
17	0.8507	0.1354	1.0636	0.9576	1.1036	1.1036	0.0628	0.8652
18	0.8529	0.132	1.0553	0.9292	1.0987	1.0987	0.0213	0.864
19	0.8497	0.131	1.0391	0.9085	0.9275	1.0527	0.1545	0.8633
20	0.836	-0.0109	1.0389	1.0308	1.0336	1.0502	0.0377	0.8515
21	0.8223	0.0609	0.9932	0.8811	1.0278	1.0278	-0.0311	0.8347
22	0.8028	0.1003	0.9491	0.777	0.9843	0.9908	-0.0645	0.8127
23	0.7739	-0.0788	0.9277	0.9068	0.9401	0.9443	-0.0825	0.7856
24	0.7401	-0.1565	0.8811	0.8746	0.8954	0.8963	-0.1384	0.7532
25	0.7015	-0.2226	0.8252	0.8146	0.8089	0.8382	-0.1703	0.7155
26	0.6579	-0.262	0.7598	0.7093	0.692	0.7726	-0.1892	0.6724

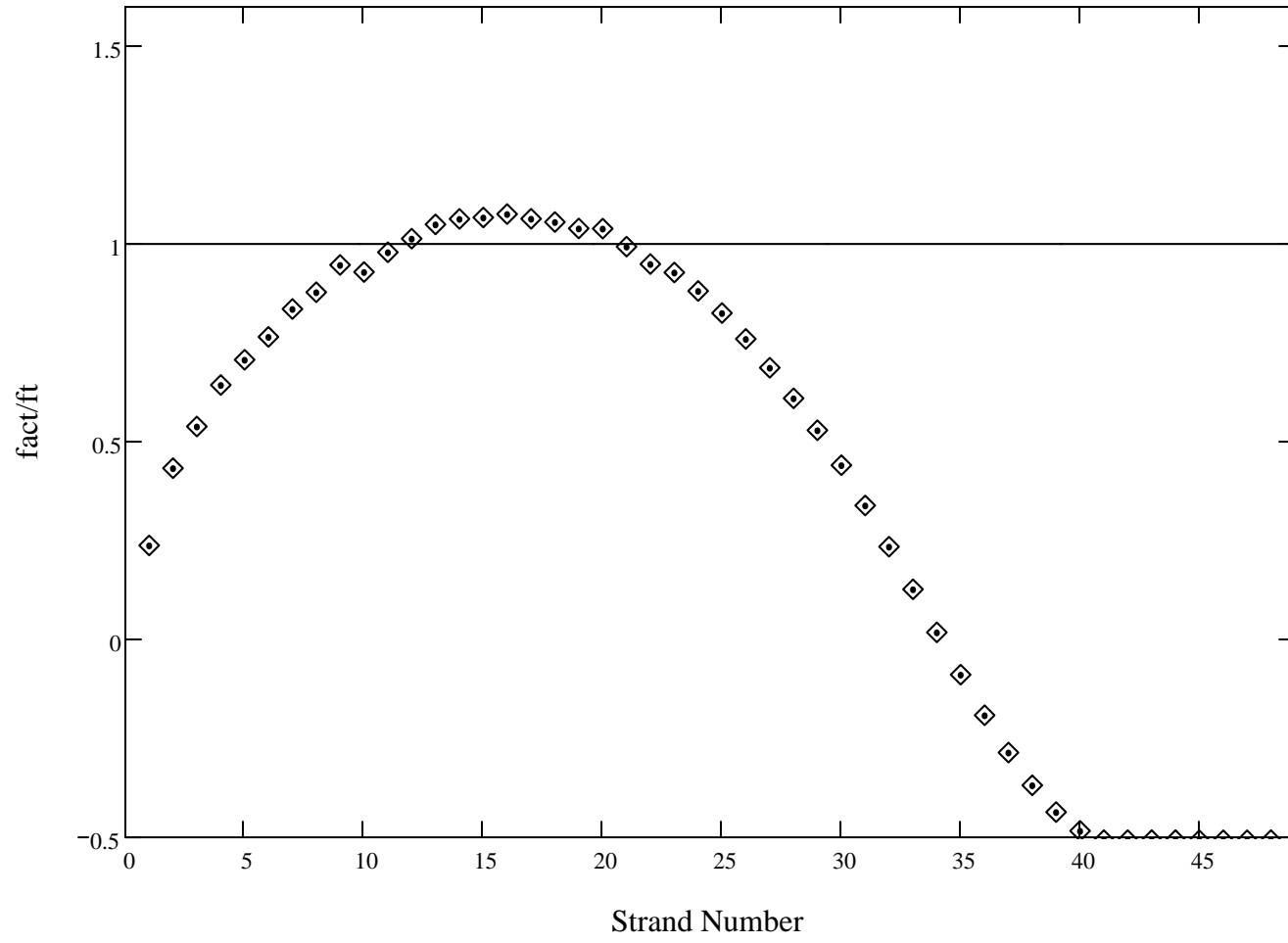
CP Beam 1 End 1



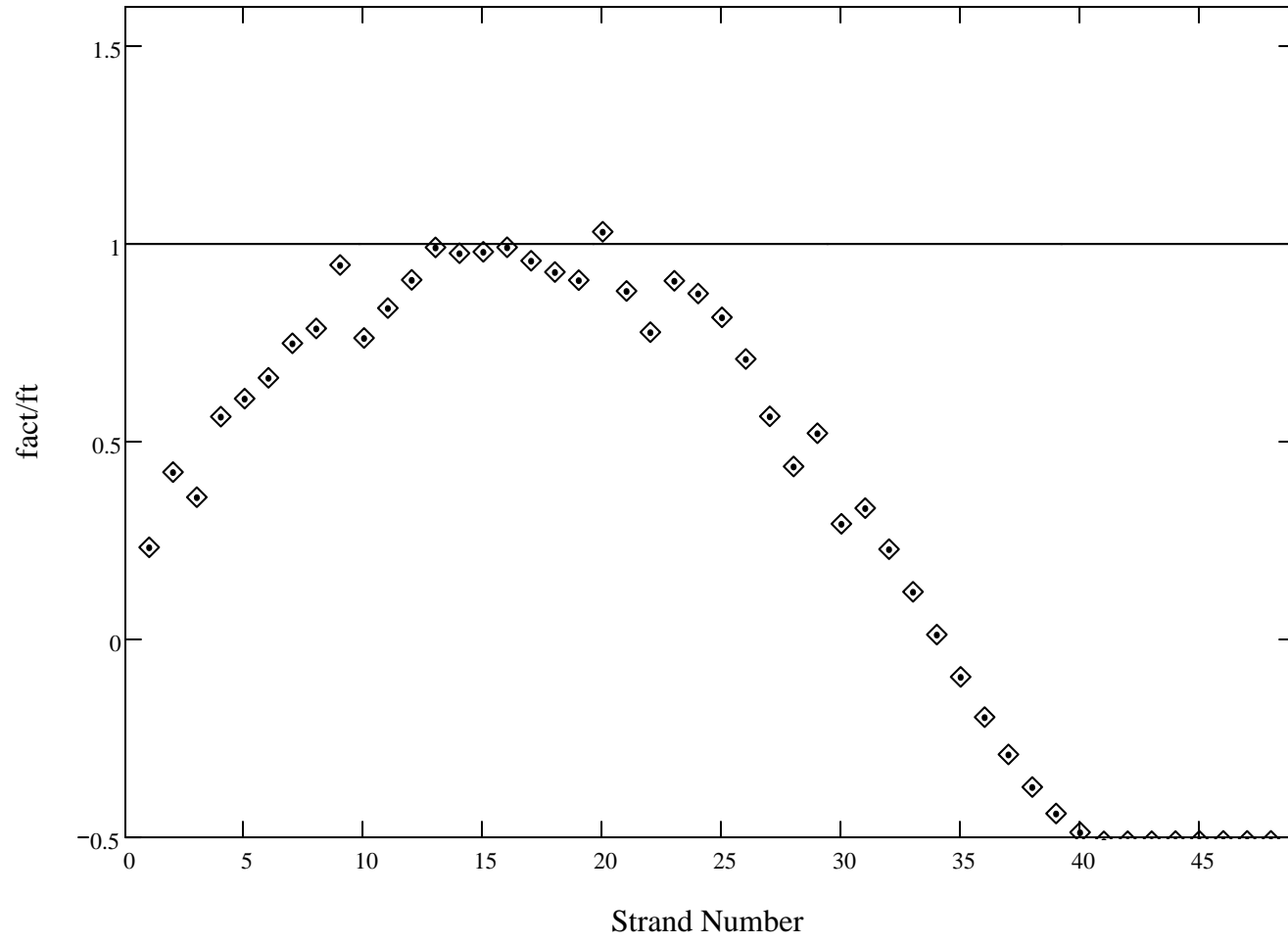
CP Beam 1 End 2



CP Beam 2 End 1

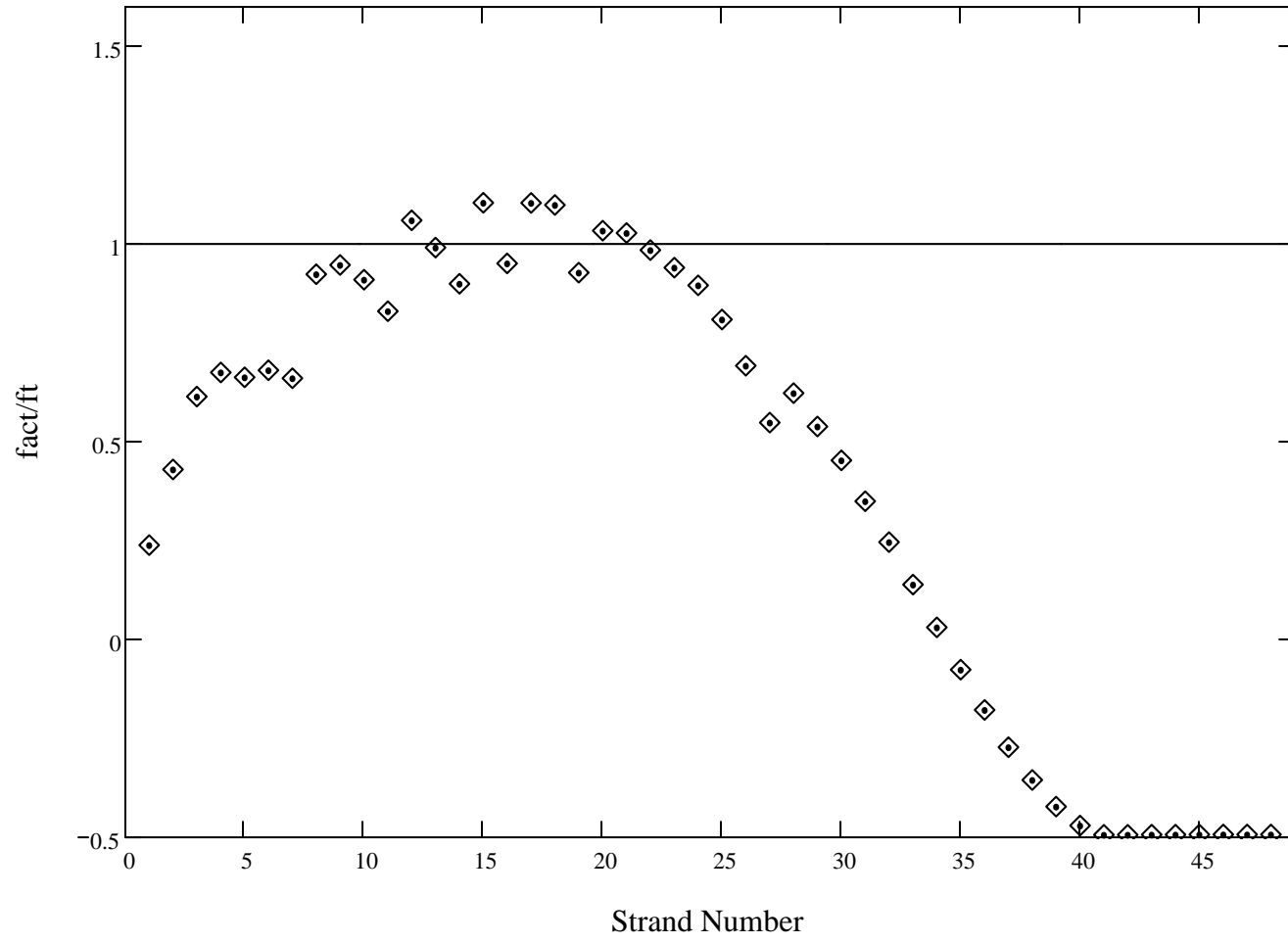


CP Beam 2 End 2

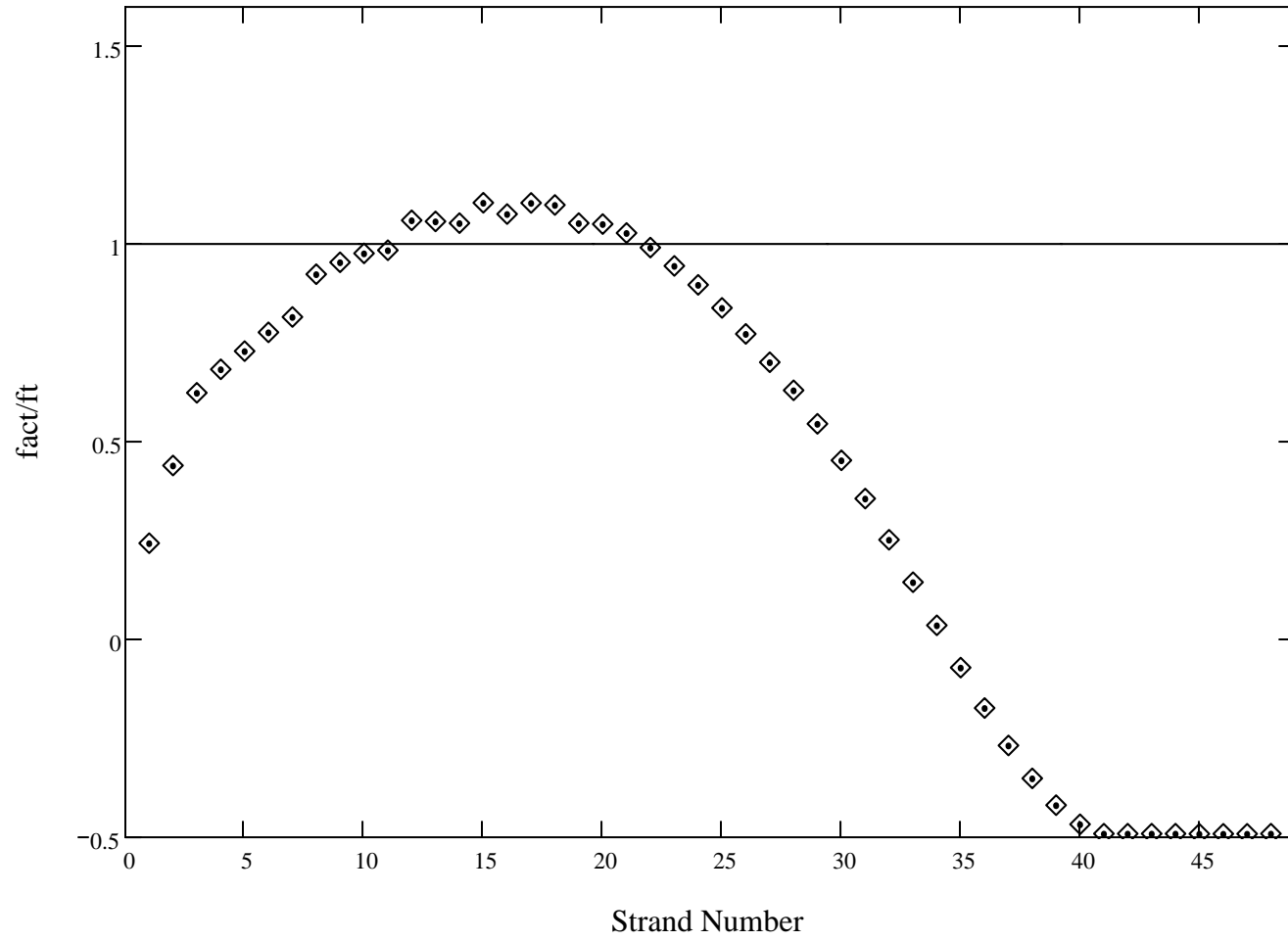




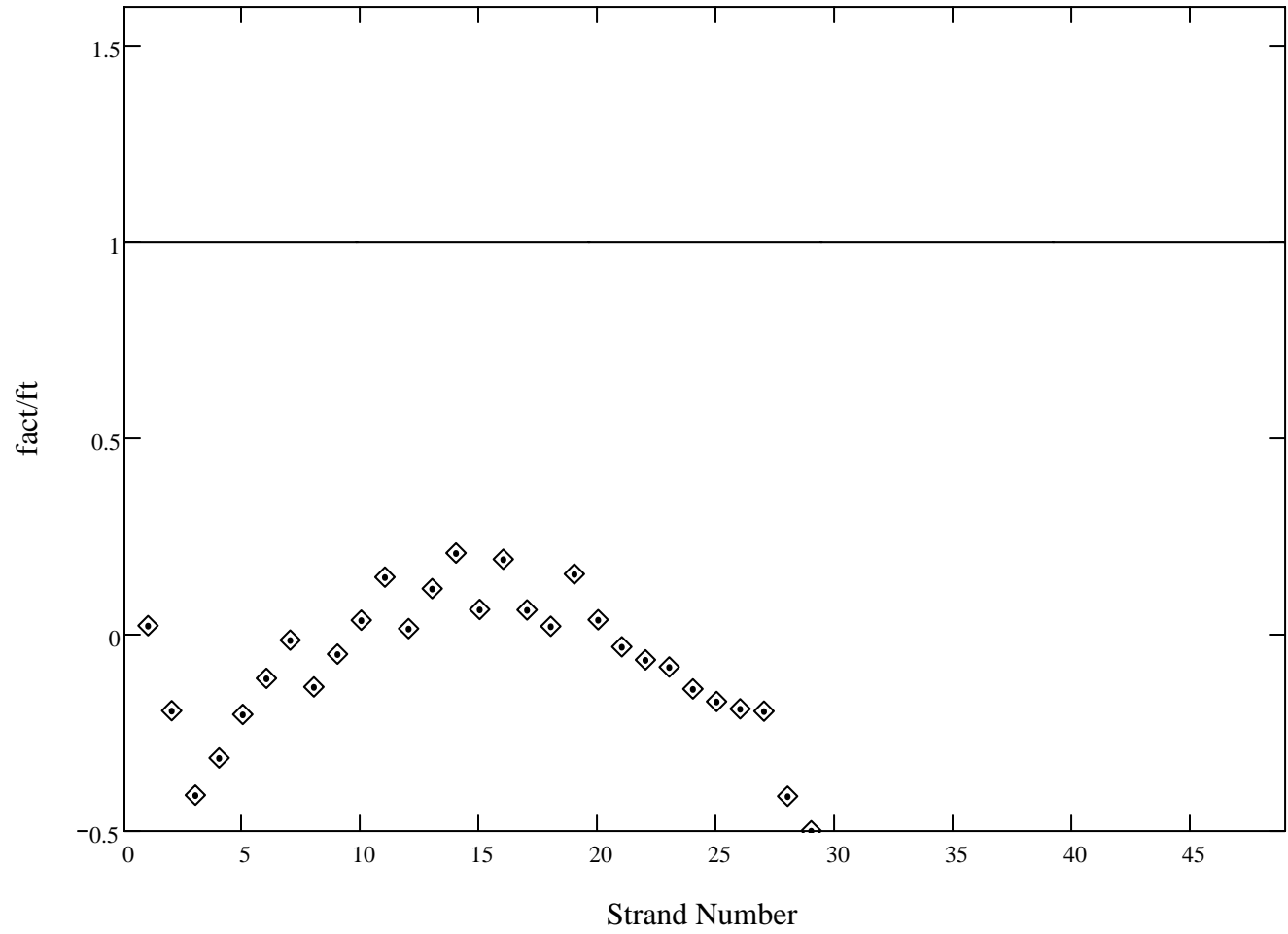
CP Beam 3 End 1



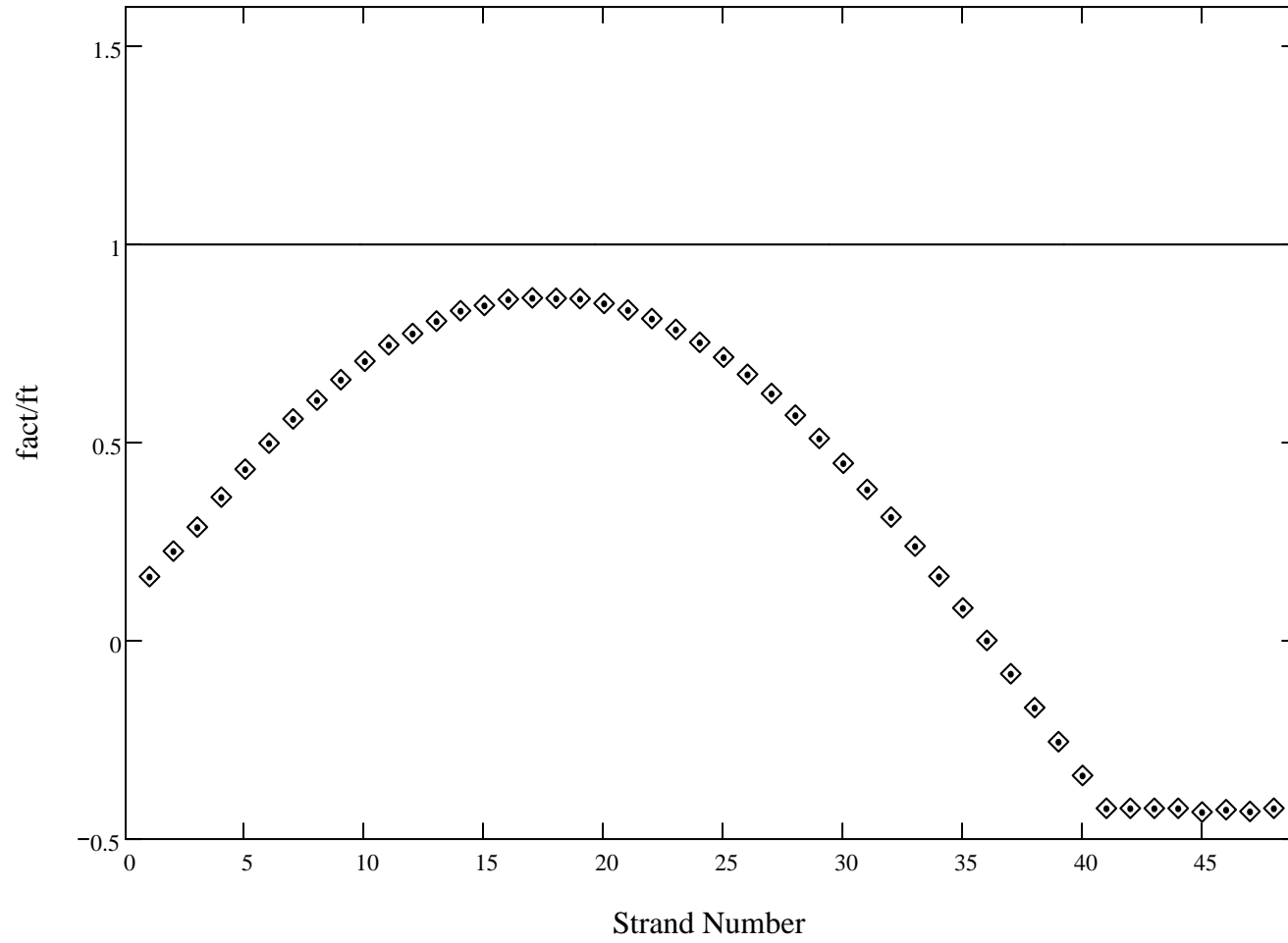
### CP Beam 3 End 2



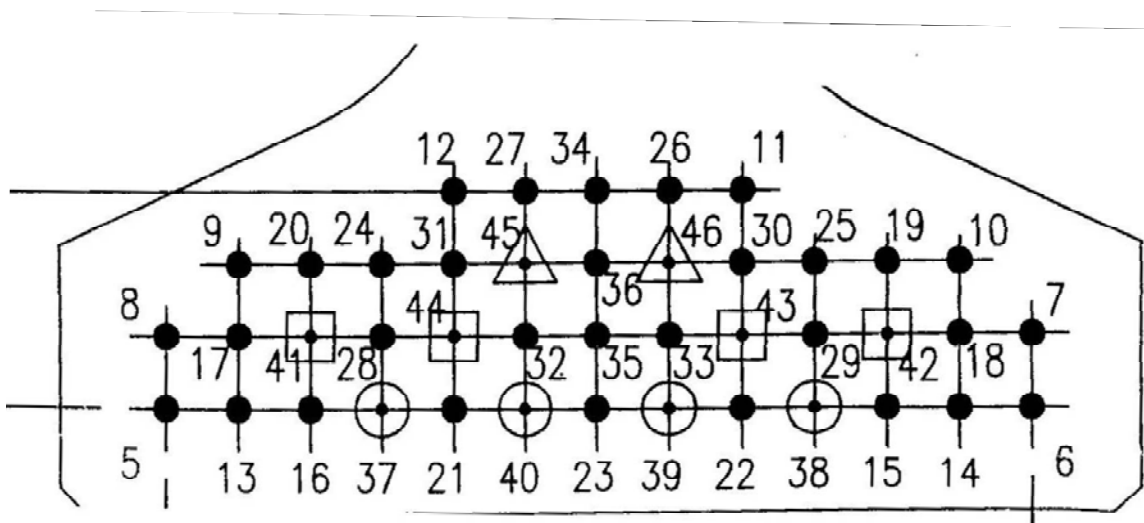
CP Beam 4 End 1



CP Beam 4 End 2



APPENDIX C  
FIELD STUDY STRAND LAYOUT



Debonded Strands:

Triangle = 15'

Square = 10'

Circle = 5'